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BEYOND THE DISPLACEMENT LIMIT: AN ANALYSIS OF SHORT-RANGE PROCESSES IN APPARENT MOTION

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Abstract—If a circle of random dots is presented in two successive displays in which the second is rotated in relation to the first, then observers are able to accurately discriminate the direction of apparent rotation as long as the rotation is small. Rotations beyond this short-range apparent motion can produce the impression of motion in the reverse direction. The performance in identifying the direction of rotation further depends on the eccentricity of stimulation and the density of the random dots. Simulations of the experiments using the Marr and Ullman model of motion detection are in good quantitative agreement with the data except for low dot density patterns and large displacements. In these situations perception seems to be dominated by the operation of long-range processes.

Apparent motion   Motion detection   Short-range processes   Marr and Ullman model

INTRODUCTION

In recent years there has been an increasing interest in the investigation of "random dot kinematograms" in which patterns of randoms dots are displaced in apparent motion (Anstis, 1970; Braddick, 1974; Lappin and Bell, 1976; Baker and Braddick, 1982a, 1982b; Chang and Julesz, 1983a, 1983b). The perception of regular patterns in these displays is mediated solely by motion detection processes since any relevant static cues are missing. An analogous experimental scheme has already been fruitfully employed in stereopsis (Julesz, 1971; Marr and Poggio, 1976, 1979; Grimson, 1981).

Experimental results indicate that the motion detection processes operating in random dot kinematograms have to be distinguished from those responsible for the perception of classical apparent motion stimuli (Wertheimer, 1912; Kolers, 1972; Ullman, 1979). The differences between the two processes, called short-range process and long-range process have been extensively discussed by Braddick (1980). But although the distinctions between the two processes seem fairly clear the nature of their operation is not. This is true not only for the question on the characteristics of the input and output of these processes but also for the question on what computational scheme these processes employ. Models of short-range processes usually assume some kind of local correlational operation on a rather early, possibly retinal representation of the visual input. The long-range processes, on the other hand, seem to operate on a more advanced, cortical representation of the visual input and their operation seems to be best described in terms of maximizing some similarity metric between tokens of the visual representation (Ullman, 1979).

The experimental investigation of short-range processes is complicated by the fact that they operate at the base of a hierarchy of processes that compute an increasingly advanced description of the visual input. Characteristics of the motion detection processes can thus be concealed by the operations of later processes. Additional difficulties arise from the fact that short-range processes are investigated with a variety of experimental paradigms such as identifying the motion of one coherent pattern (motion detection task), identifying differently moving stimulus parts (segregation task) and identifying the form of differently moving stimulus parts (form discrimination task). Differences between results obtained with the various paradigms suggest that they may reflect different levels of visual processing (Chang and Julesz, 1983a).

Using a form discrimination task Braddick (1974) showed that short-range processes operate only for short ISIs and small displacements (roughly up to 15 min of visual angle). With larger displacements performance degrades to chance level. In his experiments Braddick varied the size of the elements of the random dot pattern while keeping the retinal angle of the whole pattern constant and hence co-varied the number of elements. He found that this variation had no influence on performance if the displacement of the pattern was measured in visual angle and not in number of elements. This result was replicated and also found with a segregation task by Baker and Braddick (1982b). Braddick (1974) concluded that "the appearance of coherent motion is limited by the absolute spatial displacement of partner dots, and not by the number of pattern elements intervening between partners' positions" (p. 521f).

Using a motion detection task Lappin and Bell (1976) also found decreasing performance with increasing displacement and ISI. In contrast to Braddick they found that performance increased with increasing number of elements in the pattern, roughly proportional to the square root of this number. However, the results of their experiment cannot be directly compared with those of Baker and Braddick because they varied the spacing and the number of elements in the patterns, thus co-varying the retinal angle of the whole pattern. Lappin and Bell concluded that the operation of the short-range process was best described in terms of a cross-correlational model. Their model predicts that the spacing of elements should have no influence on performance when the displacement is measured in number of elements (scaling invariance), a prediction which was however not completely confirmed (Lappin and Bell, 1976, p. 166). Performance tended to be higher for the smaller spacing of elements. In a comparable experiment Baker and Braddick (1982b) investigated motion detection with patterns having different dot spacings and dot numbers. They also found increasing performance with increasing area of the patterns but again no scaling invariance: the performance for smaller spacings was better. A similar trend can be found in Chang and Julesz (1983a, Table 3).

Baker and Braddick explain the area effect with the fact that larger patterns stimulate motion detectors at larger eccentricities having larger receptive fields. This hypothesis was supported by the finding that occluding central portions of the patterns had no influence on performance. On the other hand Chang and Julesz (1983a) did not find an increase of the displacement limit with eccentric stimulation. One possible explanation for this discrepancy might be that Baker and Braddick used a motion detection task while Chang and Julesz used a segregation task. The eccentricity hypothesis is further supported by the investigations on the fine-grain movement illusion (Foster *et al.*, 1981; Biederman-Thorson *et al.*, 1971) which also shows a clear eccentricity effect and which seems to reflect the operation of the same mechanism as the short-range process.

