Model-Based RL

Reinforcement Learning Summer School

Martha White
University of Alberta and AMII

Comments for the lecture

- Please ask questions (this is a summer school)
- I will pause a few times and get you to answer questions/exercises
- Outcomes: you will
  - understand how models can be used to learn optimal values/policies
  - understand in-depth one strategy, called Dyna, for online setting
  - recognize some of the other ways models can be used

What is model-based RL?

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High-level Outline

- Part 1: Learning the optimal policy given the model (offline)
- Part 2: Moving to learned models (online)
  - Particularly looking at a formalism called Dyna
- Part 3: A brief discussion about other ways to use models

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Imagine we have the model

- Joint transition and reward dynamics

$$p(s', r \mid s, a)$$

- **Then**, we can learn **offline** without interacting with the world!

Bellman equations & Dynamic Programming to find the optimal policy

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$$q_*(s,a) = \sum_{s'} \sum_{r} p(s', r \mid s, a) \left[r + \gamma \max_{a'} q_*(s', a') \right]$$

Bellman equations & Dynamic Programming to find the optimal policy

- We can directly solve for the (optimal) action-values, using Bellman equations

$$q_*(s,a) = \sum_{s'} \sum_{r} p(s',r \mid s,a) \left[r + \gamma \max_{a'} q_*(s',a') \right]$$

$$q_{k+1}(s,a) = \sum_{s'} \sum_{r} p(s',r \mid s,a) \left[r + \gamma \max_{a'} q_k(s',a') \right]$$

called Value Iteration

Value Iteration, for estimating $\pi \approx \pi_*$

Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation

Initialize Q(s,a) = 0 for all s,a

Loop:
| $\Delta \leftarrow 0$
| Loop for each $s \in \mathcal{S}, a \in \mathcal{A}$
| $\quad v \leftarrow Q(s,a)$
| $\quad Q(s,a) \leftarrow \Sigma_{s',r} p(s', r \mid s,a)[r + \gamma \max_{a'} Q(s',a')]$
| $\quad \Delta \leftarrow \max(\Delta, |v - Q(s,a)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
$\pi(s) = \arg\max_a Q(s,a)$

