Increased Discrimination in Level Set Methods with Embedded Conditional Random Fields

**Dana Cobzas**  
*University of Alberta*

**Mark Schmidt**  
*University of British Columbia*

### Introduction
- We want to use training data to build an automatic segmentation tool.
- Conditional random fields (CRFs):
  - discriminative model
  - models neighbor’s correlation
  - feature-based edge regularization
  - Markov assumption on labels
- Level set segmentation:  
  - generative model
  - assumes neighbor independence  
  - data-based edge regularization  
  - allows non-Markov priors
- We embed CRFs within a level set framework:  
  - a conditional level set method  
  - a CRF that allows non-Markov priors

### Conditional Random Fields
- CRFs model the conditional probability of the labels $Y$ given features $f(X)$

$$P(Y|X)\triangleq \frac{1}{Z} \exp\left(\sum_{i \in N} y_i w_i f_i(X) + \sum_{i \neq j \in E} y_i y_j f_{ij}(X)\right)$$

- Parameter estimation:
  - is jointly convex in $w$ and $v$
  - is efficient using a conditional pseudo-likelihood
  - is discriminative; there is no image model $P(X)$
  - models correlations between neighboring pixels learns edge regularization related to labels

### Level Set Segmentation
- Represent contour implicitly as the zero level set of an embedding function
- Minimize the energy by solving the Euler-Lagrange equations

### Continuous-Domain CRFs
- The associative CRF can be embedded into a continuous model that has the same energy:

$$E(\Phi) = \int_{\Omega} -H(\Phi) \log p_1(f(x), w) - (1 - H(\Phi)) \log p_2(f(x), w) + v |\nabla H(\Phi)| g(X, \alpha)$$

- Energy functional:

$$E(\Phi) = \int_{\Omega} -H(\Phi)(w^T f) + (1 - H(\Phi))(w^T f)$$

- Euler-Lagrange equations:

$$\frac{\partial E}{\partial \Phi_i} = -2\delta(\Phi^T f) + \delta(\Phi) \left( \sum_j v_j \gamma_j \Phi_j \right)$$

Training:
- Given: a set of images $X_1, X_2, \ldots, X_t$
- Extract features $f(X_1), f(X_2), \ldots, f(X_t)$
- Compute optimal node and edge parameters $\{w, v\}$ by maximizing the constrained pseudo-likelihood of the CRF

Segmentation:
- Given: one image $X$
- Extract features $f(X)$
- Compute segmentation by evolving a curve driven by the Euler-Lagrange equations

### Shape Priors
- We can add a non-Markov shape prior to the continuous CRF as an extra term in the energy:

$$E_s(\Phi) = \int_{\Omega} \beta H(\Phi) (s\Phi - \Phi(A(x)))^2 dx$$

$A(x)$ is an affine transformation with scale $s$ of the prior level set $\Phi_s$, and $\beta$ is the shape regularization strength.

### Discussion
- Unlike most work on level set methods, we require no manual initialization or parameter tuning, and do not need a generative model of the image.
- Other non-Markov terms can easily be added, such as the intensity inhomogeneity field.