cmput 3042023 study questions 7: part 1
(part 2 will be posted by Wed Nov 15)

1. For each digraph, (a) give the postorder traversal sequence (assume node list and arc lists are stored in alphabetic order) (b) repeat (a) assuming that node and arc lists are stored in reverse alphabetic order (c) using your answer from (a), give the postorder traversal sequence of the transpose digraph (d) give the strongly connected components of the digraph (e) explain how to use your answer from (c) to answer (d).

2. Prove or disprove: if $S$ is an sec of a digraph $D$, then $S$ is an sec of the transpose of $D$.
3. In a digraph $D$, let $x$ be a node in a source strongly connected component and let $y$ be a node in some other sci such that there is an arc from $x$ to $y$. Prove/disprove: $y$ appears before $x$ in the postorder traversal sequence of $D$.
4. Prove/disprove: for a digraph $D$, the last node in the postorder traversal sequence of $D$ is in some source scc of $D$.
5. The sec algorithm from the lectures starts by taking the last node in a postorder traversal of the transpose and then using a depth-first or breadth-first search to find a sink ac. Consider this simpler algorithm: take the last node in a postorder traversal of the digraph and then use a dis or bs to find a source scc. Is the simpler algorithm correct: does it always find a source scc? Explain.
6. (a) On the nodes below, draw the implication digraph for this 2-sat formula:

$$
f=\left[\begin{array}{ll}
1 & -5
\end{array}\right]\left[\begin{array}{ll}
-2 & -3
\end{array}\right]\left[\begin{array}{ll}
3 & 4
\end{array}\right]\left[\begin{array}{ll}
-4 & -5
\end{array}\right]\left[\begin{array}{ll}
2 & 5
\end{array}\right]\left[\begin{array}{ll}
-1 & -5
\end{array}\right] .
$$


(b) Repeat for $\left[\begin{array}{ll}1 & -2\end{array}\right]\left[\begin{array}{ll}1 & 3\end{array}\right]\left[\begin{array}{ll}-2 & -3\end{array}\right]\left[\begin{array}{ll}2 & 4\end{array}\right]\left[\begin{array}{ll}-3 & -4\end{array}\right]\left[\begin{array}{ll}3 & -5\end{array}\right]\left[\begin{array}{ll}3 & 5\end{array}\right]$.


Hint: answers to this question on the next page.


