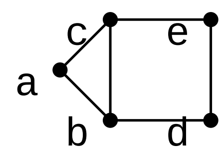
first name **last name** **id#**

each page 8 marks **30 min** **closed book** **no devices** **3 pages**

1. Below is an integer program (IP) for finding a maximum indendent set in this graph. Also below is the dual of the IP.




```
primal                                    dual
max x1 + x2 + x3 + x4 + x5 s.t.    min y1 + y2 + y3 + y4 + y5 + y6 s.t.
    x1 + x2                 <= 1       y1 + y2                     >= 1
    x1      + x3            <= 1       y1      + y3 + y4           >= 1
         x2 + x3            <= 1            y2 + y3      + y5      >= 1
         x2      + x4       <= 1                      y4      + y6 >= 1
              x3      + x5  <= 1                           y5 + y6 >= 1
                   x4 + x5  <= 1
x1, x2, x3, x4, x5 in {0, 1}           y1, y2, y3, y4, y5, y6 >= 0
```

2. What does primal variable x_3 represent?

   What does dual variable y_3 represent?

   Justify/explain the primal objective function:

   Justify/explain the dual objective function:

   Justify/explain this primal constraint: $x_1 + x_2 \leq 1$.

   Justify/explain this dual constraint: $y_1 + y_3 + y_4 \geq 1$.

3. Is $x = (1, 0, 0, 0, 0)$ primal optimal? If yes, write it below: if no, find a primal optimal solution.

   Is $y = (1, 1, 1, 1, 1, 1)$ dual optimal? If yes, write it below: if no, find a dual optimal solution.

   Your primal optimal solution:

   Your dual optimal solution:

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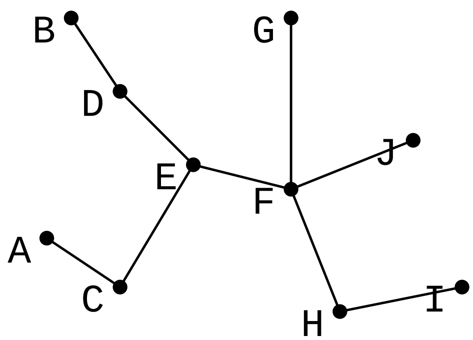
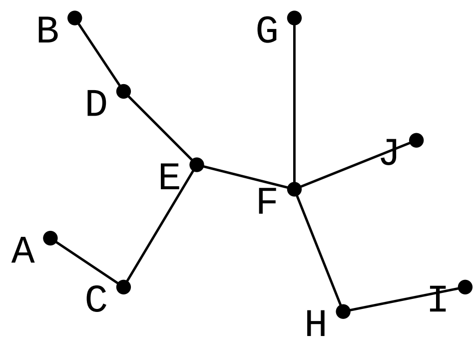
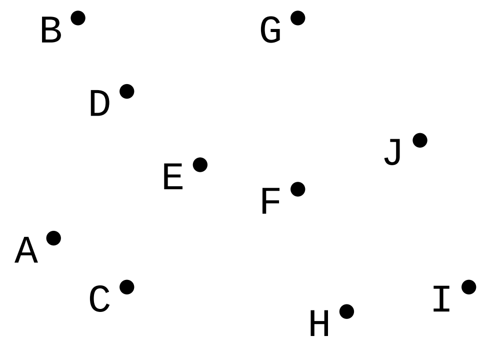
4. For the complete graph with these edge weights, give the Hamiltonian cycle found by the 1.5-approx method. When shortcutting your Eulerian tour, start at node E and whenever you have a choice, pick the alphabetically-first choice. Use below left for your rough work. Show your matching-plus-MST below middle. Show your cycle below right.

	B	C	D	E	F	G	H	I	J
A	5	2	4	4	6	7	7	9	9
B		6	2	4	6	5	9	11	8
C			4	3	4	7	5	8	7
D				2	4	4	7	9	7
E					2	4	5	7	5
F						4	3	4	3
G							7	7	4
H								3	4
I									4







5. Let G be a weighted complete graph with at least 4 nodes and positive edge weights. Let v be a node in G and let H be the graph obtained by removing v and all edges incident with v. Let h be the weight of a min-weight Hamiltonian cycle in H. Let g be the weight of a min-weight Hamiltonian cycle in G. Prove/disprove: $h \leq g$.