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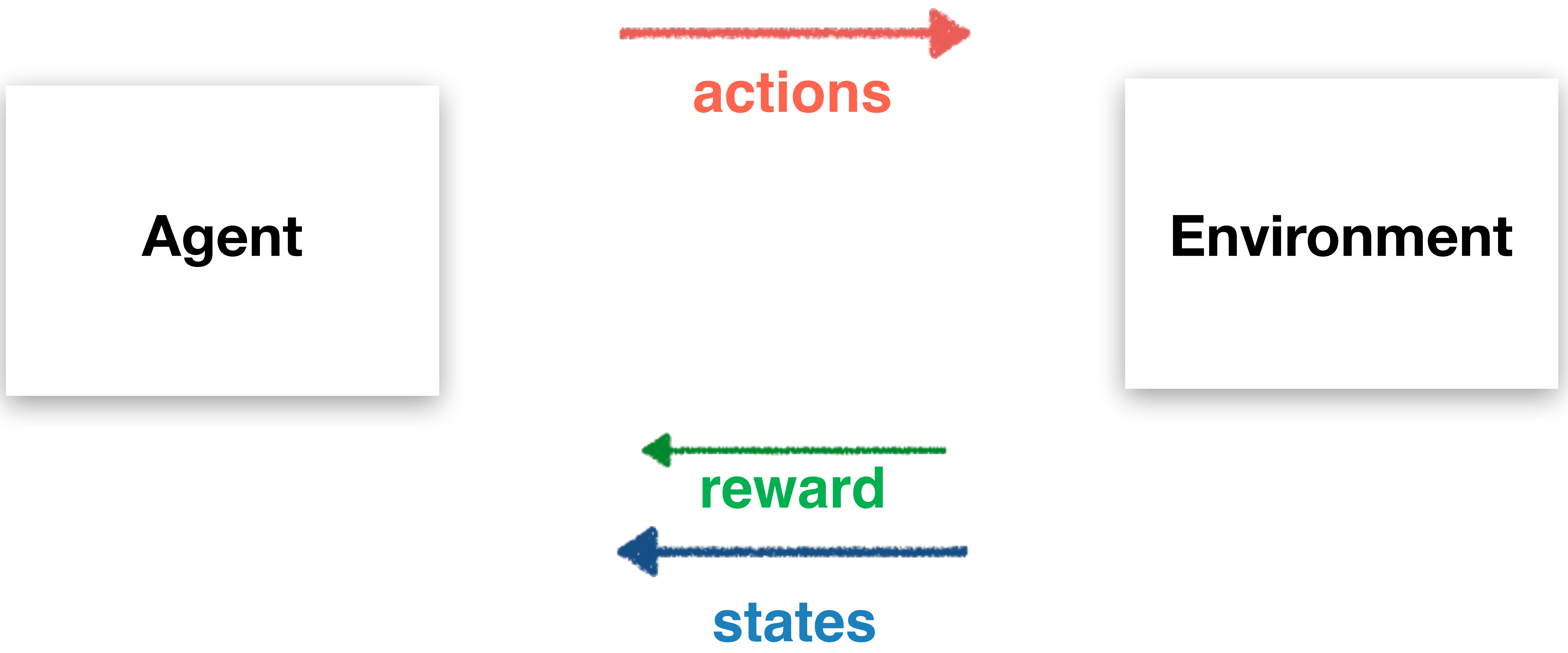
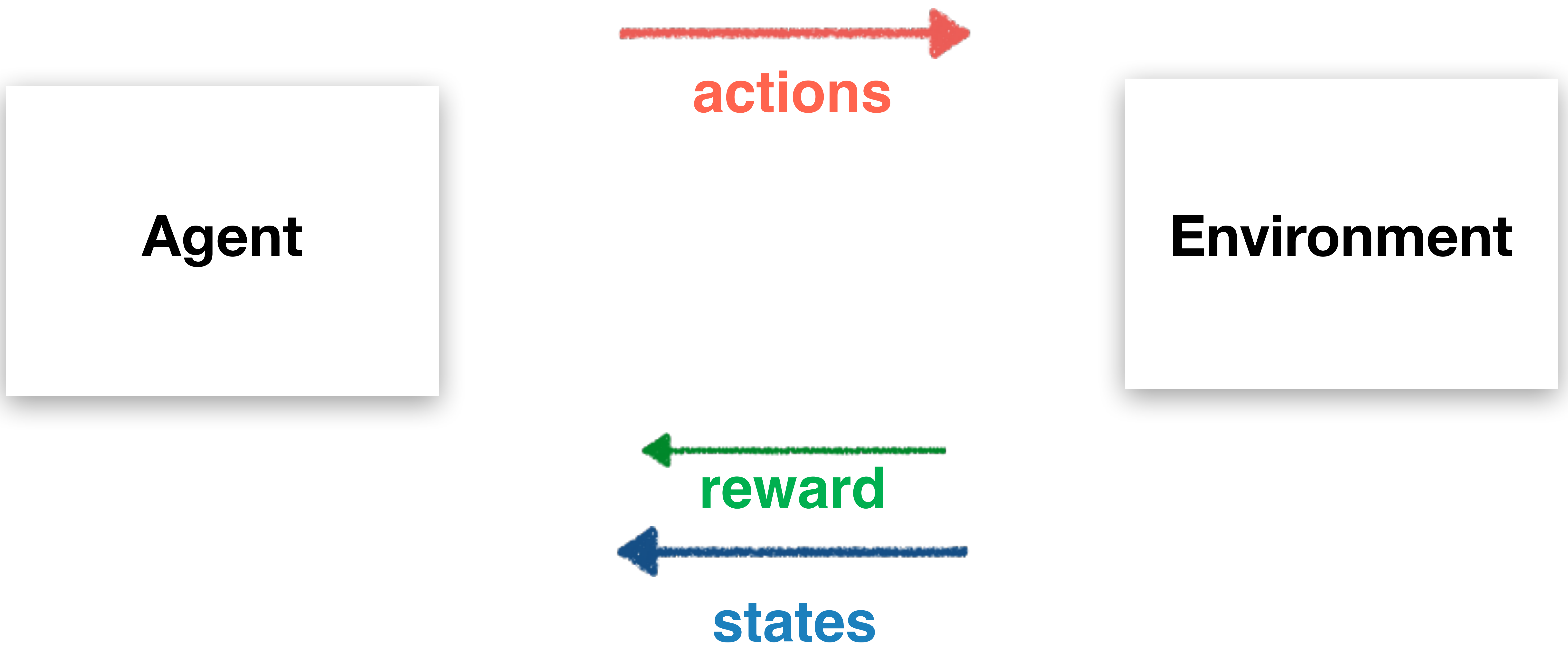
RL with learned models can use a similar approach to Dynamic Programming
but online

