CS 341, Fall 1999 due Friday Dec. 3

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ASSIGNMENT 5

Do not copy from others, and acknowledge your sources.

1. [10 points] The Tautology problem is: given a Boolean formula (using $\vee$, $\wedge$, and $\neg$), is it a tautology—i.e. does every truth value assignment satisfy it? Prove that the Tautology problem is in co-NP.

2. [10 points] Recall that a Boolean formula is 3-CNF if it is in conjunctive normal form and has at most 3 literals per clause. Recall that 3-SAT is the problem of deciding whether 3-CNF Boolean formula has a satisfying truth-value assignment. Let FIND-3-SAT be the problem of *finding*, if there is one, a satisfying truth-value assignment for a 3-CNF Boolean formula. Give a polynomial time Turing reduction from FIND-3-SAT to 3-SAT.

For problems 3 and 4, you may assume the NP-completeness of SAT, 3-SAT, Independent Set, Clique, Vertex Cover, Subset Sum, and Hamiltonian Cycle.

3. [10 points] Prove the following problem is NP-complete: given a directed graph with integer weights on the edges, and two vertices s and t, and an integer k, is there a simple path from s to t of weight at most k.

4. [10 points] Prove the following problem is NP-complete: given a set of n natural numbers can it be partitioned into two subsets with equal sums.

5. [10 points] The greedy heuristic for Vertex Cover repeatedly chooses a vertex of highest degree and deletes all edges incident to it, until no edges are left in the graph. Show that this approximation algorithm can produce a vertex cover more than twice the minimum size.

6. BONUS QUESTION. [5 points] 2-SAT is the problem of determining whether a 2-CNF Boolean formula is satisfiable. Give a polynomial time algorithm for 2-SAT or prove that it is NP-complete.