CMPUT 340—Introduction to Numerical Methods

Assignment 4

Winter 2007
Department of Computing Science
University of Alberta

Due: *Thursday, April 12 at 23:59:59 local time*
Worth: 15% of final grade

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Note: This assignment is to be submitted electronically by using the Unix "astep" command. You need to submit the answers to all written questions in hard copy to the course drop box.

Part 1 (Conversion to first order ODEs—2%)

(a) (1%) Exercise 9.1 on Page 416.

(b) (1%) Exercise 9.2 on Page 416.

Part 2 (Stability—3%)

Exercise 9.4 on Page 417.

Part 3 (Euler method—5%)

(a) (2%) Write a Matlab function `[yend] = Euler(odefun,t0,tend,y0,stepsize)` which returns the value $\mathbf{y}_{end} = \mathbf{y}(t_{end})$ for the initial value problem specified by the first order ODE system $\mathbf{y}' = \mathbf{f}(t, \mathbf{y})$ starting at initial vector $\mathbf{y}_0 = \mathbf{y}(t_0)$ and ending at time t_{end}. Implement the basic Euler method to do this. Note that you need to supply a function `odefun` such that `feval(odefun,t,y)` computes the derivatives $\mathbf{f}(t, \mathbf{y})$.

(b) (3%) Use your `Euler` method to solve the modified version of Computer Problem 9.5 given below.

Part 4 (Simulating dynamical systems 2—5%)

Computer Problem 9.7 on Page 419.

Remember to submit all written answers in the course drop box.

Appendix: Exercise descriptions from the textbook

Exercise 9.1. (Page 416)

Write each of the following ODEs as an equivalent first-order system of ODEs:

(a) $y'' = t + y + y'$.

(b) $y''' = y'' + ty$.

(c) $y''' = y'' - 2y' + y - t + 1$.

Exercise 9.2. (Page 416)

Write each of the following ODEs as an equivalent first-order system of ODEs:

(a) Van der Pol equation:
$$y'' = y'(1 - y^2) - y.$$

(b) Blasius equation:
$$y''' = -yy''.$$

(c) Newton's Second Law of Motion for the two-body problem:

$$\begin{aligned} y_1'' &= -GMy_1/(y_1^2 + y_2^2)^{3/2}, \\ y_2'' &= -GMy_2/(y_1^2 + y_2^2)^{3/2}. \end{aligned}$$

Exercise 9.4. (Page 417)

Consider the ODE $y' = -5y$ with initial condition $y(0) = 1$. We will solve this ODE numerically using a step size of $h = 0.5$.

(a) Are solutions to the ODE stable?

(b) Is Euler's method stable for this ODE using this step size?

(c) Compute the numerical value for the approximate solution at $t = 0.5$ given by Euler's method.

(d) Is the backward Euler method stable for this ODE using this step size?

(e) Compute the numerical value for the approximate solution at $t = 0.5$ given by the backward Euler method.

Modified Computer Problem 9.5. (Adapted from Page 419)

The following system of ODEs models nonlinear chemical reactions

$$\begin{aligned} y_1' &= -\alpha y_1 + \beta y_2 y_3, \\ y_2' &= \alpha y_1 - \beta y_2 y_3 - \gamma y_2^2, \\ y_3' &= \gamma y_2^2, \end{aligned}$$

where $\alpha = 4 \times 10^{-2}$, $\beta = 10^4$, and $\gamma = 3 \times 10^7$. Starting with initial conditions $y_1(0) = 1$ and $y_2(0) = y_3(0) = 0$, integrate this ODE from $t = 0$ to $t = 3$. Plot each of y_1, y_2, and y_3 as a function of t, and also plot each of the trajectories $(y_1(t), y_2(t))$, $(y_1(t), y_3(t))$, and $(y_2(t), y_3(t))$ as a function of t, each on a separate plot. Try perturbing the initial values by a tiny amount and see how much difference this makes in the final value of $\mathbf{y}(3)$.

(Note that you will have to experiment with the `stepsize` parameter in `Euler` to balance the run time and accuracy for solving this problem.)

Computer Problem 9.7. (Page 419)

An important problem in classical mechanics is to determine the motion of two bodies under mutual gravitational attraction. Suppose that a body of mass m is orbiting a second body of much larger mass M, such as the earth orbiting the sun. From Newton's laws of motion and gravitation, the orbital trajectory $(x(t), y(t))$ is described by the system of second-order ODEs

$$\begin{aligned} x'' &= -GMx/r^3, \\ y'' &= -GMy/r^3, \end{aligned}$$

where G is the gravitational constant and $r = (x^2 + y^2)^{1/2}$ is the distance of the orbiting body from the center of mass of the two bodies. For this exercise, we choose units such that $GM = 1$.

(a) Use a library routine to solve this system of ODEs with the initial conditions

$$x(0) = 1 - e, \qquad y(0) = 0,$$
$$x'(0) = 0, \qquad y'(0) = \left(\frac{1+e}{1-e}\right)^{1/2},$$

where e is the eccentricity of the resulting elliptical orbit, which has period 2π. Try the values $e = 0$ (which should give a circular orbit), $e = 0.5$, and $e = 0.9$. For each case, solve the ODE for at least one period and obtain output at enough intermediate points to draw a smooth plot of the orbital trajectory. Make separate plots of x versus t, y versus t, and y versus x. Experiment with different error tolerances to see how they affect the cost of the integration and how close the orbit comes to being closed. If you trace the trajectory through several periods, does the orbit tend to wander or remain steady?

(b) Check your numerical solutions in part a to see how well they conserve the following quantities, which should remain constant:

Conservation of energy:

$$\frac{(x')^2 + (y')^2}{2} - \frac{1}{r},$$

Conservation of angular momentum:

$$xy' - yx'.$$

(Note: you can use any of the ODE solvers in "`help funfun`" that you wish to solve this question, but please document your choice.)