Online Reinforcement Learning



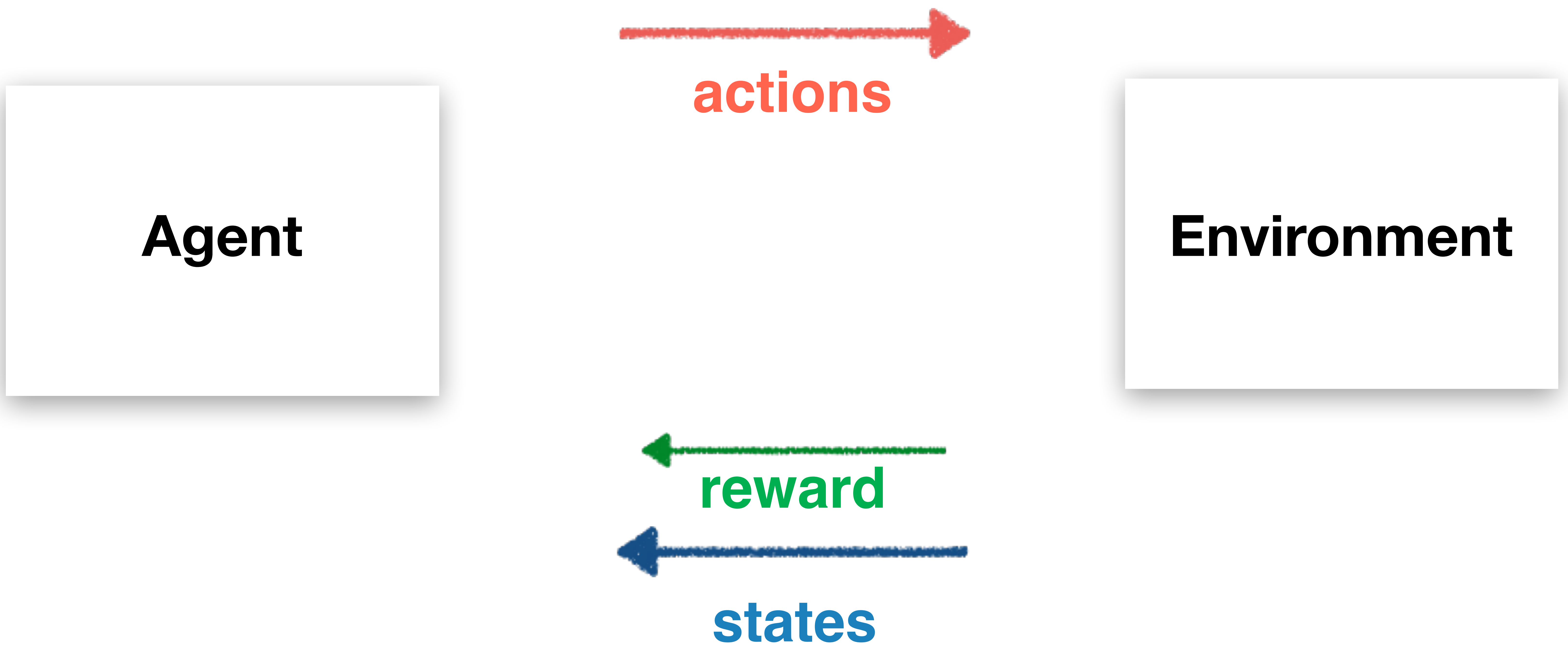
Online Reinforcement Learning

Online Reinforcement Learning



S_0 A_0

Online Reinforcement Learning



S_0 A_0 R_1 S_1

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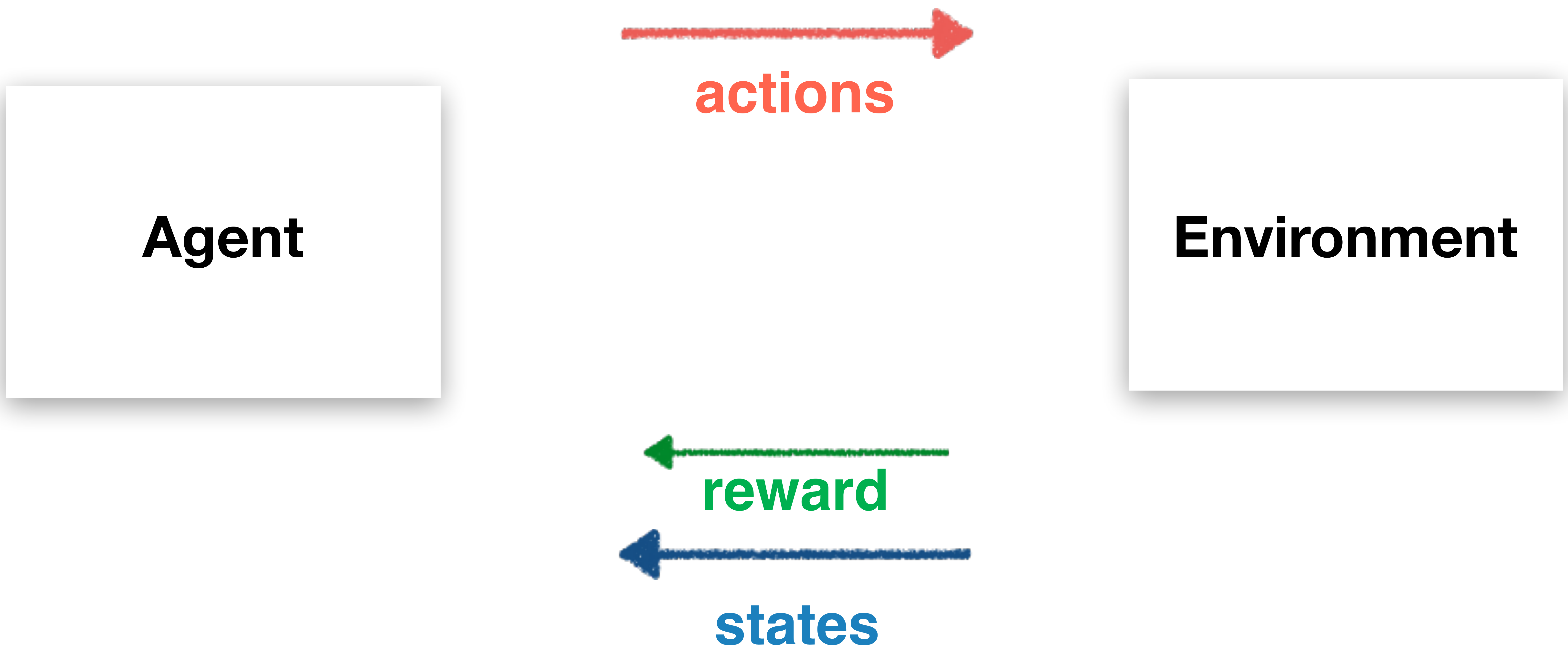

