cmput 304 2023 study questions 6

$$
\begin{aligned}
\max\ 3 * x_1 + x_2 & \\
x_1 &\leq 2 \\
x_2 &\leq 3 \\
6 * x_1 + 2 * x_2 &\leq 15 \\
x_1, x_2 &\geq 0
\end{aligned}
$$

1. For the LP above

   a) above and to the right, give the dual of the LP. Use format similar to above, i.e. with variables $y_1, y_2, \ldots$

   b) here, sketch the feasible region:

   c) give an optimal solution and value. Answer like this: (30, 40) 39.5.

   d) explain briefly how you know your answer to c) is correct

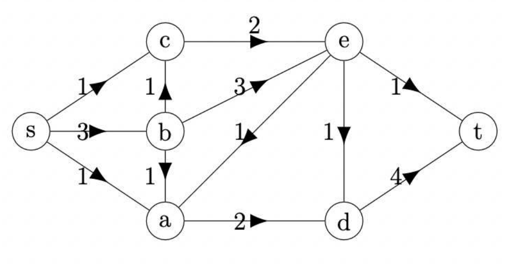
$$
\begin{aligned}
\max\ 3 * x_1 + x_2 & \\
x_1 &\geq 2 \\
x_2 &= 3 \\
6 * x_1 + 2 * x_2 &\leq 15 \\
x_2 &\geq 0
\end{aligned}
$$

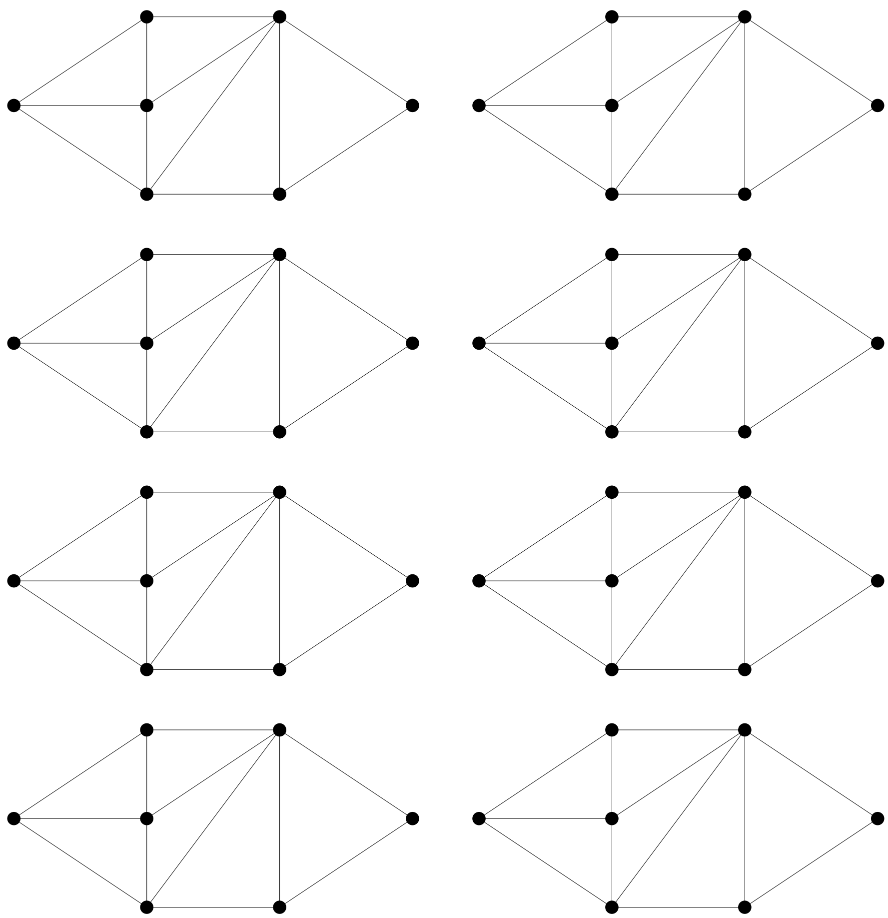
2. For the LP above

   a) above and to the right, rewrite the problem in vector format, give the vectors c, b, x, and give the matrix A.

   b) below, give the equivalent LP in less-equal format. Explain your work.

3. Using the residual network method, find a max flow and a min cut for this network. Show your work on the diagrams below. Give your final answer like this: `8 {s, a, b, d, t}`

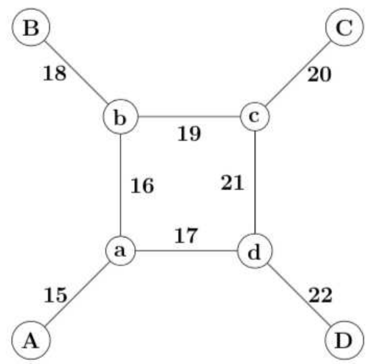




4. Formulate the above max flow problem as an LP. Hint: there will 12 variables, one for each network arc.

5. a) Explain how max flow polytime transforms to LP. b) Explain why there is always an all-integer solution to any max flow problem with all-integer capacities.

6. You manage a communications network with users A,B,C only (D is no longer involved) and bandwidths shown in the figure below. You need to establish connections between A-B, A-C, and B-C, which pay \$5, \$4, \$3 respectively per unit bandwidth. Between each pair of users at least 7 units must be routed.

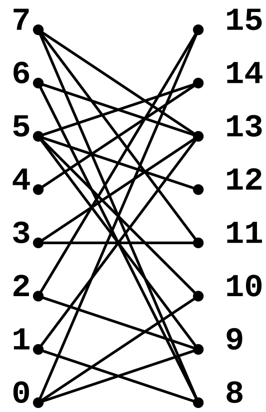


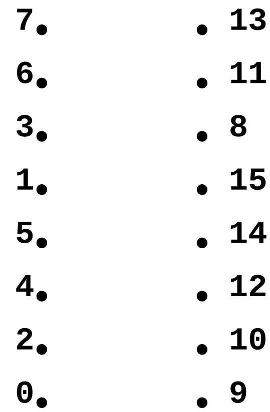
Each connection has two possible routes. For A-B, xAB is traffic volume along A-a-b-B, yAB is volume along A-a-d-c-b-B; define xBC, yBC, similarly; xAC, yAC is traffic along A-a-b-c-C, A-a-d-c-D respectively. You want to maximize this network's revenue. Using the variables above, formulate this problem as an LP:

a) Give the objective function.

b) Give the system of (in)equalities.

c) Give a feasible solution.





8. Above left is a bipartite graph $G = (V_0, V_1, E)$.

   a) In G, give V_0 and V_E.

   b) In G, give a matching of size 3. Answer like this: `{ (1,10), (3, 11), (5, 8) }` .

   c) In G, give a matching of size 7. Answer as in the previous question. Hint: redraw G on the nodes above right.

   Below is an s-t flow network H: arrows on middle arcs have been omitted, they are all from left to right; each arc has capacity 1.

   d) In H, give a cut of size 7. Hint: look at the redrawing from c).

   e) Prove that your matching in c) is a maximum matching.