Question 2

- a) Based on the definition for v'_j which is set to be $\lfloor v_j \cdot \frac{n}{\epsilon \cdot v_{max}} \rfloor$, the equation (2) holds.

- b) Consider the following:

$$v'_j = \lfloor v_j \cdot \frac{n}{\epsilon \cdot v_{max}} \rfloor \Longrightarrow v'_j \leq v_j \cdot \frac{n}{\epsilon \cdot v_{max}} \Longrightarrow v'_j \cdot \frac{\epsilon \cdot v_{max}}{n} \leq v_j$$

  Then, we can sum over any set of items, say S':

$$\sum_{j \in S'} v_j \geq \sum_{j \in S'} v'_j \cdot \frac{\epsilon \cdot v_{max}}{n}$$

- c) Based on 4, we can say

$$\sum_{j \in S'} v'_j \cdot \frac{\epsilon \cdot v_{max}}{n} = \frac{\epsilon \cdot v_{max}}{n} \cdot sum_{j \in S'} v'_j \geq \frac{\epsilon \cdot v_{max}}{n} \cdot (K \cdot \frac{n}{\epsilon \cdot v_{max}} - n)$$

  Just to mention, the optimal assignment for the shrunken problem S', has a rescaled value of at least (4), that is why we can use the replacement.