$S_0 \quad A_0 \quad R_1 \quad S_1 \quad A_1$

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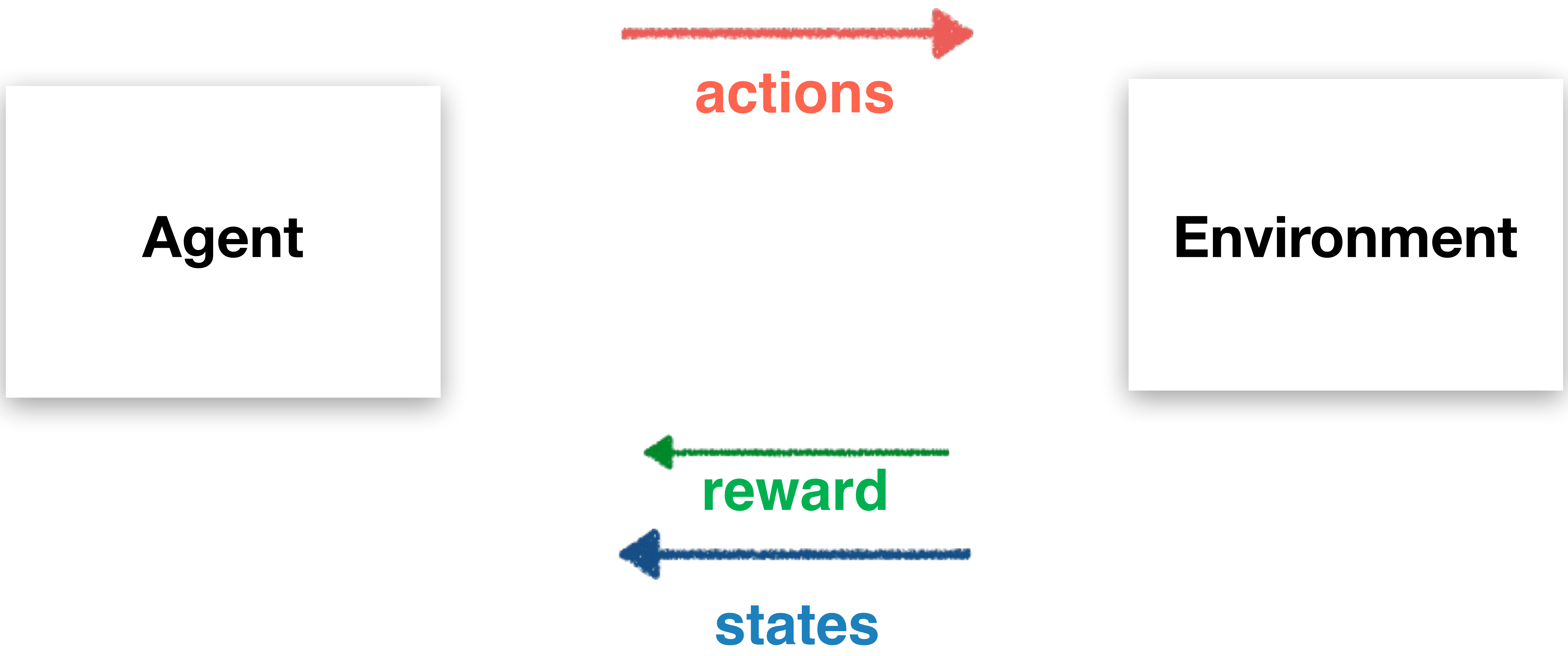