For several reasons it is difficult to decide between Braddick's eccentricity hypothesis and the predictions derived from the cross-correlational model of the other authors. First they vary different combinations of stimulus parameters which makes it difficult to compare the experimental results on a quantitative level. Second they use different experimental tasks, which may involve different levels of visual processing, and it seems that many discrepancies between experimental results can be attributed to this fact.

A further problem of these experiments is that the notion of the displacement limit is not very clear. The performance in identifying the direction of motion or in identifying the form of differently moving patches does not break down abruptly beyond a certain displacement limit but degrades slowly, depending on various factors such as dot density and eccentricity. The displacement limit is usually identified with the point of deviation from perfect performance within a certain tolerance limit. Given the relatively slow degradation in performance, the influence of the short-range processes then is effective far beyond the displacement limit.

Yet inferences about properties of the short-range processes are complicated by further facts. Due to the size of the stimuli local motion signals can be integrated over a large area. Thus the process of motion detection cannot be separated from the mechanism of motion integration. Yet we have only little knowledge about how local motion information could be integrated (Ullman and Hildreth, 1983).

There is evidence that the motion detection mechanism depends on receptive field size and thus motion information is integrated over inhomogenous receptive field sizes with extended stimuli. Relating the displacement limit of the short-range processes to the operation of the local motion detection mechanisms therefore becomes a non-trivial task.

To avoid some of these problems we investigate the behavior of the short-range processes with simplified stimuli, rotating circles consisting of random dots. Since receptive field size varies with eccentricity (Fischer, 1973; Wilson and Bergen, 1979) but is otherwise independent of retinal position we can keep the receptive field size of the relevant motion detectors approximately constant when subjects fixate the center of the circles. This simplifies finding a relation between eccentricity and the displacement limit. Furthermore our stimuli can be regarded as being essentially one-dimensional for the local motion detectors. We thus can avoid having to include a model of two-dimensional motion integration. Finally we use a pure motion detection task that does not involve the computation of motion contours or the interpretation of motion contours as in other paradigms. We thus expect inferences about the properties of the short-range processes to be easier and more direct.

METHODS

Apparatus

All stimuli were presented on a HP 1321A CRT display (P4 phosphor) controlled by a Megatek MG-552 graphics processor from display instructions held in the memory of a Data General NOVA 3/12 minicomputer. Subjects responses were recorded with push-buttons and saved together with all relevant stimulus parameters on the computer.

Stimuli

All stimuli generated on the screen were random dot circles having a diameter of 8.7 cm (17.4 cm) thus subtending a visual angle of 5 deg (10 deg) at the viewing distance of 1 m. The circles consisted of dots whose size was approximately 0.7 mm or 2.5′ of visual angle. The dot positions were spaced at 1.9 deg distance or 5′ of visual angle yielding 188 (377) dot positions on a circle. The probability that a dot was drawn at any one position was 50%. The stimulus patterns were generated randomly and a given pattern was never presented more than once.

The dot luminance was 0.3 cd/m^2, the background of the screen was less than 0.01 cd/m^2 (measured with a Spektra Pritchard photometer model 1980A-PL) and the general background illumination was approximately 0.6 lm. The relatively dark environment was necessary to keep reflections on the screen surface at a satisfactory minimum and the relatively low stimulus luminance was necessary to avoid visual persistence effects.

Each circle was presented for 100 msec with an interframe interval of 20 msec determined by the refresh interval of 20 msec, giving a total stimulus duration of 200 msec. A zero inter-stimulus interval was chosen in order to favor the operation of the short-range process (Braddick and Adlard, 1978). The second circle was identical to the first but rotated clockwise or counterclockwise by an integer multiple of the dot spacing.

Procedure

A forced-choice paradigm was used to determine subjects performance in identifying the clockwise/counterclockwise rotation of the random dot circles. Each subject gave 100 judgments per displacement. Each session was initiated with a 20 min dark adaptation period followed by 50 warmup trials.