6. On this instance, find the greedy set cover: in each step, if there are ties then pick the set with smaller index, e.g. if there is a tie between picking S5 and S9, pick S5. Also, find a minimum size set cover. Write each cover like this: {S0, S1, S2, S4}.

   a) **your greedy set cover:**

   b) **your min-size set cover:**

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S0	-	-	*	*	-	-	*	-	-	-	*	-	-	-	*	-
S1	-	-	*	*	-	-	*	-	*	-	*	*	-	*	*	-
S2	-	-	-	*	-	-	-	-	-	-	*	-	-	-	*	-
S3	-	-	*	-	*	-	-	*	-	-	*	-	-	-	*	*
S4	-	-	-	-	*	*	-	*	*	*	-	-	-	-	-	-
S5	-	-	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S6	-	*	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S7	*	-	-	*	-	*	-	*	*	-	-	-	-	*	-	*
S8	-	*	*	-	*	-	*	-	-	*	-	*	*	-	*	-
S9	*	-	-	*	-	*	-	*	*	-	*	-	-	*	-	*

Rough work here

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S0	-	-	*	*	-	-	*	-	-	-	*	-	-	-	*	-
S1	-	-	*	*	-	-	*	-	*	-	*	*	-	*	*	-
S2	-	-	-	*	-	-	-	-	-	-	*	-	-	-	*	-
S3	-	-	*	-	*	-	-	*	-	-	*	-	-	-	*	*
S4	-	-	-	-	*	*	-	*	*	*	-	-	-	-	-	-
S5	-	-	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S6	-	*	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S7	*	-	-	*	-	*	-	*	*	-	-	-	-	*	-	*
S8	-	*	*	-	*	-	*	-	-	*	-	*	*	-	*	-
S9	*	-	-	*	-	*	-	*	*	-	*	-	-	*	-	*

7. Consider the greedy set cover algorithm on an instance with 50 elements and minimum cover size 9. Let n_t be the number of elements not yet in the cover after t iterations of the algorithm.

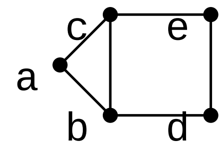
   a) Give the best upper bound you can for n_1. Justify briefly.

   a) Give the best upper bound you can for n_2. Justify briefly.

first name **last name** **id#**

each page 8 marks **30 min** **closed book** **no devices** **3 pages** **page 1**

1. Below is an integer program (IP) for finding a maximum indendent set in this graph. Also below is the dual of the IP.




```
primal
max x1 + x2 + x3 + x4 + x5 s.t.
    x1 + x2                <= 1
    x1      + x3           <= 1
         x2 + x3           <= 1
         x2      + x4      <= 1
              x3      + x5 <= 1
                   x4 + x5 <= 1
x1, x2, x3, x4, x5 in {0, 1}
```

```
dual
min y1 + y2 + y3 + y4 + y5 + y6 s.t.
    y1 + y2                     >= 1
    y1      + y3 + y4           >= 1
         y2 + y3      + y5      >= 1
                   y4      + y6 >= 1
                        y5 + y6 >= 1
    y1, y2, y3, y4, y5, y6 >= 0
```

2. What does primal variable x_3 represent?

   What does dual variable y_3 represent?

   Justify/explain the primal objective function:

   Justify/explain the dual objective function:

   Justify/explain this primal constraint: $x_1 + x_2 \leq 1$.

   Justify/explain this dual constraint: $y_1 + y_3 + y_4 \geq 1$.

3. Is $x = (1, 0, 0, 0, 0)$ primal optimal? If yes, write it below: if no, find a primal optimal solution.

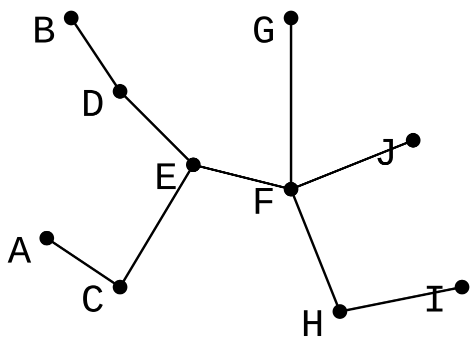
   Is $y = (1, 1, 1, 1, 1, 1)$ dual optimal? If yes, write it below: if no, find a dual optimal solution.

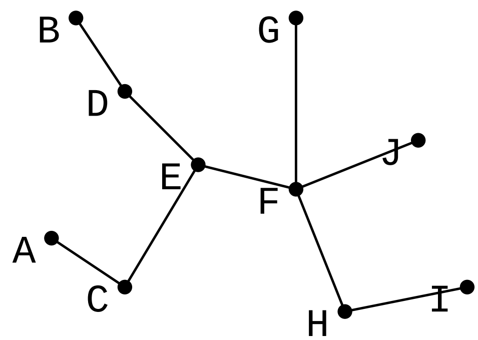
   Your primal optimal solution:

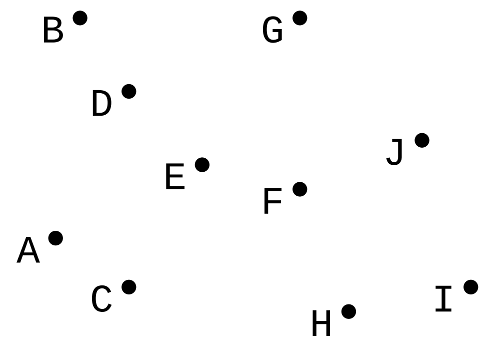
   Your dual optimal solution:

4. For the complete graph with these edge weights, give the Hamiltonian cycle found by the 1.5-approx method. When shortcutting your Eulerian tour, start at node A and whenever you have a choice, pick the alphabetically-first choice. Use below left for your rough work. Show your matching-plus-MST below middle. Show your cycle below right.

