quiz 7 cmput 304 fall 2022

first name last name id#

2022 c304 sq7 30 min closed book no devices 2 pages

1. Suppose we use the knapsack DPV FPTAS to find an approximate solution with $\varepsilon = .1$ for this knapsack instance P with $n = 10$ and $W = 100$:

wt	22	18	19	25	21	17	23	20	21	25
val	5371	6650	10000	7671	9967	8436	8032	6435	5652	6350

a) [**2 marks**] What is the scaling factor? Show your work.

$$\text{Scaling Factor} = \frac{n}{\varepsilon . V_{max}} = \frac{10}{0.1 \times 10000} = 0.01$$

$V_{max} = 10\,000$

b) [**2 marks**] Give vectors w (first 3 only) and v (first 3 only) for the knapsack instance of the scaled problem P':

wt	22	18	19	→	22	18	19
val	$\lfloor 53.71 \rfloor$	$\lfloor 66.50 \rfloor$	$\lfloor 100 \rfloor$		53	66	100

Assume that {2,3,5,6,7} is an optimal knapsack for P'.

c) [**2 marks**] As a sum, give the value of the corresponding approximate solution to P (you do not have to add the numbers.)

I accepte both solutions:

Solution 1: $6650 + 10000 + 9967 + 8436 + 8032$

Solution 2: $(66 + 100 + 99 + 84 + 80) \times \frac{\varepsilon . V_{max}}{n} = (66 + 100 + 99 + 84 + 80) \times 100$

d) [**1 marks**] Let T be your answer to c). Let K be the value of an optimal solution to P. Give a lower and upper bound for K in terms of T.

lower bound: Solution 1 in part (c) is a Feasible solution for the problem. So $T \leq K$

Solution 2 in part (c) is even smaller than solution 1. So even in this case $T \leq K$.

upper bound: From the second inequality in the textbook, we have

$$T \geq K(1-\varepsilon) \Rightarrow K \leq \frac{1}{1-\varepsilon} . T = \frac{1}{0.9} T = \frac{10}{9} T.$$