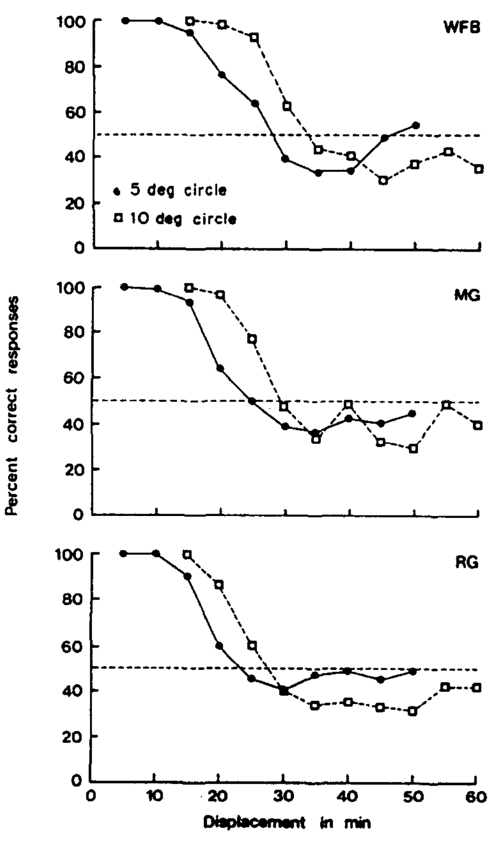

Fig. 1. Performance in identifying the clockwise/counterclockwise rotation of random dot circles as a function of their rotational displacement in min of visual angle. Data are from 3 subjects, each judging the rotation of a 5 deg and a 10 deg diameter circle. Each data point is based on 100 judgments.

Each trial started with an auditory signal immediately followed by a fixation cross (30′ visual angle) in the middle of the screen. After a delay of 500 msec the stimulus was presented. The subjects then indicated the perceived direction of rotation of the circle by pressing one of two buttons. In case of an ambiguous percept the subjects were instructed to indicate the dominant direction of rotation. After an intertrial interval of 5 sec the next trial was started.

The direction of rotation and the amount of displacement were chosen randomly with equal probability. Subjects gave 100 judgments per experimental condition, i.e. per displacement and circle size.

EXPERIMENT 1

The first experiment was designed to investigate the effect of eccentricity of stimulation on the displacement limit of the short-range processes. The stimuli used were circles having a diameter of 5 deg and 10 deg thus stimulating motion detectors at 2.5 deg and 5 deg eccentricity. The 5 deg circle was rotated by 1–10 dot positions equivalent to 5′–50′ visual angle and the 10 deg circle was rotated by 3–12 dot positions equivalent to 15′–60′ visual angle. Three subjects participated in this experiment. The results of this experiment are presented in Fig. 1.

Results

Performance with the larger circle is consistently superior to that with the smaller circle but both curves have the same general shape. For small displacements (up to 5′–10′ and 15′–20′ respectively) performance is practically perfect. For these conditions the subjects report the impression of a clear, unambiguous motion of the circles. For larger displacements performance slowly degrades but does not just reach chance level. Rather we find a performance level significantly below chance level for displacements in the range of 30′–45′ for the 5 deg circle and of 35′–60′ for the 10 deg circle. Displacements in this range thus produce an illusory motion percept in the reverse direction of the actual physical displacement. This motion illusion is not very strong which is related to the fact that for displacements in this range the motion percept is no more clear and unambiguous, but is rather comparable to visual "noise" whose motion may dominate more or less in one direction. By further increasing the displacements beyond this range, performance reaches chance level asymptotically.

Discussion

The performance curve for the 10 deg circle is systematically better than the performance curve for the 5 deg circle. The curves of Fig. 1 look the same when the displacement is measured in number of elements since the dot spacing is the same for both circles. The improved performance for the larger circle can be attributed to two possible factors, one is the increased eccentricity of stimulation and the other is the increased number of dots in the pattern (the larger circle has twice as many dots on the average). This problem will be further investigated in Experiment 2.

In a similar experiment Bell and Lappin (1979) had investigated the apparent rotation of random dot patterns. In contrast to the present experiment they used circles whose whole area was covered with random dots. A comparison of the two studies shows several differences besides the common finding of decreasing performance with increasing displacement. Bell and Lappin did not find significant differences in performance for circles of different size. They also found no perception of reversed rotation. This could be an effect of not using large enough displacements. However, pilot experiments of ours with the same stimuli and larger displacements also showed no motion illusion. Performance just degraded to chance level. The motion illusion found with circle lines may be due to the nature of these stimuli whose essentially one-dimensional character may reduce the effectiveness of the processes integrating the local motion signals. We believe that these processes level out false motion signals generated by the motion detectors, be it due to the fact that they process the motion signals in all directions or to the fact that they integrate motion signals over different receptive field sizes. Both possibilities are excluded with our stimuli. A definite answer requires, however, more detailed knowledge of the motion integration process.

More interesting is the question why the short-range system generates false motion signals. Among the models we have considered the Marr and Ullman (1981) model has been particularly successful in predicting the experimental data and in our further discussion we will concentrate on this model. The Marr and Ullman model is formulated for motion analysis of two-dimensional visual input but with our stimuli it can be simplified to one dimension by making the analysis along the perimeter of the circles. The Marr and Ullman model involves three major steps in the generation of motion signals. The first step, the construction of the Primal Sketch, involves the convolution of the image with a G'' profile.

$$G''(x, \sigma) = (1 - x^2/\sigma^2)\exp(-x^2/2\sigma^2)$$

and the localization of all zero-crossings in the convolution (for details of the model and its simulation consult the appendix). In a second step the time derivative of the Primal Sketch is computed. In the third step local units compute a motion signal for each zero-crossing from the signs of the signals $S = G''*I$ and $T = \partial S/\partial t$ at and nearby each zero-crossing.

