

# Comparative Study of Approximate Strategies for Playing Sum Games Based on Subgame Types

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# Contents

- Sum games
- Strategies
- Subgame types
- Results and comments

# Combinatorial games

- Simple model (Cazenave)
- Games  $A \mid B \parallel C \mid D$
- $D = \text{rnd}(50)$ ,  $C = D + \text{rnd}(50)$ ,  
 $B = C + \text{rnd}(50)$ ,  $A = B + \text{rnd}(50)$
- Example:  $102 \mid 74 \parallel 50 \mid 13$

# Sum games

- Play sum of 5 random A|B||C|D games
- Example:  $g_1 + g_2 + g_3 + g_4 + g_5$ ,  
where  $g_1 = 102 | 74 || 50 | 13$ ,  
 $g_2 = 122 | 91 || 69 | 42, \dots$
- Remark: a similar, more general model was used by Müller+Li

# Algorithms

- Problem: given sum of games, find a good move quickly
- Exact solution: global minimax search
- Local solution: compute simple properties of each subgame, then make global decision based on properties

# Hotstrat

- Compute temperature of each subgame
- Play in hottest subgame
- *Hotstrat+* refinement by Müller+Li: try to play sente moves early

# Thermostrat

- Compute thermograph of each subgame
- Idea: optimize 1-ply minimax score at each temperature, select optimum
- More informed than hotstrat
- Better asymptotic properties

# Game Types

- Type 1: sente for left
- Type 2: gote
- Type 3: sente for right
- For  $A|B||C|D$  games: compute  $S=A+B+C+D$ 
  - Type 1:  $S > 4B$
  - Type 2:  $4B \geq S \geq 4C$
  - Type 3:  $S < 4C$



# Type Distribution

- In random model: almost all are type 2
- E.g. in sum of 5 games:
  - 43% all 5 are type 2
  - 40% have 4 type 2
  - 11% have 3 type 2
- What happens with more type 1 and 3 games present?

# Experiment

- Generate sums with specific mix of subgame types
- Test algorithms specifically for those sums
- Measure percentage of optimal moves

# Overall Results

(a) Percentage Coincidence

Strategy	% of Optimal
BMove	58.70%
SenteQ	64.70%
MaxMove	66.80%
Sente	72.30%
MaxThreat	87.70%
ThermoStrat	89.70%
HotStrat	91.50%
HotStrat+	91.60%
Optimal	100.00%

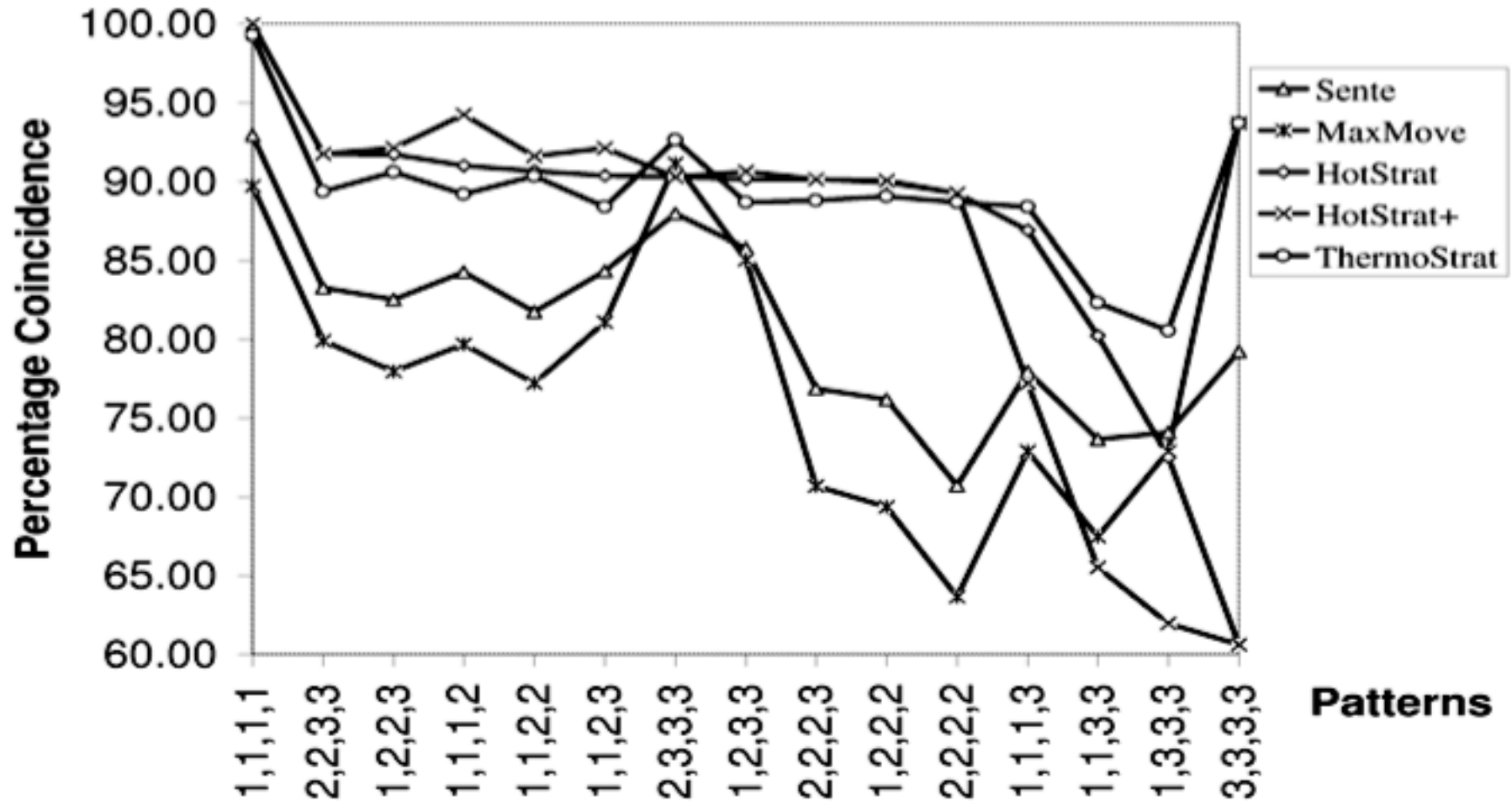
(b) Results from [3]

