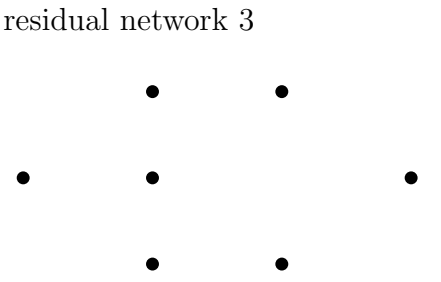
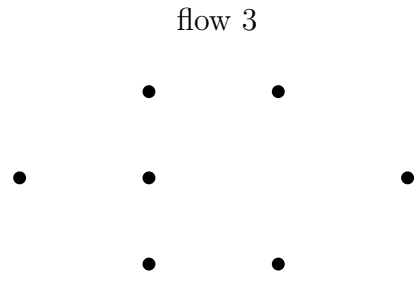
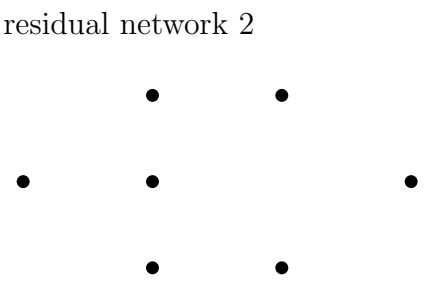
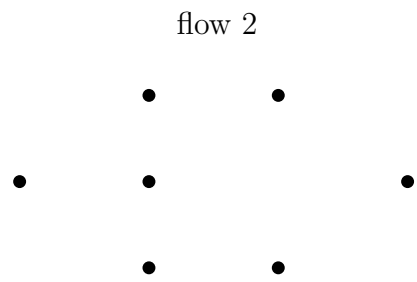
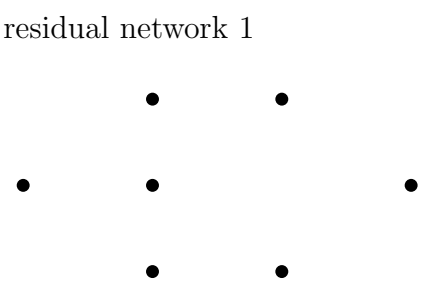
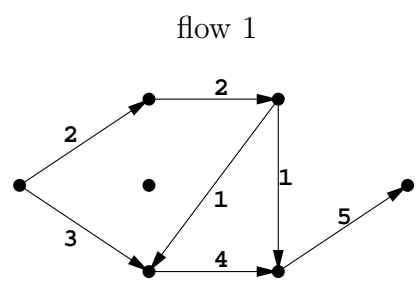
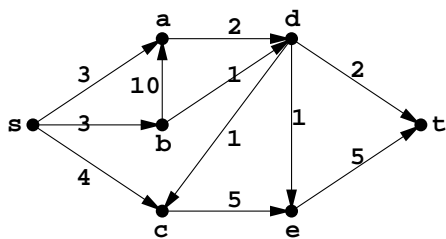


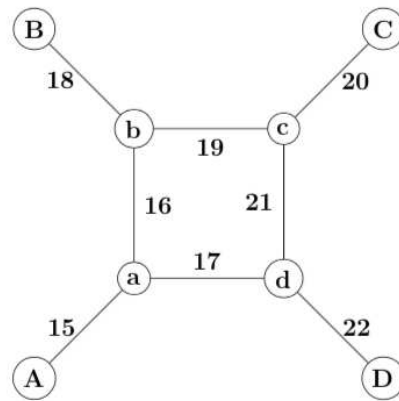
1. For this network, starting with flow 1, use the residual flow method to find a max flow: show residual networks 1,2,3 and flows 2 and 3.



2. a) For the above network, give a cut whose capacity is equal to that of the flow that you found in the previous question. Answer like this: $S = \{s, a\}$ $T = \{b, c, d, e, t\}$

b) Prove that your cut is a min cut.