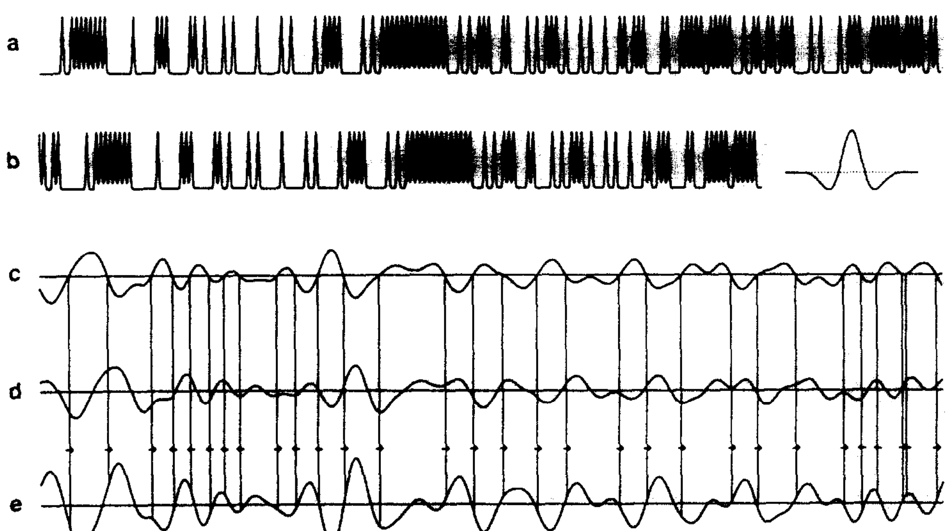

Fig. 2. Illustration of the operation of the Marr and Ullman model on the circle stimuli. In our simulations the analysis is performed along the perimeter of the circles. Figures 2(a) and 2(b) show the intensity profiles I_1 and I_2 of the two circles presented successively. Figures 2(c) and 2(d) show the S_1 and S_2 profiles, i.e. the products of convolving I_1 and I_2 with a G'' profile. The G'' profile is shown in appropriate size in the inset in 2(a). Figure 2(e) shows the temporal derivative T of the S_1 signal approximated by the difference $S_2 - S_1$. The vertical lines connecting the S_1 and T signal indicate the zero-crossing positions of the S_1 signal and the little arrows on the lines indicate the direction of motion signalled by the Marr and Ullman motion detectors. The curves are based on a 5 deg circle using a filter size of 13.78′. The circles dot spacing is 5′ and the dot probability is 50%. The second circle is rotated by 6 dot positions equivalent to 30′ of visual angle.

The application of the Marr and Ullman model requires the determination of the size of the G'' filter which we have done using the data of Wilson and Bergen (1979). Assuming that the performance limits of the short-range processes are determined by the coarsest channel, Wilson and Bergen's U-channel, we get a filter size of $\sigma(0) = 10.5'$ foveally where σ is the standard deviation of the Gaussian (Wilson and Bergen, 1979, p. 22, 24). Using the Wilson and Bergen eccentricity factor

$$\sigma(\epsilon) = \sigma(0)\cdot(1 + 0.125\cdot|\epsilon|)$$

where ϵ is the eccentricity in deg, we get $\sigma(2.5) = 13.78'$ for the 5 deg circle and $\sigma(5) = 17'$ for the 10 deg circle.

The operation of the Marr and Ullman motion detectors on our stimuli is illustrated in Fig. 2. Figure 2(a) and (b) show the intensity profiles I_1 and I_2 along the perimeter of the two circles presented successively, Fig. 2(c) and (d) show the corresponding Primal Sketch profiles S_1 and S_2. Figure 2(e) shows the profile $T = \partial S/\partial t$ which is approximated by the difference $S_2 - S_1$. The vertical lines connecting the S_1 and T profiles indicate the location of the zero-crossings and the little arrows indicate the direction of motion signalled by the Marr and Ullman motion detectors. Note that the Marr and Ullman motion detectors signal only the sign of motion but not the velocity magnitude (although it could be estimated from the gradients of the S and T signals). The local motion signals are integrated into a global motion field by a motion integration process. We simply used the proportion of motion signals in the correct direction and the total number of motion signals (given by the number of zero-crossings) as a predictor of the probability that motion will be seen in the correct direction, i.e. the predicted probability of identifying the correct direction equals the proportion of motion signals in the correct direction.

