Learning with hierarchical features

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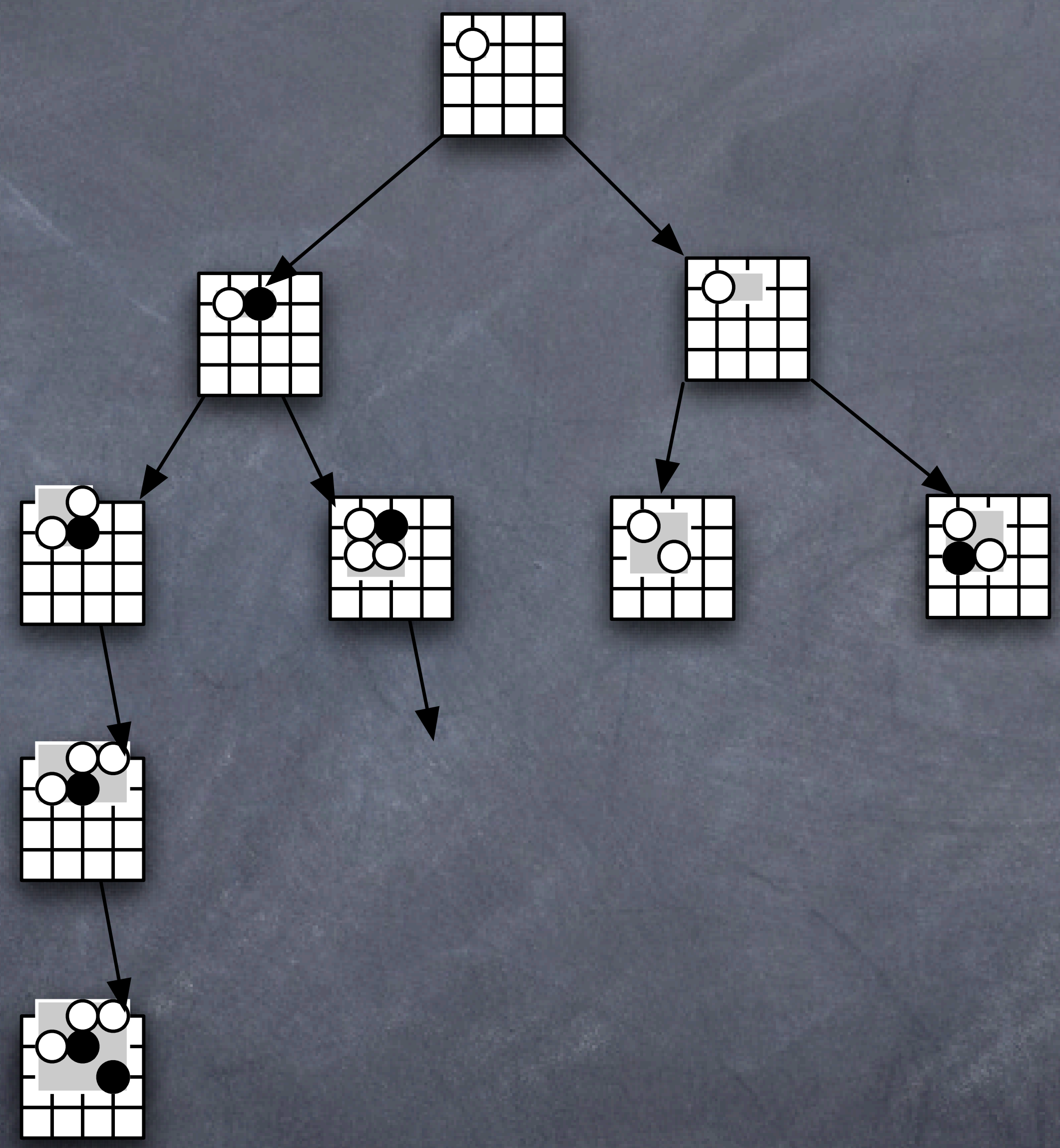
Outline

- Introduction to hierarchical features
- Learning issues with hierarchical features
- Experimental setup
- Preliminary results
- Scaling up to Go