$S_0 \quad A_0 \quad R_1 \quad S_1 \quad A_1 \quad R_2 \quad S_2$

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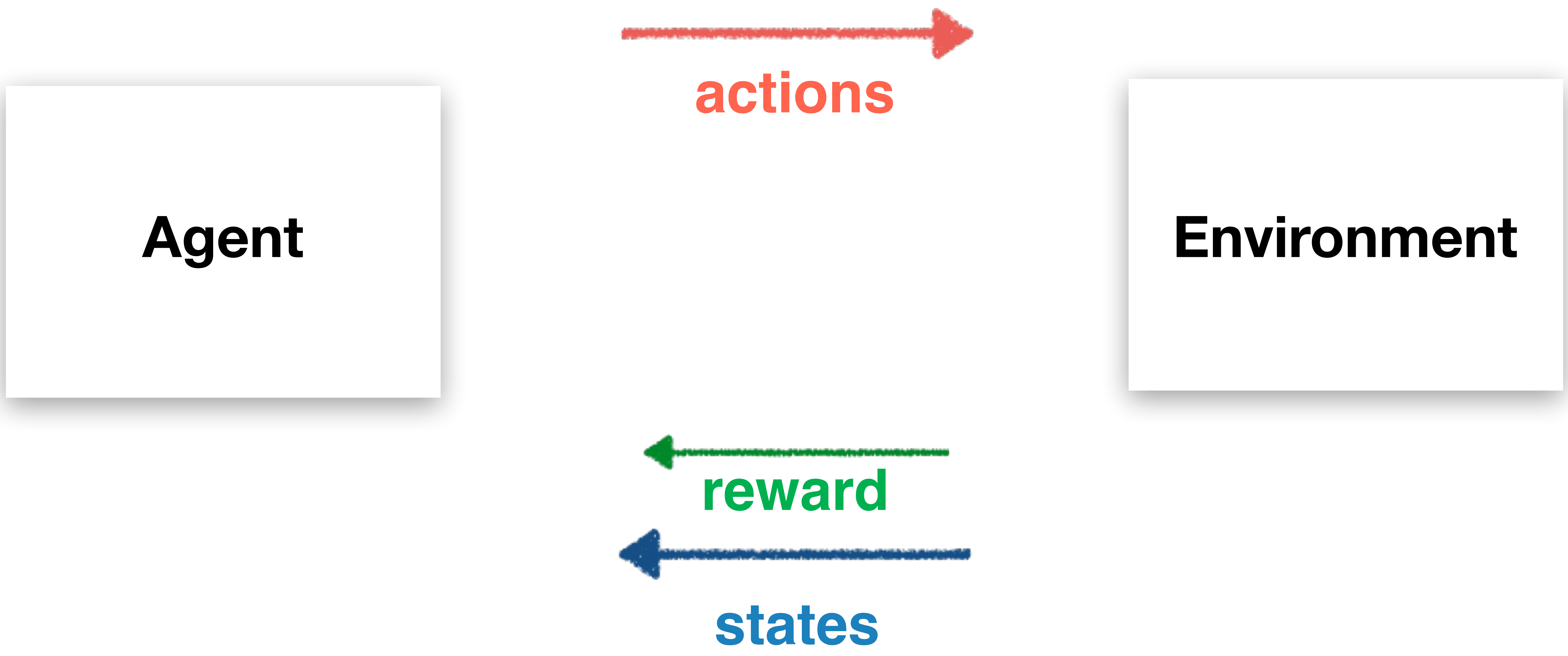

