first name	la	ast name		$\mathrm{id}\#$	
each page 8 marks	<b>30</b> min	closed book	no devi	ces 3 page	s page 1
1. Recall: $f(n)$	= 0, 1, f(n -	2) + f(n-1)	when $n$ is	<pre>def ifib(n):</pre>	
respectively (	$0, 1, \ge 2$ . For each $1, \ge 2$ .	ach non-negative	integer $j$ ,	a,b = 0,1	#1
C(j) is this	claim: after line	e 3 has executed	exactly $j$	for _ in r	ange(n): #2
times, $a, b$ eq	ual $f(j), f(j+1)$	) respectively.		a,b = b,	a+b #3
				return a	#4

**Prove** C(0), i.e. after line 3 has executed exactly 0 times, a,b equal resp. f(0), f(1).

Let w be a non-negative integer, and assume C(w). Prove C(w+1).

first name				last name				$\mathrm{id}\#$					
each page 8 marks		5	30 min closed book			ook	no devices			ages	page 2		
2.	Rec subs	all: for sequenc	a seque e (LIS)	ence $S$ , ending	for eac at posi	tion $j$ .	$\begin{array}{l} \text{ for } j, \ L[j] \\ \text{ Below,} \end{array}$	is the show t	e length he valu	of a lo es for L	ongest i	ncreasing	
	j	0	1	2	3	4	5	6	7	8	9		
	S	12	0	4	8	5	11	2	9	6	3		
	L												
3.	Let	S be a	sequenc	$ee(s_0, s_1)$	$1,\ldots,s_{7}$	7). Ass	ume tha	at $f(7)$ :	= 4. As	sume t	hat $(s_1,$	$(s_3,s_4,s_7)$	
	is a	n increa	sing su	bsequen	ice. Foi	each	j below,	give th	he set $Z$	f(j) of p	possible	values of	

L[j] consistent with the above information.

j 0 1 2 3 4 5 6 7 Z(j) \_\_\_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_

Your justification for your answer to Z(0).

Your justification for your answer to Z(1).

You do not have to justify your other answers.

first name	1	ast name	$\mathrm{id}_{7}$		
each page 8 marks	<b>30 min</b>	closed book	no devices	3 pages	page 3
4. Below, is $\{0,A\}$	· an unhappy	couple? Explain	carefully.		

Below, is  $\{2,C\}$  an unhappy couple? Explain carefully.

Below, is  $\{3,D\}$  an unhappy couple? Explain carefully.



5. a) For each assignment of values to u,v,w,x below, is the bipartite system valid and the matching stable? Explain each answer.

u 0, v 1, w 1, x 1 ?

u 0, v 1, w 0, x 1 ?

u 0, v 1, w 1, x 0 ?



first name	la	ast name		$\mathrm{id}\#$	
each page 8 marks	<b>30</b> min	closed book	no devi	ces 3 pages	s page 1
1. Recall: $f(n)$	= 0, 1, f(n -	2) + f(n-1)	when $n$ is	<pre>def ifib(n):</pre>	
respectively (	$0, 1, \ge 2$ . For each $1, 2 \ge 2$ .	ch non-negative	integer $k$ ,	a,b = 0,1	#1
C(k) is this of	claim: after line	e 3 has executed	exactly $k$	for _ in r	ange(n): #2
times, $a, b$ equ	ual $f(k), f(k+1)$	l) respectively.		a,b = b,	a+b #3
				return a	#4

**Prove** C(0), i.e. after line 3 has executed exactly 0 times, a,b equal resp. f(0), f(1).

Let x be a non-negative integer, and assume C(x). Prove C(x+1).

