quiz 3 cmput 304 fall 2022

1. Solution:

$1) f = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_2 \vee x_3 \vee \neg x_4) \wedge (x_1 \vee x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee \neg x_4)$
$(x_1 \vee \neg x_2 \vee \neg x_3) \wedge (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_4) \wedge (\neg x_1 \vee x_2 \vee \neg x_3)$

Then Let $x_1 = F, x_2 = T, x_3 = F, x_4 = T$, which makes f satisfied and hence, we can find the independent set to be node x_2 from first triangle, x_2 from second triangle, x_4 from third triangle, $\sim x_1$ from the fourth triangle, $\sim x_3$ from the fifth triangle, x_2 from the sixth triangle, x_4 from the seventh triangle, and $\sim x_2$ from the eighth triangle.

2)

$f = (x_2 \vee \neg x_3 \vee \neg x_4) \wedge (x_2 \vee x_3 \vee \neg x_4) \wedge (\neg x_2 \vee \neg x_3 \vee x_1) \wedge (\neg x_2 \vee x_3 \vee \neg x_4$
$(\neg x_2 \vee x_3 \vee x_4) \wedge (x_3 \vee x_4 \vee \neg x_1) \wedge (x_2 \vee x_4 \vee x_1) \wedge (\neg x_2 \vee \neg x_3 \vee \neg x_1)$

Then Let $x_1 = T, x_2 = F, x_3 = T, x_4 = F$, which makes f satisfied and hence, we can find the independent set to be node $\sim x_4$ from first triangle, x_3 from second triangle, x_1 from third triangle, x_3 from the fourth triangle, x_3 from the fifth triangle, x_3 from the sixth triangle, x_1 from the seventh triangle, and x_2 from the eighth triangle.

3)

$f = (x_3 \vee x_1 \vee x_2) \wedge (\neg x_3 \vee \neg x_4 \vee \neg x_2) \wedge (\neg x_3 \vee x_4 \vee x_1) \wedge (x_4 \vee x_1 \vee \neg x_2) \wedge$
$(\neg x_3 \vee \neg x_4 \vee x_2) \wedge (\neg x_3 \vee x_4 \vee \neg x_1)(x_3 \vee \neg x_4 \vee \neg x_1) \wedge (x_3 \vee x_4 \vee \neg x_1)$

Then Let $x_1 = F, x_2 = T, x_3 = F, x_4 = T$, which makes f satisfied and hence, we can find the independent set to be node x_2 from first triangle, $\sim x_3$ from second triangle, x_4 from third triangle, x_4 from the fourth triangle, x_2 from the fifth triangle, x_4 from the sixth triangle, $\sim x_1$ from the seventh triangle, and x_4 from the eighth triangle.

2. **Solution:**

   (a) To prove that a problem is in NP-complete we need to shows 2 things i.e it is in NP class and it can be reduced to an already existing NP-hard problem.
   Step 1: 3-SAT is in NP class (Already given in the question)
   Step 2: Let's try to take a already NP-hard problem which is a SAT problem.We need to show that it is reducible to 3-SAT problem.
   Now, f = $(X \wedge Y \vee Z)$
   $= (X \vee Y) \wedge (X \vee Z)$
   $= (X \vee Y \vee Z * Z') \wedge (X \vee Z \vee Y * Y')$
   $= (X \vee Y \vee Z') \wedge (X \vee Y \vee Z) \wedge (X \vee Z \vee Y) \wedge (X \vee Z \vee Y')$
   $= (X \vee Y \vee Z) \wedge (X \vee Y \vee Z') \wedge (X \vee Y' \vee Z)$

   Since SAT is reducible to 3-SAT hence 3-SAT is NP hard problem.
   From step 1 and 2 we can prove that 3-SAT is in NP-Complete.

   (b) Here we can use transitive property to show that k-independent set to sat is a poly-time answer preserving transformation.
   Transitive property : $P_1 \propto P_2$ and $P_2 \propto P_3$ then $P_1 \propto P_3$.
   Now we know there exist poly-time answer preserv-

ing reduction of SAT to k-clique and similarly from k-clique to k-independent set. Hence using transitive property we can say, there exists an answer preserving poly-time transformation from k-independent set to SAT.