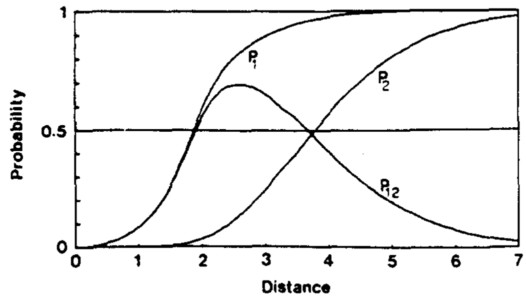

Fig. 4. Distribution of zero-crossing distances. Curves P_1 and P_2 give the probability that the distance between any zero-crossing and respectively its first and second neighbouring zero-crossing is less than or equal to d. $P_{12}(= P_2 - P_1)$ gives the probability that d lies between the first and second neighbouring zero-crossing. The curves were obtained in simulations using 5 deg circles with a dot probability of 50% and using a filter size of $\sigma = 13.78'$. The curves are based on approximately 34,000 zero-crossings.

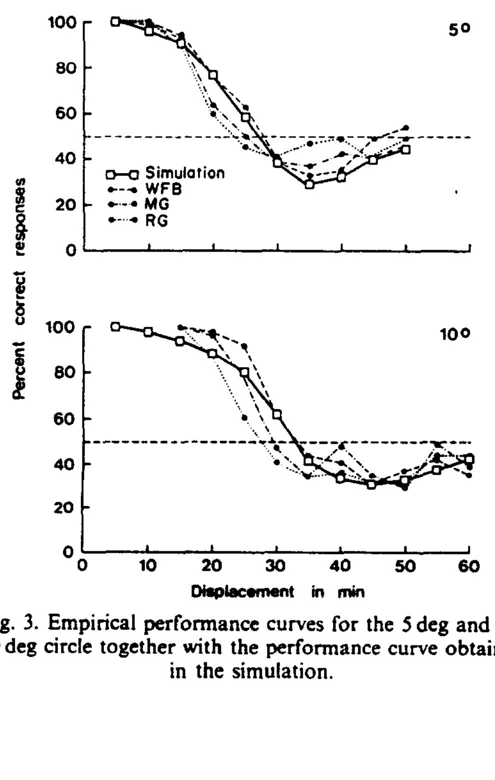

Fig. 3. Empirical performance curves for the 5 deg and the 10 deg circle together with the performance curve obtained in the simulation.

All experimental conditions were simulated with the Marr and Ullman model, using the same stimuli that were presented to the subjects. The results are presented in Fig. 3 which shows the experimental curves from Fig. 1 and the simulated curves for both conditions of one subject (W.F.B.). The simulated curves obtained for the different subjects do not vary much due to the large number of stimuli used. (The differences in predictions derive simply from random variations in the patterns presented to each subject.)

The results of the simulation are very close to the empirical curves given that the model contains no free parameters! The simulated performance curves are generally too low for small displacements which we attribute to the missing motion integration part of the model. Note that the model predicts a less-than-perfect performance for a stimulus even if only one local motion signal is false. Any reasonable motion integration model could correct such errors. We have deliberately not done so in order to show the basic strength of the Marr and Ullman model. A closer look at the Marr and Ullman model shows why the false motion signals are generated. Let Z_0, Z_1 and Z_2 be adjacent zero-crossings in the Primal Sketch of a pattern with the distances $d(Z_0, Z_1) = d_1$ and $d(Z_0, Z_2) = d_2$. If the pattern is displaced by an amount $d < d_1$ the motion signal generated at Z_0 is correct but for a displacement $d_1 < d < d_2$ the sign of the T-signal and consequently the sign of the motion signal generated at Z_0 changes. The proportion of correct motion signals generated for any displacement d therefore depends on the distribution of zero-crossings.

Figure 4 shows the probability distributions for the distance $d_1 = d(Z_0, Z_1)$ in curve P_1, the distance $d_2 = d(Z_0, Z_2)$ in curve P_2 and the probability that distance d lies between Z_1 and Z_2 in curve P_{12} which is simply the difference between P_2 and P_1. The horizontal axis is scaled in units of the Gaussian filter (σ). The curve P_{12} shows that distances between 1.9 and 3.6σ have a probability $p > 0.5$ of lying between the first and second zero-crossing and displacements in that range are thus likely to lead to a false motion percept. These curves are only an approximation to what actually happens because the sign reversal of motion signals repeats at every zero-crossing. Including more and more zero-crossings in the analysis eventually leads to the curve obtained in the simulation of the experiment.

The probability curves of Fig. 4 have been obtained in simulations using 5 deg circles and a filter size of $\sigma = 13.78'$ and are based on approximately 34,000 zero-crossings. Marr and Poggio (1979) have used the Longuet-Higgins (1962) approximation to the distribution of zero-crossings but it is not very useful in our situation since the approximation is already well off the empirical values in the range we are interested in (2–5σ), and since it is not appropriate for low dot density patterns.

