first name

each page 8 marks

last name

closed book

no devices

id#

3 pages

page 1

1. node\_subset\_is\_clique (NSIC) is this problem:

instance: a graph G = (V, E) and a subset K of V.

30 min

query: is K a clique of G?

Give a polytime algorithm for NSIC.

2. Recall that k-clique is this problem:

instance: a graph G = (V, E) and an integer k.

query: does G have a clique of size k?

Prove/disprove: k-clique is in the class NP. In your answer, use NSIC.

first name last name id# each page 8 marks 30 min closed book no devices 3 pages page 2

1. Let t be the transformation we saw in the lectures from cnf-sat to 3-sat. For the cnf-sat formula f represented below, give the corresponding 3-sat formula t(f). In this question, write boolean clauses like this [-1 2 4] instead of like this  $(\neg x_1 \lor x_2 \lor x_4)$ .

clauses of f corresponding clauses of t(f)

[-1 3 5 6]

[2 3 -4 -5 -6]

[1 -2 3 -4 5 6]

----- ROUGH WORK BELOW THIS LINE -----

2. Does there exist a polytime answer-preserving transformation from k-independent set to sat (satisfiability)? Write your answer here (yes/no/not known):

Justify your answer here:

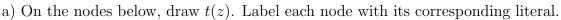
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page 3

5. Recall from the lectures the polytime answer-preserving transformation t() that maps any 3-sat instance z with k clauses to a k-independent set instance t(z) where t(z) is a graph.



$$z = (\neg x_1 \lor x_2 \lor x_3) \land (x_1 \lor \neg x_2 \lor x_3) \land (\neg x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor \neg x_3)$$



b) Assume that t(z) has an independent set I of size k. For each statement below, write T (true) or F (false) and justify your answer.

i) I includes exactly one node from the first clause of z.

T/F? \_\_\_\_\_ Reason?

ii) I can contain a node labelled  $\neg x_j$  and a node labelled  $x_j$ .

T/F? \_\_\_\_\_ Reason?

iii) Let A be the truth assignment that sets each literal that is a node of I to true. A satisfies z.

T/F? \_\_\_\_\_ Reason?

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page 3

- 5. Recall from the lectures the polytime answer-preserving transformation t() that maps any 3-sat instance z with k clauses to a k-independent set instance t(z) where t(z) is a graph.
  - a) On the nodes below, draw t(z). Label each node with its corresponding literal.

$$z = (x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_3)$$



- b) Assume that t(z) has an independent set I of size k. For each statement below, write T (true) or F (false) and justify your answer.
- i) If I contains a node labelled  $\neg x_i$ , then I does not contain a node labelled  $x_i$ .

T/F? \_\_\_\_\_ Reason?

ii) I includes exactly one node from the first clause of z.

T/F? \_\_\_\_\_ Reason?

iii) Let A be the truth assignment that sets each literal that is a node of I to true. A satisfies z.

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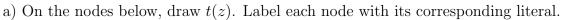
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page 3



$$z = (x_1 \lor x_2 \lor \neg x_3) \land (x_1 \lor \neg x_2 \lor x_3) \land (\neg x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor \neg x_3)$$



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