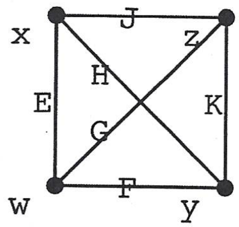
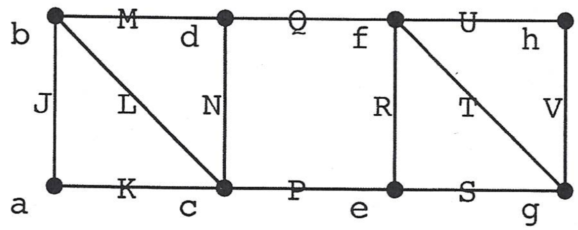
QUIZ 8 — SOLUTIONS TO QUESTIONS 1-3

first name | last name | id#

2022 c304 sq8 | 30 min | closed book | no devices | 2 pages |



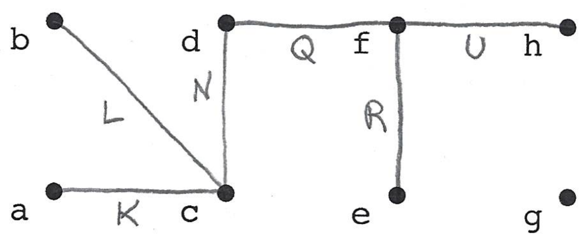


Recall: a *cut* of a graph is a partition of the node set into two non-empty subsets. E.g. on the small graph (above left), {{w,x}, {y,z}} is a cut with cross-edges {F,G,H,J}. RKMC is the randomized Kruskal min cut algorithm: unless otherwise stated, its input is a uniform-random permutation of the edges.

1. **[2 marks]** For the big graph, give each min cut (partition and cross-edges) . . .

#	Partition	Cross-edges
1	{{a}, {b, c, d, e, f, g, h}}	{J, K}
2	{{h}, {a, b, c, d, e, f, g}}	{U, V}
3	{{a, b, c, d}, {e, f, g, h}}	{P, Q}

2. **[3 marks]** . . . and give the forest (draw on the nodes below) and cut (partition and cross-edges) found by RKMC when edges are input in order LUQRKNTVSJMP.



Partition

{{g}, {a, b, c, d, e, f, h}}

Cross-edges

{S, T, V}

3. **[2 marks]** Let G be a connected graph with a cut $\{X, Y\}$ with $G[X]$ (the subgraph of G on the node set X) connected but $G[Y]$ disconnected, with exactly two components $G[Y_1]$ and $G[Y_2]$. Prove or disprove: $\{X, Y\}$ is a min cut of G.

$\boxed{\text{False.}}$ Proof:

Because G is connected and $G[Y]$ is disconnected, there must be at least one edge connecting $G[X]$ and $G[Y_1]$. Denote these edges as E_1.

Similarly, there must be at least one edge connecting $G[X]$ and $G[Y_2]$. Denote these edges as E_2.

From the above, we see that both $\{X \cup Y_2, Y_1\}$ & $\{X \cup Y_1, Y_2\}$ are cuts of G, with sizes $|E_1|$ and $|E_2|$, respectively. We also see that cut $\{X, Y\}$ has size $|E_1| + |E_2|$. Since $|E_1| + |E_2| > |E_1|$ and $|E_1| + |E_2| > |E_2|$, $\{X, Y\}$ cannot be a min cut. (There are at least two smaller ones.)