The successful application of the Marr and Ullman model leads to the question whether other models could also predict the motion illusion found with random dot circles. Foster (1969, 1971) found a related effect with spatially-periodic anular stimuli and was able to predict the effect using a combination of low-pass filtering mechanism and the Hassenstein–Reichardt model. As far as we can see his model should also be applicable to our stimuli but we cannot judge how close the quantitative predictions would be. The advantage of the Marr and Ullman model lies in the fact that it is completely specified computationally and could be applied in our situation with a minimal number of assumptions. But one has to admit that it is difficult to find test cases that would allow a clear decision between different models of motion perception (see e.g. Marr and Ullman, 1981).

EXPERIMENT 2

In Experiment 1, subjects showed a consistently superior performance with the 10 deg circle as compared to the 5 deg circle. This result can be considered an effect of retinal eccentricity as it is done by Baker and Braddick (1982b). The Marr–Ullman model successfully predicts the data for the two circles by using two different filter sizes, both of which have been computed according to the linear relationship between retinal eccentricity and size of receptive fields found by Wilson and Bergen (1979). However, it could be argued that the difference in performance with the two circles is only due to the difference in number of points on the circle line since we co-varied eccentricity and number of points. If the latter hypothesis is true then presenting the same circle at different eccentricities should not affect the performance curve.

METHODS

The experimental setup and procedure of this experiment were the same as in Experiment 1. The stimulus was identical to the 5 deg circle except that the centre of the circle was positioned at 2.5 deg eccentricity to the right. The circles were displaced by 1–10 dot positions. Three subjects participated in the experiment each giving 100 judgments per data point.

Results and discussion

Figure 5 shows the results obtained with the eccentric circle compared with the circle presented foveally in Experiment 1, for three subjects and a simulation (using the stimuli of subject W.F.B.). The results show that increased eccentricity of stimulation leads to increased performance in identifying the direction of motion for medium size displacements and to a shift of the range of displacements for which the motion illusion is likely to be seen. This is in accordance with the results of Baker and Braddick (1982b) but contrasts with the findings of Chang and Julesz (1983a). One possible explanation for this discrepancy could be that both, Baker and Braddick and we, used a motion detection task while Chang and Julesz obtained their result using a segregation task.

The simulations using the Marr and Ullman model were identical to those for Experiment 1 except that the size of the G'' filter was continuously adjusted at each retinal position of the circle according to the Wilson and Bergen equation. The predictions of the model agree well with the experimental results except that the predictions for small displacements are too low as in Experiment 1. Furthermore the model predicts a slightly weaker motion illusion for the eccentric circle while all subjects show a slightly stronger motion illusion.

EXPERIMENT 3

Baker and Braddick (1982b) have found that the displacement limit of the short-range processes remains invariant for a large range of dot densities. There are two limiting conditions to this dot density invariance principle. One occurs in paradigms employing a form detection or form discrimination task where the form of a patch of differently moving dots has to be determined. If the dot density is too low then the patch and the background consist of a few dots only and the critical percept, the motion contour between patch and background, breaks down. This problem does not arise if a pure motion detection task is used. However, patterns with very few dots may fulfill the conditions for the long-range processes to become effectively operational and then motion can be seen for very large displacements. But these conditions are not known and one precondition for

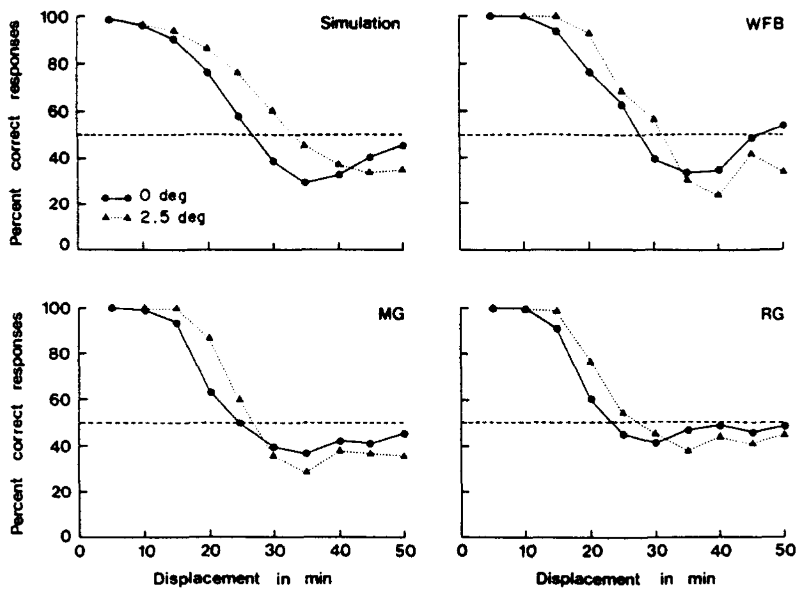

Fig. 5. Performance curve for 5 deg circles presented at 0 deg and 2.5 deg eccentricity (center of circle) for three subjects and the simulation using the stimuli of subject W.F.B. All data points are based on 100 judgments.

a proper investigation is a better knowledge about the output of the short-range processes for low density patterns. The third experiment was designed to investigate this question.