	B	C	D	E	F	G	H	I	J
A	5	2	4	4	6	7	7	9	9
B		6	2	4	6	5	9	11	8
C			4	3	4	7	5	8	7
D				2	4	4	7	9	7
E					2	4	5	7	5
F						4	3	4	3
G							7	7	4
H								3	4
I									4







5. Let G be a weighted complete graph with at least 4 nodes and positive edge weights. Let v be a node in G and let H be the graph obtained by removing v and all edges incident with v. Let h be the weight of a min-weight Hamiltonian cycle in H. Let g be the weight of a min-weight Hamiltonian cycle in G. Prove/disprove: $h \leq g$.

6. On this instance, find the greedy set cover: in each step, if there are ties then pick the set with smaller index, e.g. if there is a tie between picking S5 and S9, pick S5. Also, find a minimum size set cover. Write each cover like this: {S0, S1, S2, S4}.

   a) **your greedy set cover:**

   b) **your min-size set cover:**

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S0	-	-	*	*	-	-	*	-	-	-	*	-	-	-	*	-
S1	-	-	*	*	-	-	*	-	*	-	*	*	-	*	*	-
S2	-	-	*	-	*	-	-	*	-	-	*	-	-	-	*	*
S3	-	-	-	-	*	*	-	*	*	*	-	-	-	-	-	-
S4	-	-	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S5	-	*	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S6	*	-	-	*	-	*	-	*	*	-	-	-	-	*	-	*
S7	-	*	*	-	*	-	*	-	-	*	-	*	*	-	*	-
S8	-	-	-	*	-	-	-	-	-	-	*	-	-	-	*	-
S9	*	-	-	*	-	*	-	*	*	-	*	-	-	*	-	*

Rough work here

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S0	-	-	*	*	-	-	*	-	-	-	*	-	-	-	*	-
S1	-	-	*	*	-	-	*	-	*	-	*	*	-	*	*	-
S2	-	-	*	-	*	-	-	*	-	-	*	-	-	-	*	*
S3	-	-	-	-	*	*	-	*	*	*	-	-	-	-	-	-
S4	-	-	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S5	-	*	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S6	*	-	-	*	-	*	-	*	*	-	-	-	-	*	-	*
S7	-	*	*	-	*	-	*	-	-	*	-	*	*	-	*	-
S8	-	-	-	*	-	-	-	-	-	-	*	-	-	-	*	-
S9	*	-	-	*	-	*	-	*	*	-	*	-	-	*	-	*

7. Consider the greedy set cover algorithm on an instance with 50 elements and minimum cover size 8. Let n_t be the number of elements not yet in the cover after t iterations of the algorithm.

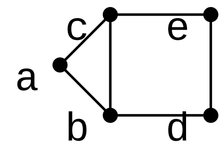
   a) Give the best upper bound you can for n_1. Justify briefly.

   a) Give the best upper bound you can for n_2. Justify briefly.

first name **last name** **id#**

each page 8 marks **30 min** **closed book** **no devices** **3 pages**

1. Below is an integer program (IP) for finding a maximum indendent set in this graph. Also below is the dual of the IP.




```
primal                                  dual
max x1 + x2 + x3 + x4 + x5 s.t.         min y1 + y2 + y3 + y4 + y5 + y6 s.t.
    x1 + x2                <= 1             y1 + y2                     >= 1
    x1      + x3           <= 1             y1      + y3 + y4           >= 1
         x2 + x3           <= 1                  y2 + y3      + y5      >= 1
         x2      + x4      <= 1                            y4      + y6 >= 1
              x3      + x5 <= 1                                 y5 + y6 >= 1
                   x4 + x5 <= 1
x1, x2, x3, x4, x5 in {0, 1}                y1, y2, y3, y4, y5, y6 >= 0
```