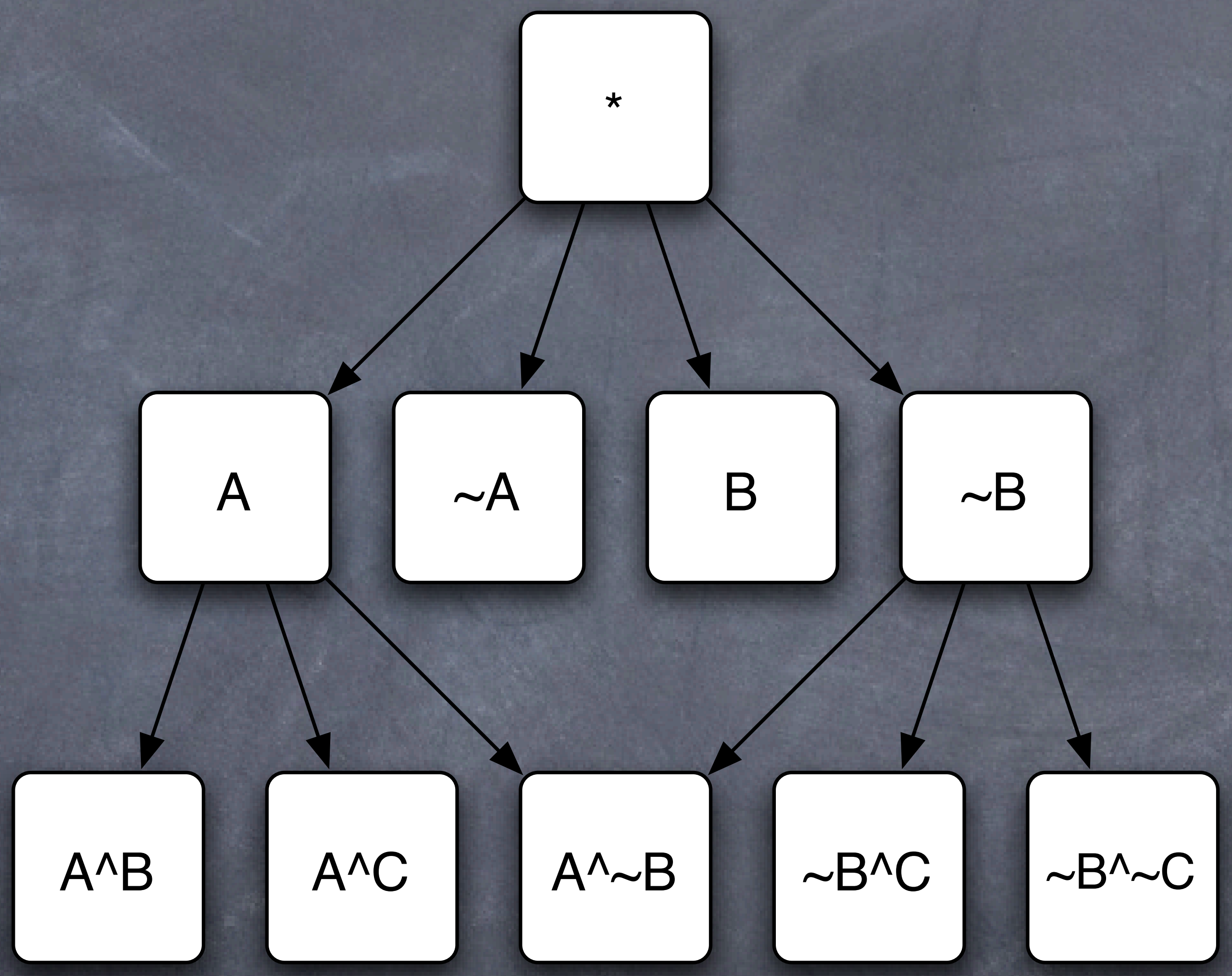
Hierarchical features

- Binary features are either on or off in any state.
- Define partial ordering between features
- If feature A => feature B, then A is a child of B