Method

The experimental setup and the procedure of this experiment were exactly the same as in Experiment 1. The stimuli used were 5 deg circles with dot probabilities of 25, 12.5 and 6.25% thus having 47, 23 and 12 dots on the average. The circles were displaced by 1–10 dot positions equivalent to 5′–50′ of visual angle. Three subjects participated in the study each giving 100 judgments per conditions and displacement.

Results and discussion

The results of Experiment 3 are presented in Fig. 6 together with the curves obtained in the simulation (using the stimuli of subject W.F.B.). The results

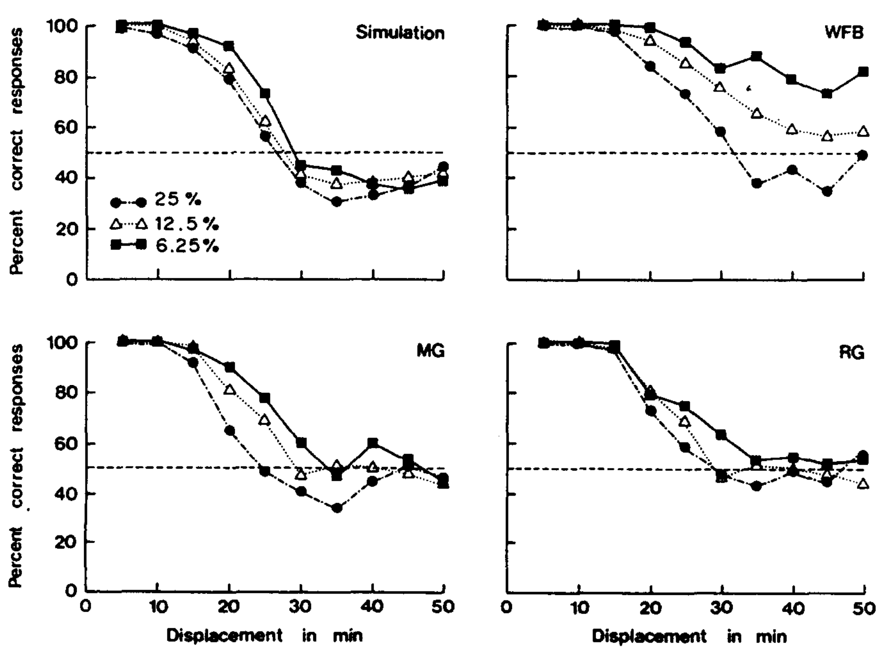

Fig. 6. Performance curves for circles with dot probabilities of 25, 12.5 and 6.25% for three subjects and the simulation using the stimuli of subject W.F.B. All data points are based on 100 judgments.

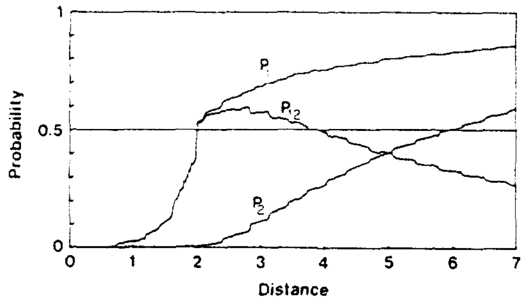

Fig. 7. Distribution of zero-crossing distances for circles with a dot density of 6.25%. The curves are analogous to those in Fig. 4. They were obtained in simulations using 5 deg circles with a filter size of $\sigma = 13.78'$ and are based on approximately 17.000 zero-crossings.

show that the displacement limit of the short-range processes—defined as the point of deviation from perfect performance—remains relatively stable over different dot densities, in accordance with the findings of Baker and Braddick (1982b). However, for displacements beyond this limit we find that the performance in identifying the direction of rotation improves with decreasing dot density. The motion illusion we found in Experiment 1 with large displacements is still present with patterns having a dot probability of 25% but disappears for patterns with a lower dot density.

Subjects differ strongly in the amount they improve in performance with decreasing dot density. While subject W.F.B. shows a strong improvement the others improve much less and in fact perform on chance level for large displacements. We suscept that this difference is due to a learning effect and informal experiments have provided supporting evidence for this hypothesis.

The simulation of the Marr and Ullman model also shows systematic improvement in performance with decreasing dot density but the effect is weak. In particular we find that the model predicts the motion illusion effect even for low density patterns and the strength of the motion illusion does not differ much between the 50 and the 6.25% dot density patterns. Lower dot densities lead to a greater variability in zero-crossing distances but it is mainly the region of very large displacements that is affected. Figure 7 which is analogous to Fig. 4 shows the cumulative probability distributions of the first (P_1) and second (P_2) zero-crossing for 6.25% dot probability patterns and the probability that the position d lies between the first and second zero-crossings (P_{12}). The curves were obtained using 5 deg circles and are based on approximately 17,000 zero-crossings. The notched shape of the curves is not due to sampling errors but results from the low dot density and the relatively large dot spacing. Comparing Figs 4 and 7 we see that the P_{12}-curve is not strongly affected by the lower dot density for the range of displacements we are interested in.

