

1. `node_subset_is_clique` (NSIC) is this problem:

instance: a graph  $G = (V, E)$  and a subset  $K$  of  $V$ .

query: is  $K$  a clique of  $G$ ?

Give a polytime algorithm for NSIC.

2. Recall that  $k$ -clique is this problem:

instance: a graph  $G = (V, E)$  and an integer  $k$ .

query: does  $G$  have a clique of size  $k$ ?

Prove/disprove:  $k$ -clique is in the class NP. In your answer, use NSIC.

1. Let  $t$  be the transformation we saw in the lectures from cnf-sat to 3-sat. For the cnf-sat formula  $f$  represented below, give the corresponding 3-sat formula  $t(f)$ . In this question, write boolean clauses like this  $[-1 \ 2 \ 4]$  instead of like this  $(\neg x_1 \vee x_2 \vee x_4)$ .

clauses of  $f$

corresponding clauses of  $t(f)$

$[-1 \ 3 \ 5 \ 6]$

$[2 \ 3 \ -4 \ -5 \ -6]$

$[1 \ -2 \ 3 \ -4 \ 5 \ 6]$

----- ROUGH WORK BELOW THIS LINE -----

2. Does there exist a polytime answer-preserving transformation from  $k$ -independent set to sat (satisfiability)? Write your answer here (yes/no/not known):

Justify your answer here:

5. Recall from the lectures the polytime answer-preserving transformation  $t()$  that maps any 3-sat instance  $z$  with  $k$  clauses to a  $k$ -independent set instance  $t(z)$  where  $t(z)$  is a graph.

a) On the nodes below, draw  $t(z)$ . Label each node with its corresponding literal.

$$z = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$$



b) Assume that  $t(z)$  has an independent set  $I$  of size  $k$ . For each statement below, write T (true) or F (false) and justify your answer.

i)  $I$  includes exactly one node from the first clause of  $z$ .

T/F? \_\_\_\_\_ Reason?

ii)  $I$  can contain a node labelled  $\neg x_j$  and a node labelled  $x_j$ .

T/F? \_\_\_\_\_ Reason?

iii) Let  $A$  be the truth assignment that sets each literal that is a node of  $I$  to **true**.  $A$  satisfies  $z$ .

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clauses of  $f$ corresponding clauses of  $t(f)$  $[2 \ 3 \ -5 \ -6]$  $[2 \ 3 \ -4 \ -5 \ -6]$  $[1 \ -2 \ 3 \ -4 \ 5 \ 6]$ 

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