3. You manage a communications network with users B,C,D only (A is no longer involved) and bandwidths as shown in the diagram. You need to establish connections between B-C, B-D, and C-D: these pay you \$3, \$5, \$4 respectively per unit bandwidth. Between each pair of users at least 6 units must be routed.



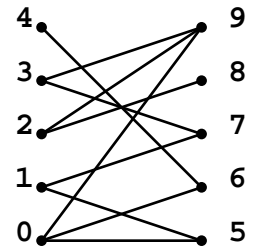
Each connection has two possible routes. For B-C: x_{BC} is traffic volume along B-b-c-C, y_{BC} is volume along B-b-a-d-c-C. For C-D: define x_{CD} , y_{CD} , similarly. For B-D: x_{BD} , y_{BD} is traffic along B-b-c-d-D, B-b-a-d-D respectively. You want to maximize the amount you are paid. Using these variables, formulate this problem as an LP:

a) Give the objective function.

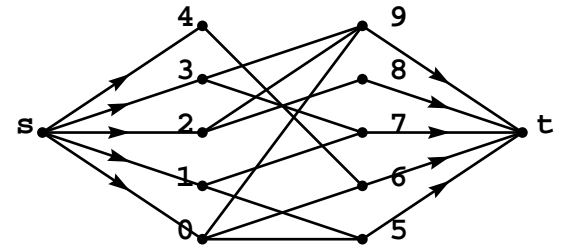
b) Give the system of (in)equalities.

c) Give a feasible solution.

5. a) Give a maximum matching for this bipartite graph G . For example, $\{ (0,5) \}$ is a matching with size 1.

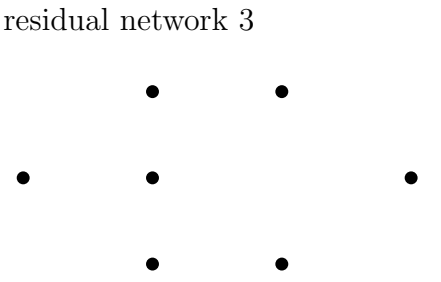
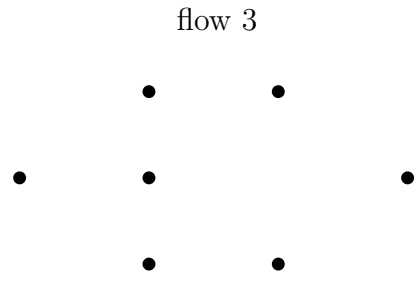
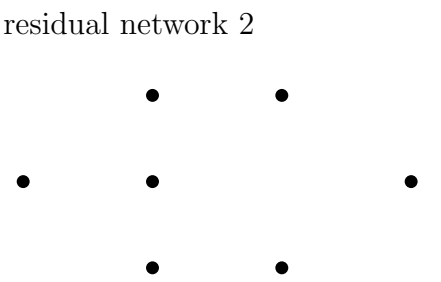
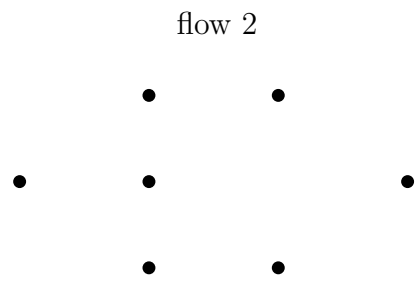
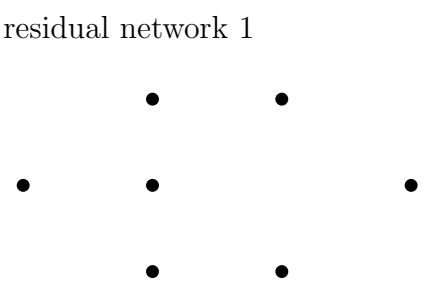
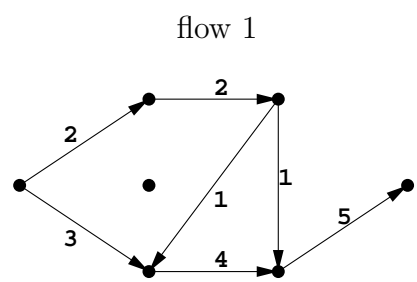
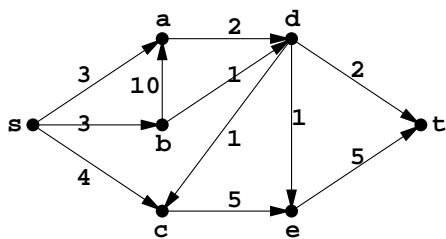


