1 _ Prove $C(0)$. After line 3 executed 0 times

: ① $a = f(0)$
② $b = f(1)$

Answer: by initialization of a and b on line #1

and using the fact that our program is just only on line #2. Then, a and b has not changed yet $\Rightarrow$ $a = 0 = f(0)$
$b = 1 = f(1)$

Prove $C(n+1)$, assuming $C(n)$.

Answer: Induction!

We are about to execute line #3 for the $n+1$-th time. Since we assumed $C(n)$, we know
① $a = f(n)$ ② $b = f(n+1)$

Then, after we execute #3 again, we have

$$a = f(n+1) \quad \text{AND} \quad b = f(n) + f(n+1) = f(n+2)$$

$\square$

2 - Fill the table.

j	0	1	2	3	4	5	6	7	8	9	
S	0	4	8	5	11	2	9	6	3	12	
L	1	2	3	3	4	2	4	4	3	5	

j	0	1	2	3	4	5	6	7	8	9	
S	8	5	11	2	9	6	3	12	0	4	
L	1	1	2	1	2	2	2	3	1	3	

j	0	1	2	3	4	5	6	7	8	9	
S	12	0	4	8	5	11	2	9	6	3	
L	1	1	2	3	3	4	2	4	4	3	

3 _ If (S_1, S_2, S_5, S_7) is an increasing subseq.

j	0	1	2	3	4	5	6	7
$Z(j)$	1	1	2	3, 2, 1	4, 3, 2, 1	3	4, 3, 2, 1	4

$Z(0) \rightarrow$ only 1. since the LIS ending at S_0 is only S_0.

$Z(1) \rightarrow 1$ we know $S_0 > S_1$ o.w $S_0\ S_1\ S_2\ S_5\ S_7$ would be increasing and $f(7) \geq 5$

If (S_1, S_3, S_4, S_7) is an increasing subseq.

j	0	1	2	3	4	5	6	7
$Z(j)$	1	1	1, 2	2	3	1, 2, 3, 4	1, 2, 3, 4	4

$Z(0) \rightarrow$ only 1. since the LIS ending at S_0 is only S_0.

$Z(1) \rightarrow 1$ we know $S_0 > S_1$ o.w $S_0\ S_1\ S_3\ S_4\ S_7$ would be increasing and $f(7) \geq 5$

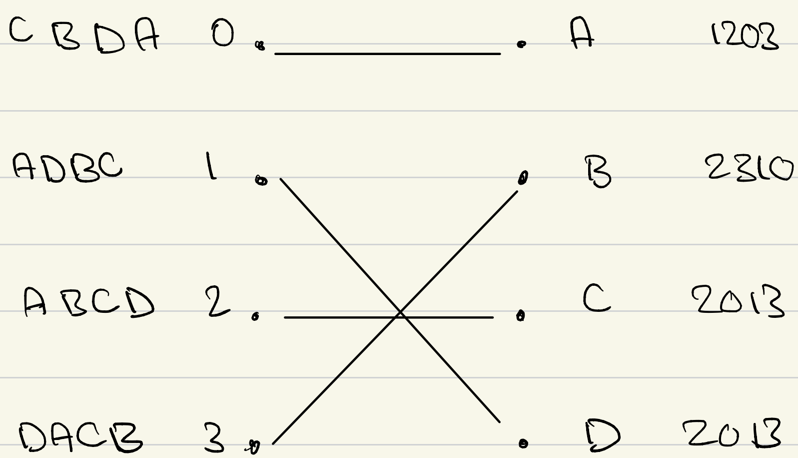
If $S_1 S_4 S_5 S_7$ is an increasing subseqn

j	0	1	2	3	4	5	6	7
$Z(j)$	1	1	1, 2	1, 2, 3	2	3	1, 2, 3, 4	4

$Z(0) \to$ only 1. since the LIS ending at S_0 is only S_0.

$Z(1) \to 1$ we know $S_0 > S_1$ o.w $S_0 S_1 S_4 S_5 S_7$ would be increasing and $f(7) \geq 5$

4 -



a) is $\{3, A\}$ unhappy?

3 prefers A to B

A does not prefer 3 to 0

$\Rightarrow$ not an unhappy couple

b) is $\{1, D\}$ unhappy?

They are matched already.

So by def, they can't be unhappy.

c) is $\{2, A\}$ unhappy?

2 prefers A to C

A prefers 2 to 0

$\Rightarrow$ they are unhappy

d) is $\{2, B\}$ unhappy?

2 prefers B to C

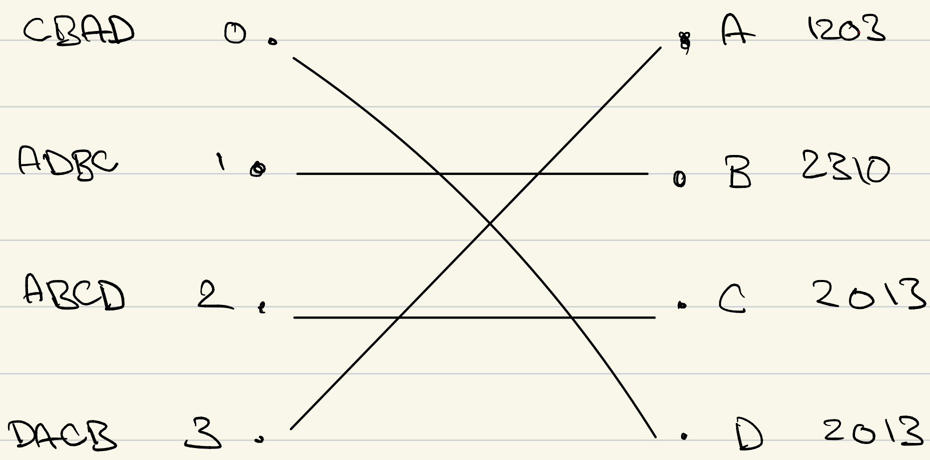
B prefers 2 to 3

$\Rightarrow$ they are unhappy

e) is $\{0, A\}$ unhappy?

They are matched already.

So by def, they can't be unhappy.



a) is {0,A} unhappy?

A prefers 0 to 3
0 prefers A to D
⇒ unhappy

b) is {2,C} unhappy?

They are matched already.
So by def, they can't be unhappy.

c) is {3,D} unhappy?

3 prefers D to A
D does not prefer 3 to 0
⇒ not unhappy

5) is $u=0, v=1, w=1, x=1$ valid? NO!

Since $w=1$ $x=1$ is not a preference list.

is $u=0$ $v=1$ $w=0$ $x=1$ valid? yes

unstable? yes

we have {0, A} as unhappy!

is $u=0$ $v=1$ $w=1$ $x=0$ valid? yes

unstable? yes

{0, A} → unhappy

is $u=0$ $v=0$ $w=1$ $x=1$ valid? NO!

this is not a preference list.

is $u=1$ $v=0$ $w=1$ $x=0$ valid? yes

unstable? yes

{1, B} is unhappy.

is $u=1$ $v=1$ $w=1$ $x=0$ valid? NO