2. What does primal variable x_3 represent?

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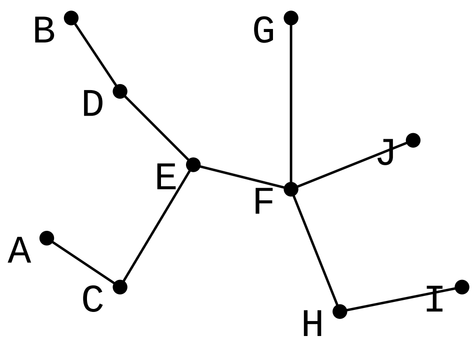
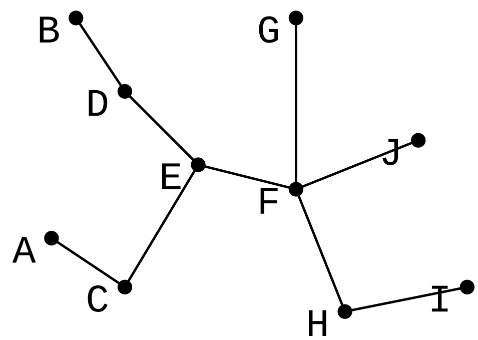
   Your primal optimal solution:

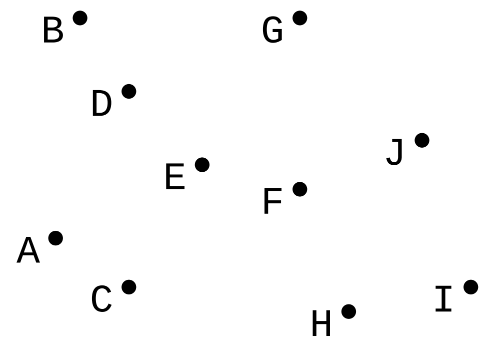
   Your dual optimal solution:

4. For the complete graph with these edge weights, give the Hamiltonian cycle found by the 1.5-approx method. When shortcutting your Eulerian tour, start at node C and whenever you have a choice, pick the alphabetically-first choice. Use below left for your rough work. Show your matching-plus-MST below middle. Show your cycle below right.

	B	C	D	E	F	G	H	I	J
A	5	2	4	4	6	7	7	9	9
B		6	2	4	6	5	9	11	8
C			4	3	4	7	5	8	7
D				2	4	4	7	9	7
E					2	4	5	7	5
F						4	3	4	3
G							7	7	4
H								3	4
I									4







5. Let G be a weighted complete graph with at least 4 nodes and positive edge weights. Let v be a node in G and let H be the graph obtained by removing v and all edges incident with v. Let h be the weight of a min-weight Hamiltonian cycle in H. Let g be the weight of a min-weight Hamiltonian cycle in G. Prove/disprove: $h \leq g$.

6. On this instance, find the greedy set cover: in each step, if there are ties then pick the set with smaller index, e.g. if there is a tie between picking S5 and S9, pick S5. Also, find a minimum size set cover. Write each cover like this: {S0, S1, S2, S4}.

   a) **your greedy set cover:**

   b) **your min-size set cover:**

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S0	-	-	*	*	-	-	*	-	-	-	*	-	-	-	*	-
S1	-	-	*	*	-	-	*	-	*	-	*	*	-	*	*	-
S2	-	-	*	-	*	-	-	*	-	-	*	-	-	-	*	*
S3	-	-	-	-	*	*	-	*	*	*	-	-	-	-	-	-
S4	-	-	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S5	-	*	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S6	-	-	-	*	-	-	-	-	-	-	*	-	-	-	*	-
S7	*	-	-	*	-	*	-	*	*	-	-	-	-	*	-	*
S8	-	*	*	-	*	-	*	-	-	*	-	*	*	-	*	-
S9	*	-	-	*	-	*	-	*	*	-	*	-	-	*	-	*

Rough work here

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S0	-	-	*	*	-	-	*	-	-	-	*	-	-	-	*	-
S1	-	-	*	*	-	-	*	-	*	-	*	*	-	*	*	-
S2	-	-	*	-	*	-	-	*	-	-	*	-	-	-	*	*
S3	-	-	-	-	*	*	-	*	*	*	-	-	-	-	-	-
S4	-	-	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S5	-	*	*	-	*	-	*	-	-	*	-	*	*	-	-	-
S6	-	-	-	*	-	-	-	-	-	-	*	-	-	-	*	-
S7	*	-	-	*	-	*	-	*	*	-	-	-	-	*	-	*
S8	-	*	*	-	*	-	*	-	-	*	-	*	*	-	*	-
S9	*	-	-	*	-	*	-	*	*	-	*	-	-	*	-	*

7. Consider the greedy set cover algorithm on an instance with 50 elements and minimum cover size 7. Let n_t be the number of elements not yet in the cover after t iterations of the algorithm.

   a) Give the best upper bound you can for n_1. Justify briefly.

   a) Give the best upper bound you can for n_2. Justify briefly.