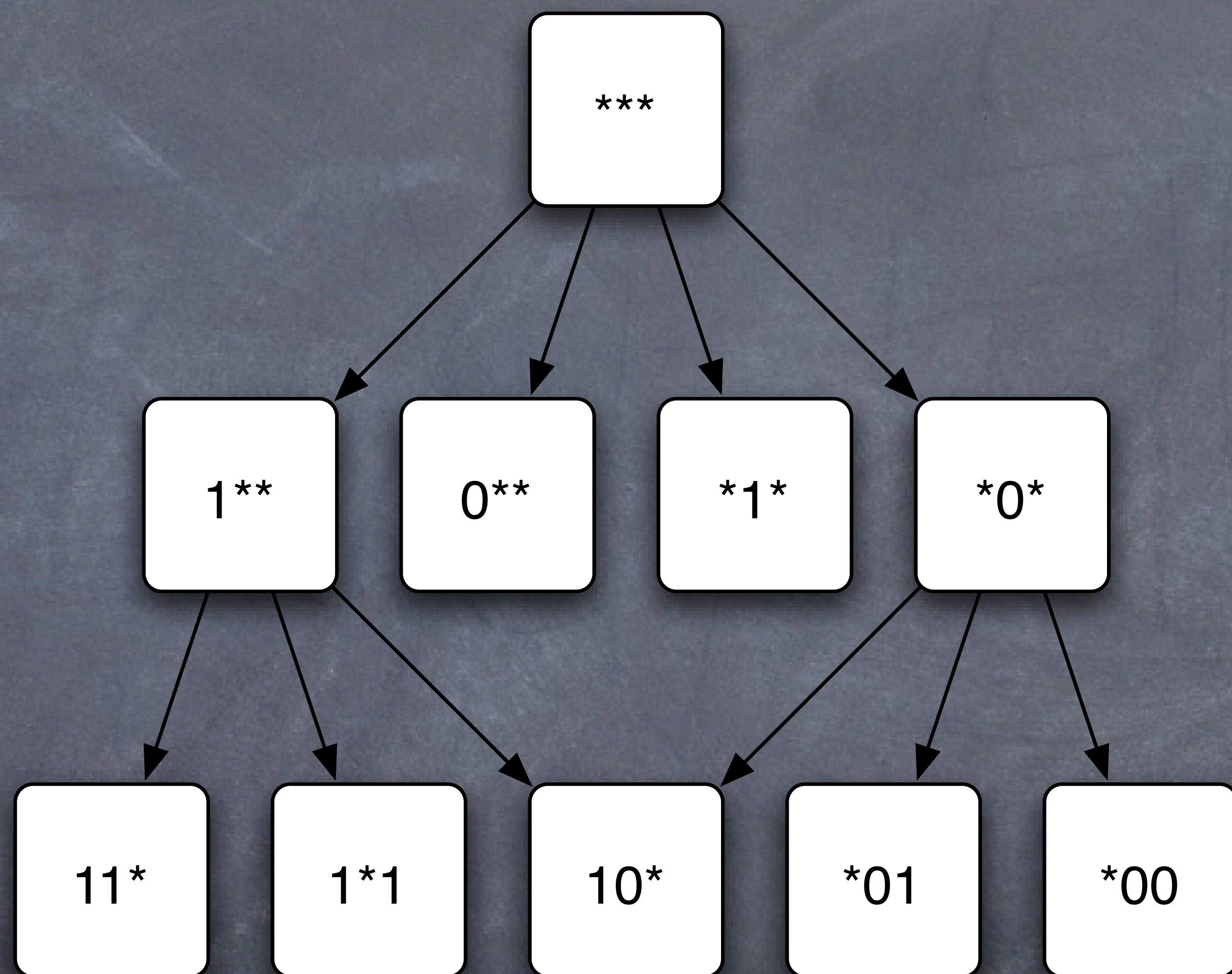
Example: Go shapes



Example: logic



Example: bit features



Terminology

- If feature A => feature B
  - B is **more general** than A
  - A is **more specific** than B
- A **strict** hierarchy has a strict partial order
  - If feature A => feature B
  - then feature B $\not\Rightarrow$ feature A

Terminology

- In an **exclusive** hierarchy
  - If parent is on, exactly one child is on
  - A single path through hierarchy is on

Motivation

- More general features learn quickly
- More specific features learn accurately
- How to combine features to get the best of both worlds?

Linear function approximation

- Prediction is a linear combination of features
  - $y = \Sigma w_j x_j$
- One weight for each feature
- Find weights that minimise squared error between prediction and target
  - $w = \text{argmin } E[(z-y)^2]$

Hierarchical function approximation

- Many solutions exist
- In general a large subspace of solutions
- Internal nodes of the hierarchy are 'redundant'
- Leaves are sufficient to describe any solution

Cascade learning

- Define a particular set of weights as optimal:
  - Weight for feature A minimises error given all features more general than A
  - Weight for feature A minimises total magnitude of children of A

Two algorithms

- Online cascade
  - Gradient descent using error of all more general features
- Floating average
  - Gradient descent at leaves
  - Average weights up hierarchy

Step-size control

- "All features are equal. But some are more equal than others"
- Frequencies of hierarchical features span many orders of magnitude
- Don't learn all features at the same rate

Step-sizes and uncertainty

- Each weight w_j can be viewed as an estimate of the optimal weight w_j*
- There is an **uncertainty** associated with this estimate: the estimation error
- Estimation error should determine step-size.
  - Uncertain features => high step-size
  - Certain features => low step-size

Measuring uncertainty

- Observed error = estimation error + approximation error + noise
- How to separate out estimation error?
  - Kalman filter
  - Linear regression
  - Gradient descent
  - Hierarchical bounds

Hierarchical bounds

- If feature B is a child of feature A (B => A)
  - Approximation error at A > B
  - Estimation error at B > A
  - Noise at B = A
- Use these bounds to track error intervals

Experimental setup

- The world is binary, e.g. 01101000
- Features recognise binary patterns
  - 1-bit features: e.g. **1*****, *****0**
  - 2-bit features: e.g. **10****, ****00**
  - 3-bit features: e.g. *110****, ***100**

Using uncertainty

- Diagonal Kalman filter assigns step-size in proportion to uncertainty
- Are there better strategies for hierarchies?
  - Weighted uncertainty
  - Resource pot
- Does floating average resolve this issue?

Experimental setup

- Target really is linear in features
  - $z = \sum w_j{}^{*} x_j$
  - $w_j{}^{*} \sim N(0, 2^{-l_j})$
- No added noise
- States (binary sequences) selected with uniform probability

Experimental setup

- 32 bit world
- All 1-bit to 8-bit features used in target
- Different subsets of features used during learning:
  - = n-bit features
  - ≥ n-bit features
  - ≤ n-bit features

Some results

Scaling up to Computer Go

- Same form of features, same issues
- Temporal sequences
- Non-stationary target
- Less structured target
- Location invariant features