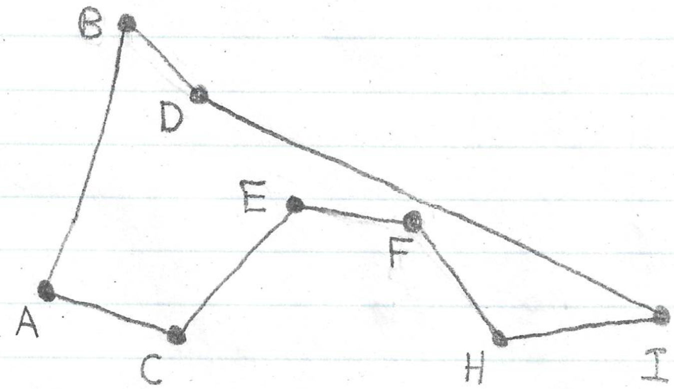
Quiz 6, Question 1, Version 1 Solution

a)



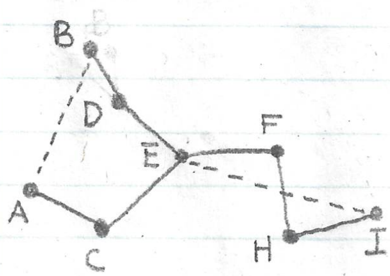
Showing your work means, e.g., showing something like this:

C E F H I ~~H~~ ~~F~~ ~~E~~ D B ~~D~~ ~~E~~ ~~C~~ A C

or stating that you removed repeated vertices from the Eulerian tour.

b)

MST + M



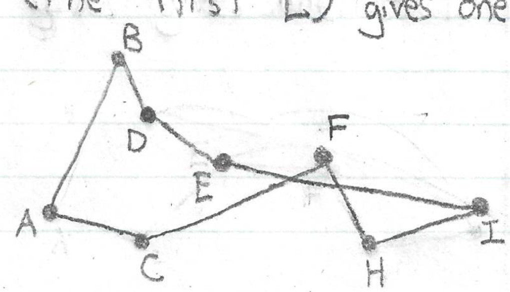
The solid edges are from MST; the dashed edges are from M

One of two possible tours starting at C

MST + M Eulerian Tour

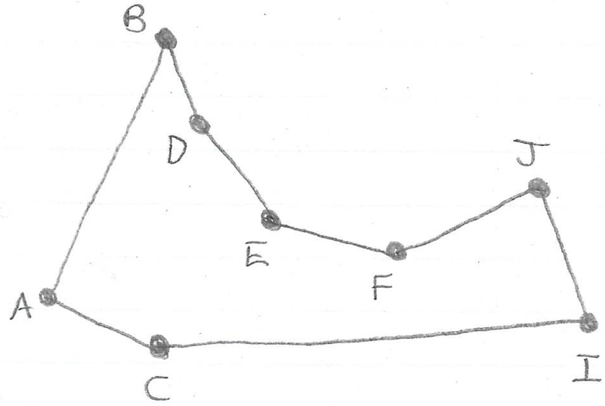
C E F H I E D B A C

Removing a repeated vertex (the first E) gives one solution:



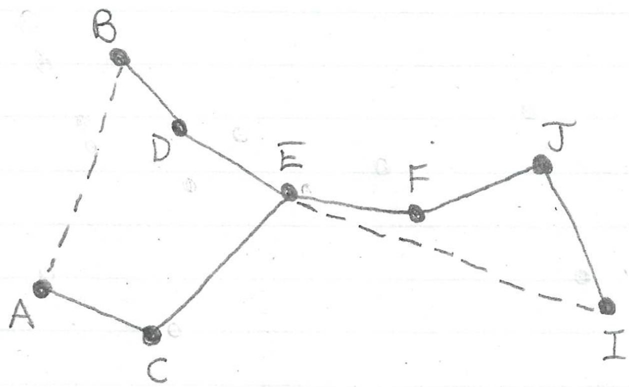
Quiz 6, Question 1, Version 2 Solution

a)



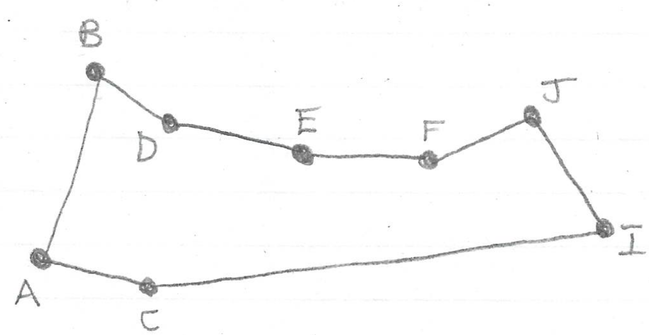
b)

MST + M



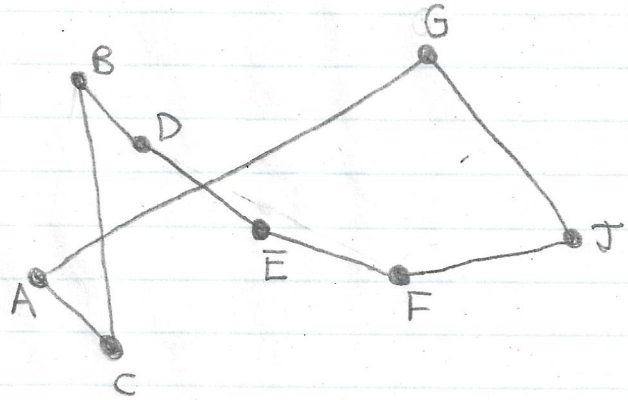
Hamiltonian Cycle

(one of several solutions)