first name			last name				$\mathrm{id}\#$					
each page 8 marks		S	30 min	с	closed book			no devices		ages	page 2	
2	. Reca subs	all: for sequenc	a sequ e (LIS)	ience $S$ , ) ending	for eac at posi	th indexition $j$ .	$\begin{array}{l} \mathbf{x} \ j, \ L[j\\ \mathbf{Below}, \end{array}$	[] is the show t	e length the valu	of a lo es for L	ongest 4.	increasing
	j	0	1	2	3	4	5	6	7	8	9	
	S	8	5	11	2	9	6	3	12	0	4	
	L											
3	. Let	S be a	sequen	$ce(s_0,s)$	$1,\ldots,s^r$	7). Assi	ume tha	at $f(7)$	= 4. As	ssume t	hat $(s_1$	$, s_2, s_5, s_7)$
	is ar	n increa	using su	ıbsequer	ice. Foi	r each j	below,	give t	ne set Z	$(j)$ of $\mathbf{I}$	20SS1D10	e values of

L[j] consistent with the above information.

j 0 1 2 3 4 5 6 7 Z(j) \_\_\_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_

Your justification for your answer to Z(0).

Your justification for your answer to Z(1).

You do not have to justify your other answers.

first name	1	ast name	$\mathrm{id}_7$		
each page 8 marks	<b>30</b> min	closed book	no devices	3 pages	page 3
4. Below, is $\{2,B\}$	an unhappy	couple? Explain	carefully.		

Below, is {3,A} an unhappy couple? Explain carefully.

Below, is  $\{0,A\}$  an unhappy couple? Explain carefully.



5. a) For each assignment of values to u,v,w,x below, is the bipartite system valid and the matching stable? Explain each answer.

u 0, v 1, w 1, x 0 ?

u 1, v 0, w 1, x 0 ?

u 1, v 1, w 1, x 0 ?



first name	la	ist name		$\mathrm{id}\#$	
each page 8 marks	s 30 min	closed book	no devi	ces 3 page	s page 1
1. Recall: $f(r$	h) = 0, 1, f(n - 1)	2) + f(n-1)	when $n$ is	<pre>def ifib(n):</pre>	
respectively	$0, 1, \ge 2$ . For each $0, 1, \ge 2$ .	ach non-negativ	e integer $t$ ,	a,b = 0,1	#1
C(t) is this	s claim: after line	3 has execute	d exactly $t$	for _ in r	ange(n): #2
times, $a, b \in$	equal $f(t), f(t+1)$	) respectively.		a,b = b,	a+b #3
				return a	#4

**Prove** C(0), i.e. after line 3 has executed exactly 0 times, a,b equal resp. f(0), f(1).

Let y be a non-negative integer, and assume C(y). Prove C(y+1).

first name				last name								
each	page 8	marks	3	0 min	clos	ed book	Σ.	no devi	ices	3 pag	ges page 2	2
2.	Recall subseq	: for a uence	sequer (LIS) e	nce $S$ , for $n$ and	or each t positi	index $_{j}$ on $j$ . B	j, L[j] selow, sl	is the l how the	ength o e values	of a lon s for L.	ngest increasing	r >
	j	0	1	2	3	4	5	6	7	8	9	
	S	0	4	8	5	11	2	9	6	3	12	
	L											
3.	Let S	be a se	quence	$(s_0, s_1,$	$\ldots, s_7).$	Assum	ne that	f(7) =	4. Assu	ume tha	at $(s_1, s_4, s_5, s_7)$	)

is an increasing subsequence. For each j below, give the set Z(j) of possible values of L[j] consistent with the above information.

j 0 1 2 3 4 5 6 7 Z(j) \_\_\_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_ \_\_\_

Your justification for your answer to Z(0).

Your justification for your answer to Z(1).

You do not have to justify your other answers.

first name	]	ast name	$\operatorname{id}_{\overline{7}}$		
each page 8 marks	<b>30 min</b>	closed book	no devices	3 pages	page 3
4. Below, is $\{3,A\}$	• an unhappy	couple? Explain	carefully.		

Below, is  $\{1,D\}$  an unhappy couple? Explain carefully.

Below, is  $\{2,A\}$  an unhappy couple? Explain carefully.



5. a) For each assignment of values to u,v,w,x below, is the bipartite system valid and the matching unstable? Explain each answer.

u 0, v 1, w 1, x 0 ?

u 0, v 0, w 1, x 1 ?

u 0, v 1, w 0, x 1 ?

