| each page 8 marks | 30 min | closed book | no devices | 3 pages |
| :--- | :--- | :--- | :--- | :--- | page 1

1. Recall: $f(n)=0,1, f(n-2)+f(n-1)$ when $n$ is def ifib( n$)$ : respectively $0,1, \geq 2$. For each non-negative integer $j, \quad \mathrm{a}, \mathrm{b}=0,1 \quad \# 1$ $C(j)$ is this claim: after line 3 has executed exactly $j$ for _ in range ( n ): \#2 times, $a, b$ equal $f(j), f(j+1)$ respectively.

Prove $C(0)$, i.e. after line 3 has executed exactly 0 times, a,b equal resp. $f(0), f(1)$.

Let $w$ be a non-negative integer, and assume $C(w)$. Prove $C(w+1)$.
first name
each page 8 marks
last name
id\#
30 min
2. Recall: for a sequence $S$, for each index $j, L[j]$ is the length of a longest increasing subsequence (LIS) ending at position $j$. Below, show the values for L .

| $j$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $S$ | 12 | 0 | 4 | 8 | 5 | 11 | 2 | 9 | 6 | 3 |

3. Let S be a sequence $\left(s_{0}, s_{1}, \ldots, s_{7}\right)$. Assume that $f(7)=4$. Assume that $\left(s_{1}, s_{3}, s_{4}, s_{7}\right)$ is an increasing subsequence. For each $j$ below, give the set $Z(j)$ of possible values of $L[j]$ consistent with the above information.
$\begin{array}{lllllllll}j & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\end{array}$

Z(j)


Your justification for your answer to $\mathrm{Z}(0)$.

Your justification for your answer to $\mathrm{Z}(1)$.

You do not have to justify your other answers.
each page 8 marks $\quad 30 \mathrm{~min} \quad$ no devices $\quad 3$ pages booke 3
4. Below, is $\{0, \mathrm{~A}\}$ an unhappy couple? Explain carefully.

Below, is $\{2, C\}$ an unhappy couple? Explain carefully.

Below, is $\{3, \mathrm{D}\}$ an unhappy couple? Explain carefully.

5. a) For each assignment of values to $u, v, w, x$ below, is the bipartite system valid and the matching stable? Explain each answer.
u 0 , v 1 , w 1 , x 1 ?
u 0 , v 1 , w 0 , x 1 ?
u 0 , v 1 , w 1 , x 0 ?

each page 8 marks $\quad 30 \mathrm{~min} \quad$ closed book $\quad$ no devices $\quad 3$ pages $\quad$ page 1

1. Recall: $f(n)=0,1, f(n-2)+f(n-1)$ when $n$ is $\operatorname{def} \operatorname{ifib}(\mathrm{n})$ : respectively $0,1, \geq 2$. For each non-negative integer $k, \quad \mathrm{a}, \mathrm{b}=0,1 \quad \# 1$ $C(k)$ is this claim: after line 3 has executed exactly $k$ for _ in range ( n ): \#2 times, $a, b$ equal $f(k), f(k+1)$ respectively.
$\mathrm{a}, \mathrm{b}=\mathrm{b}, \mathrm{a}+\mathrm{b} \quad \# 3$
return a \#4
Prove $C(0)$, i.e. after line 3 has executed exactly 0 times, a,b equal resp. $f(0), f(1)$.

Let $x$ be a non-negative integer, and assume $C(x)$. Prove $C(x+1)$.
first name
each page 8 marks
last name
id\#
30 min
closed book
2. Recall: for a sequence $S$, for each index $j, L[j]$ is the length of a longest increasing subsequence (LIS) ending at position $j$. Below, show the values for L .

| $j$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $S$ | 8 | 5 | 11 | 2 | 9 | 6 | 3 | 12 | 0 | 4 |

L
3. Let S be a sequence $\left(s_{0}, s_{1}, \ldots, s_{7}\right)$. Assume that $f(7)=4$. Assume that $\left(s_{1}, s_{2}, s_{5}, s_{7}\right)$ is an increasing subsequence. For each $j$ below, give the set $Z(j)$ of possible values of $L[j]$ consistent with the above information.
$\begin{array}{lllllllll}j & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\end{array}$

Z(j)
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Your justification for your answer to $\mathrm{Z}(0)$.