Strategy	% of Optimal
BMove	88.39%
MaxMove	95.40%
SenteQ	97.63%
Sente	97.71%
MaxThreat	99.17%
ThermoStrat	99.60%
Optimal	100.00%
HotStrat	100.33%

Pattern	BMove	Sente	SenteQ	MaxMove	HotStrat	HotStrat+	ThermoStrat	Optimal
1,1,1,1	92.77	92.92	94.76	89.70	100	100	99.29	100
2,2,3,3	54.93	83.29	62.96	79.91	<b>91.74</b>	<b>91.74</b>	89.36	100
1,2,2,3	60.39	82.52	64.74	77.96	<b>91.72</b>	<b>92.14</b>	90.60	100
1,1,1,2	80.48	84.27	72.90	79.68	<b>91.01</b>	<b>94.25</b>	89.20	100
1,1,2,2	71.27	81.76	66.66	77.22	<b>90.67</b>	<b>91.60</b>	90.34	100
1,1,2,3	61.79	84.33	68.90	81.08	<b>90.36</b>	<b>92.12</b>	88.39	100
2,3,3,3	47.96	87.95	59.91	<b>91.14</b>	90.28	90.28	<b>92.65</b>	100
1,2,3,3	52.26	85.78	64.17	85.05	<b>90.13</b>	<b>90.64</b>	88.69	100
2,2,2,3	57.03	76.88	65.43	70.70	<b>90.13</b>	<b>90.13</b>	88.80	100
1,2,2,2	63.88	76.19	65.76	69.37	<b>89.94</b>	<b>90.06</b>	89.06	100
2,2,2,2	57.41	70.72	65.93	63.66	<b>89.25</b>	<b>89.25</b>	88.69	100
1,1,1,3	57.97	77.96	74.40	72.89	<b>86.89</b>	77.20	<b>88.40</b>	100
1,1,3,3	46.66	73.65	64.30	67.48	<b>80.23</b>	65.52	<b>82.33</b>	100
1,3,3,3	45.03	<b>74.08</b>	55.62	72.90	72.54	61.94	<b>80.55</b>	100
3,3,3,3	48.60	79.21	48.76	<b>93.72</b>	60.59	60.59	<b>93.72</b>	100
<b>Average</b>	59.90	80.77	66.35	78.16	<b>87.03</b>	85.16	<b>89.34</b>	100
<b>Std. Dev.</b>	13.15	<b>5.99</b>	10.04	8.94	9.45	12.53	<b>4.32</b>	0

# Results by Subgame Type

# Same with Diagrams



# Discussion

- Thermostrat is always good
- Hotstrat, Hotstrat+ often a bit better, but terrible with many (reverse) sente positions