- b) Here is an s - t flow network H . Each arc has capacity 1. Arrows on middle arcs have been omitted, they are all from left to right. Give a min cut for H . For example, $\{ s, 1 \}$ is a cut with capacity 2.



- c) Prove that your matching in a) is maximum.

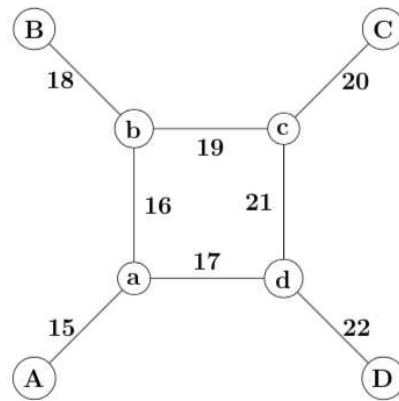
1. For this network, starting with flow 1, use the residual flow method to find a max flow: show residual networks 1,2,3 and flows 2 and 3.



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3. You manage a communications network with users B,C,D only (A is no longer involved) and bandwidths as shown in the diagram. You need to establish connections between B-C, B-D, and C-D: these pay you \$4, \$5, \$3 respectively per unit bandwidth. Between each pair of users at least 7 units must be routed.



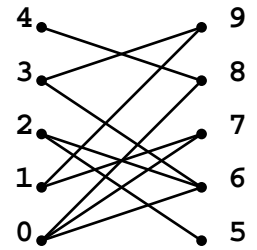
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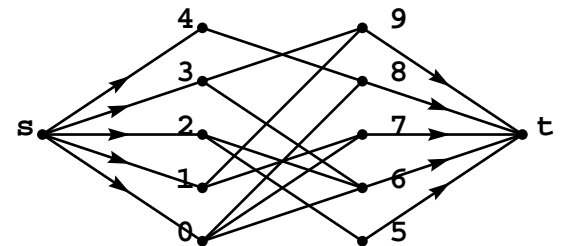
b) Give the system of (in)equalities.

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5. a) Give a maximum matching for this bipartite graph G . For example, $\{ (0,6) \}$ is a matching with size 1.