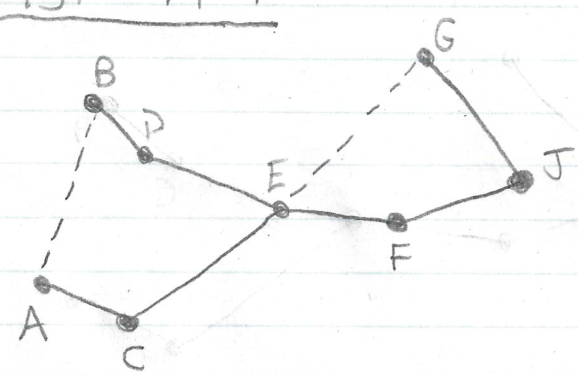
Quiz 6, Question 1, Version 3 Solution

a)

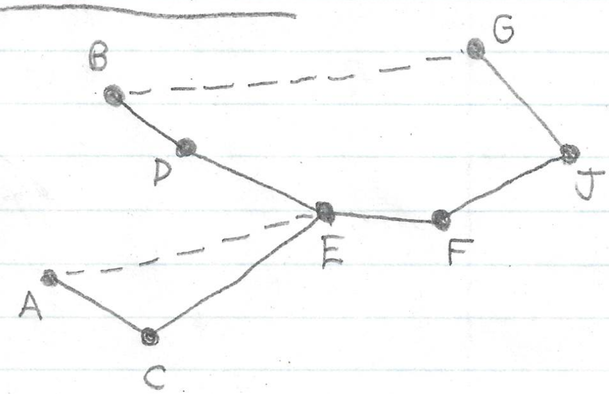


b)

MST + M 1



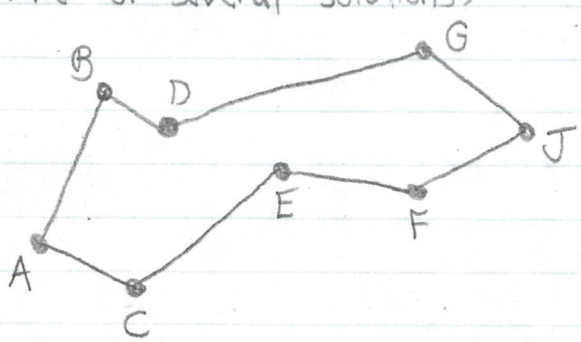
MST + M 2



Either is acceptable.

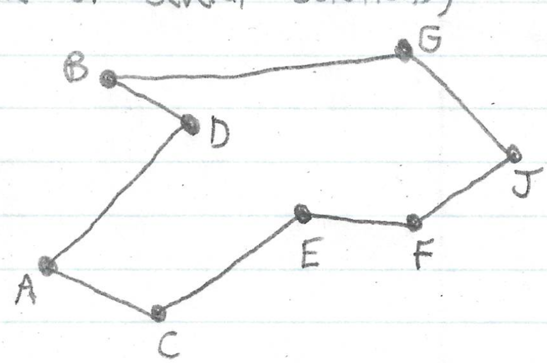
Hamiltonian Cycle from 1

(one of several solutions)



Hamiltonian Cycle from 2

(one of several solutions)



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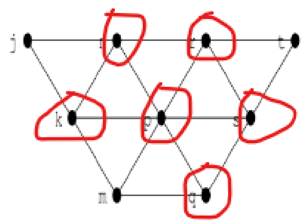
2. **[1 marks]** Recall that a *vertex cover* of a graph $G = (V, E)$ is a subset C of V that covers all edges. Give a minimum size set cover of this graph: 6



Q2 Prove that the complement of a vertex cover (the nodes of the graph that are not in the cover) is an independent set.

Q2 Solution: if S is a vertex cover, if any two nodes u and v in $V - S$ were connected by an edge e, hen neither u nor v would be in S, which contradicts the initial assumption that S is a vertex over. That is, no two nodes in $V - S$ can be adjacent and hence $V - S$ is an independent set.

Q3 Recall k-VC: given a graph G and an integer k, does G have a vertex cover of size at most k? Recall q-IS: given a graph G and an integer q, does G have an independent set of size at least q? Give a polytime answer-preserving transformation T from instances of k-VC to instances of q-IS. Explain why your T is answer-preserving (you do not have to explain why it is polytime).

Q3 Solution: Vertex Cover is $\leq_p$ Independent Set. Suppose that we have an efficient algorithm for solving Independent Set, it can simply be used to decide whether G has a vertex cover of size at most k, by asking it to determine whether G has an independent set of size at least $n - k$. So if $q < n - k$ then the answer is no else the answer is yes.

Q4 Does your answer to the previous question and the fact that q-IS is in the class NP-complete imply that k-VC is in the class NP-complete? Explain briefly.

Q4 Solution: No; Recall the definition of what it means to prove a problem X to be NP-complete from a known problem Y which is NP-complete.

If Y is NP-complete, and

- X is in NP
- $Y \leq_P X$

then X is NP-complete.

But from the antecedent and the previous question we have $Y = q$-IS and $X = k$-VC and we are given Y to be NP-complete but we showed the poly-time reduction such that $X \leq_P Y$ and not $Y \leq_p X$. Hence X is not proven to be NP-complete