Your justification for your answer to $\mathrm{Z}(1)$.

You do not have to justify your other answers.
each page 8 marks $\quad 30 \mathrm{~min} \quad$ no devices $\quad 3$ pages book 3
4. Below, is $\{2, B\}$ an unhappy couple? Explain carefully.

Below, is $\{3, \mathrm{~A}\}$ an unhappy couple? Explain carefully.

Below, is $\{0, \mathrm{~A}\}$ an unhappy couple? Explain carefully.

5. a) For each assignment of values to $u, v, w, x$ below, is the bipartite system valid and the matching stable? Explain each answer.
u 0 , v 1 , w 1 , x 0 ?
u 1 , v 0 , w 1 , x 0 ?
u 1 , v 1 , w 1 , x 0 ?

each page 8 marks $\quad 30 \mathrm{~min} \quad$ closed book $\quad$ no devices $\quad 3$ pages $\quad$ page 1

1. Recall: $f(n)=0,1, f(n-2)+f(n-1)$ when $n$ is $\operatorname{def} \operatorname{ifib}(\mathrm{n})$ : respectively $0,1, \geq 2$. For each non-negative integer $t, \quad \mathrm{a}, \mathrm{b}=0,1$ \#1 $C(t)$ is this claim: after line 3 has executed exactly $t$ times, $a, b$ equal $f(t), f(t+1)$ respectively.

| for _ in range (n) : | $\# 2$ |
| :--- | :--- |
| $\mathrm{a}, \mathrm{b}=\mathrm{b}, \mathrm{a}+\mathrm{b}$ | $\# 3$ |
| return a | $\# 4$ |

Prove $C(0)$, i.e. after line 3 has executed exactly 0 times, a,b equal resp. $f(0), f(1)$.

Let $y$ be a non-negative integer, and assume $C(y)$. Prove $C(y+1)$.
first name
each page 8 marks
last name
id\#
30 min
closed book
2. Recall: for a sequence $S$, for each index $j, L[j]$ is the length of a longest increasing subsequence (LIS) ending at position $j$. Below, show the values for L .

| j | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S | 0 | 4 | 8 | 5 | 11 | 2 | 9 | 6 | 3 | 12 |

L
3. Let S be a sequence $\left(s_{0}, s_{1}, \ldots, s_{7}\right)$. Assume that $f(7)=4$. Assume that $\left(s_{1}, s_{4}, s_{5}, s_{7}\right)$ is an increasing subsequence. For each $j$ below, give the set $Z(j)$ of possible values of $L[j]$ consistent with the above information.
$\begin{array}{lllllllll}j & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\end{array}$

Z(j)
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Your justification for your answer to $\mathrm{Z}(0)$.

Your justification for your answer to $\mathrm{Z}(1)$.

You do not have to justify your other answers.
each page 8 marks $\quad 30 \mathrm{~min} \quad$ no devices $\quad 3$ pages booke 3
4. Below, is $\{3, \mathrm{~A}\}$ an unhappy couple? Explain carefully.

Below, is $\{1, \mathrm{D}\}$ an unhappy couple? Explain carefully.

Below, is $\{2, \mathrm{~A}\}$ an unhappy couple? Explain carefully.

5. a) For each assignment of values to $u, v, w, x$ below, is the bipartite system valid and the matching unstable? Explain each answer.
u 0 , v 1 , w 1 , x 0 ?
u 0 , v 0 , w 1 , x 1 ?
u 0 , v 1 , w 0 , x 1 ?