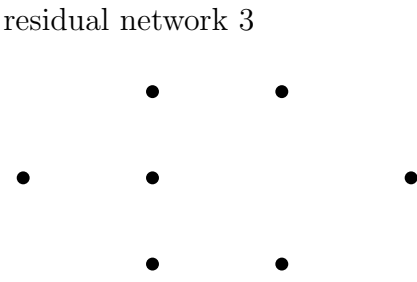
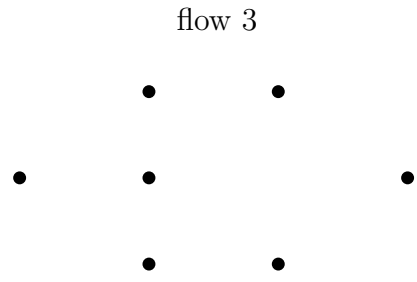
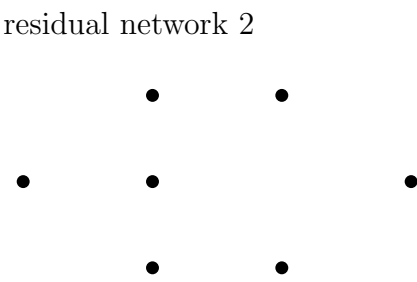
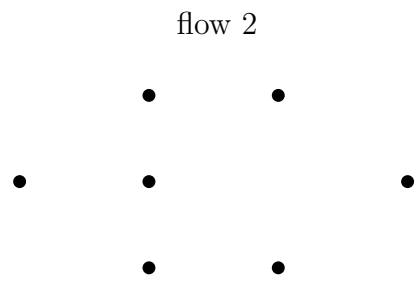
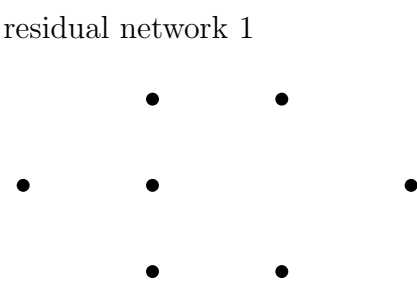
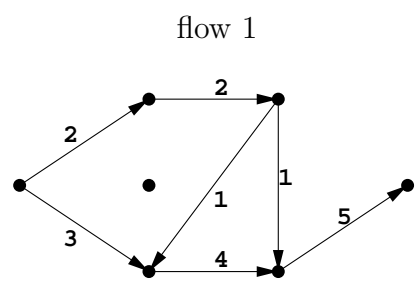
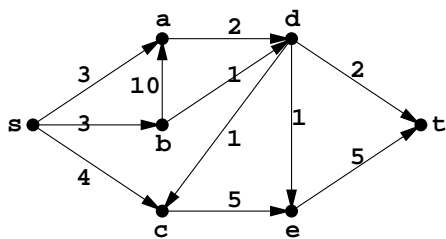


- b) Here is an s - t flow network H . Each arc has capacity 1. Arrows on middle arcs have been omitted, they are all from left to right. Give a min cut for H . For example, $\{ s, 1 \}$ is a cut with capacity 2.



- c) Prove that your matching in a) is maximum.

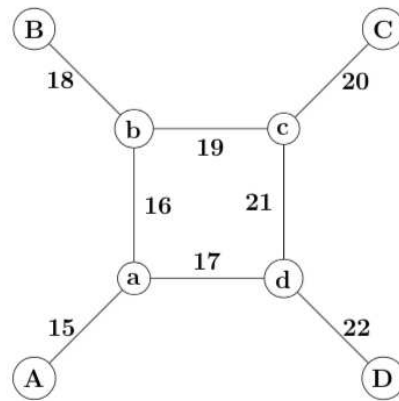
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3. You manage a communications network with users B,C,D only (A is no longer involved) and bandwidths as shown in the diagram. You need to establish connections between B-C, B-D, and C-D: these pay you \$5, \$3, \$4 respectively per unit bandwidth. Between each pair of users at least 5 units must be routed.



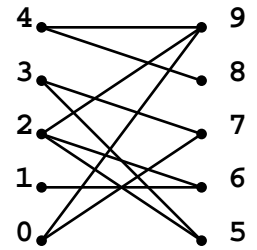
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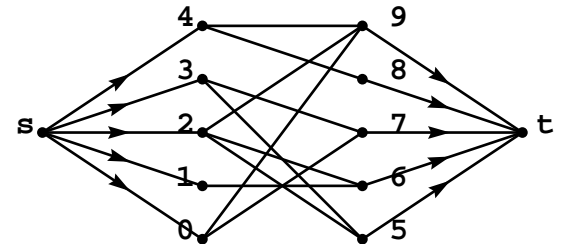
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- b) Here is an s - t flow network H . Each arc has capacity 1. Arrows on middle arcs have been omitted, they are all from left to right. Give a min cut for H . For example, $\{s, 0\}$ is a cut with capacity 2.



- c) Prove that your matching in a) is maximum.