Comparing the effects of eccentricity of stimulation and dot density predicted by the Marr and Ullman model we find that the performance of the model is primarily influenced by eccentricity of stimulation and only secondarily by the stimulus pattern, i.e. dot spacing and dot density. Simulations have shown that decreasing dot spacing below the 5 min used in these experiments has virtually no effect on the predictions of the model. Only when the dot spacing is large (>20 min) or the dot probability low we find that the predictions of the model are affected. This agrees well with the empirical results: we find similar performance curves for 50 and 25% dot probability using a dot spacing of 5 min.

We tried several modifications of the Marr and Ullman model to improve on the performance for low density patterns. One variant involved the idea that the Marr and Ullman motion detectors do not signal motion if the strength of the T-signal is below a certain threshold. This modification, however, primarily affected the models performance for short displacements, leading to a strong degradation in performance. Another variant involved the idea that the Marr and Ullman motion detectors signal motion only at zero-crossings having a steep slope in the S-signal, an idea which is connected to Binford's (1981) idea of zero-crossings without significance. This variant affected, however, motion signals for all displacements in the same way. While these and other modifications of the Marr and Ullman model are certainly worth investigating for other reasons we now do not think that they will lead to better predictions for low dot density patterns.

We believe that the performance for low dot density patterns is primarily influenced by the operation of the long-range processes. If we take the dot density of random dot patterns to the extreme case of a single dot (or very few dots) we end up in the situation of classical apparent motion with perfect performance over many degrees of visual angle. The perception of these patterns is mediated by the operation of long-range processes. The circles with a dot probability of 6.25% are well within this range, given that they consist of only 12 dots on the average, whose apparent number may be reduced even further through grouping processes. On the other hand the long-range processes do not effectively operate for normal (high dot density) random dot patterns. Determining the relative contribution of the two systems to the perception of any pattern turns out to be difficult and the factors governing their relative dominance are only partially understood (Petersik and Pantle, 1979). This is mainly because we do not know what representation of the visual input the long-range processes operate on and thus under what conditions they can produce consistent motion signals. A proper investigation of these conditions requires a good understanding of the short-range process. The Marr and Ullman model has been shown to be a good starting point on this line.

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APPENDIX

The simulations of the experiments were done for each subject and each experimental condition using the same patterns that were presented to each subject. The simulation of a single experimental trial involved six steps.

(1) Compute the image intensity distribution of the circle along the perimeter. Dots were assumed to have a Gaussian intensity distribution (Watt and Morgan, 1983) with $\sigma = 1'$. The intensity distribution was computed in 20″ intervals.

(2) Convolve the image with the G'' filter

$$G''(x, \sigma) = (1 - x^2/\sigma^2)\exp(-x^2/2\sigma^2) = -d^2G/dx^2$$

where σ is the standard deviation of the Gaussian G. Based on the data of Wilson and Bergen (1979) and assuming that the limits of the short-range processes are determined by the coarsest, the U-channel, we used $\sigma = 13.78'$ for the 5 deg circle and $\sigma = 17'$ for the 10 deg circle.

The G'' profile is very close to the DOG profile

$$\mathrm{DOG}(x, \sigma) = \sigma^{-1}\exp(-x^2/2\sigma^2) - k^{-1}\sigma^{-1} \times \exp(-x^2/2k^2\sigma^2)$$

using $k = 1.6$. Note that Wilson and Bergen obtained $k = 3.1$ for the U-channel under T-stimulation. This value removes, however, the balance between the excitatory and inhibitory components leading to a d.c. offset in the convolution. This is difficult to interpret computationally since it strongly affects the position and number of zero-crossings obtained. The convolution $S = I*G$ was again computed in 20″ intervals. The convolution S_2 of the second circle was obtained by shifting S_1 by the displacement distance.

(3) Locate all zero-crossings of S_1 to within 20″.

(4) Compute the signal $T = \partial S/\partial t$ approximately by the difference $S_2 - S_1$.

(5) Determine the motion signals for all zero-crossings in S_1 from the sign of T at the zero-crossing position (Batali and Ullman, 1979) using the following rules:

If $T > 0$ then signal motion towards the negative side of S_1.

If $T < 0$ then signal motion towards the positive side of S_1.

If $T = 0$ then signal no motion (or ambiguous motion).

The last rule was not implemented in the simulation because we did not know what range of T-signals should be regarded as being effectively zero.

(6) The strength of the perceived motion in one direction is computed as the proportion of zero-crossings signalling motion in that direction to the total number of zero-crossings.