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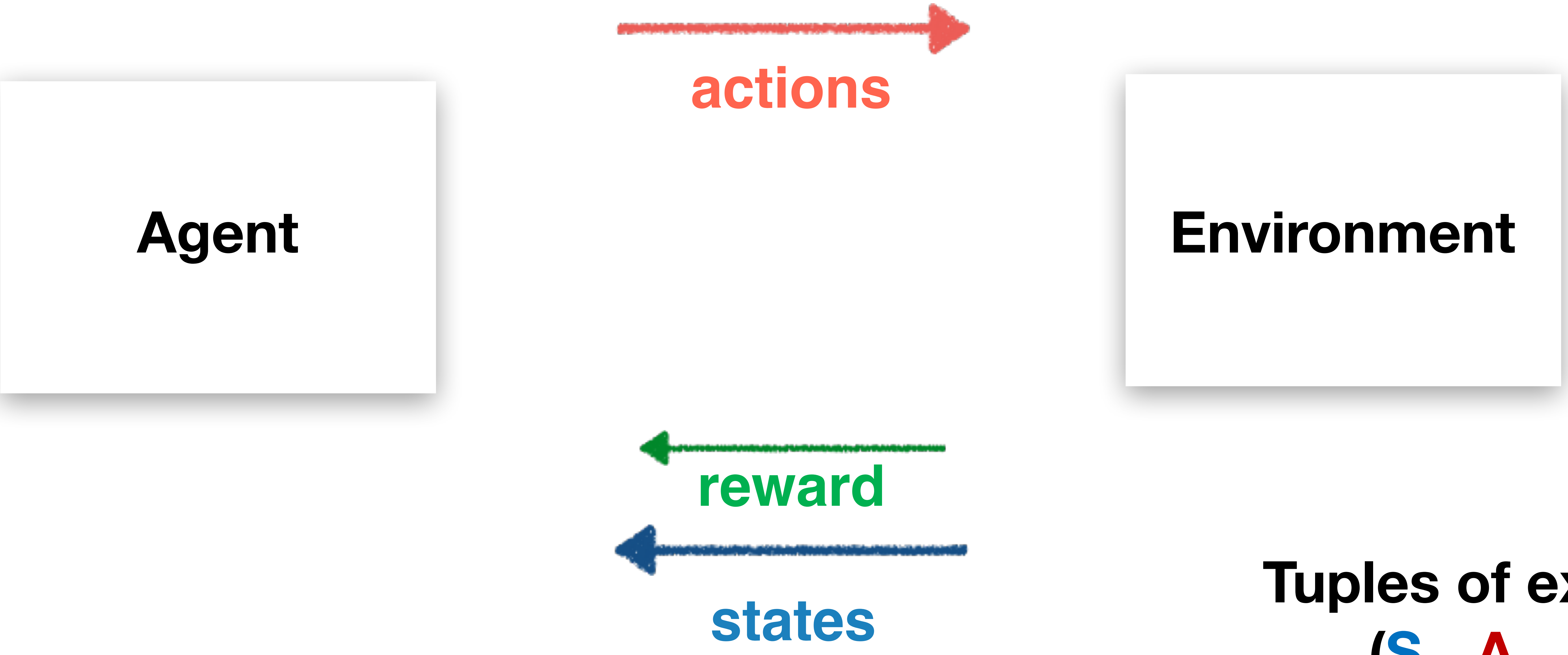


$S_0 \quad A_0 \quad R_1 \quad S_1 \quad A_1 \quad R_2 \quad S_2 \quad A_2 \ldots$

Tuples of experience:

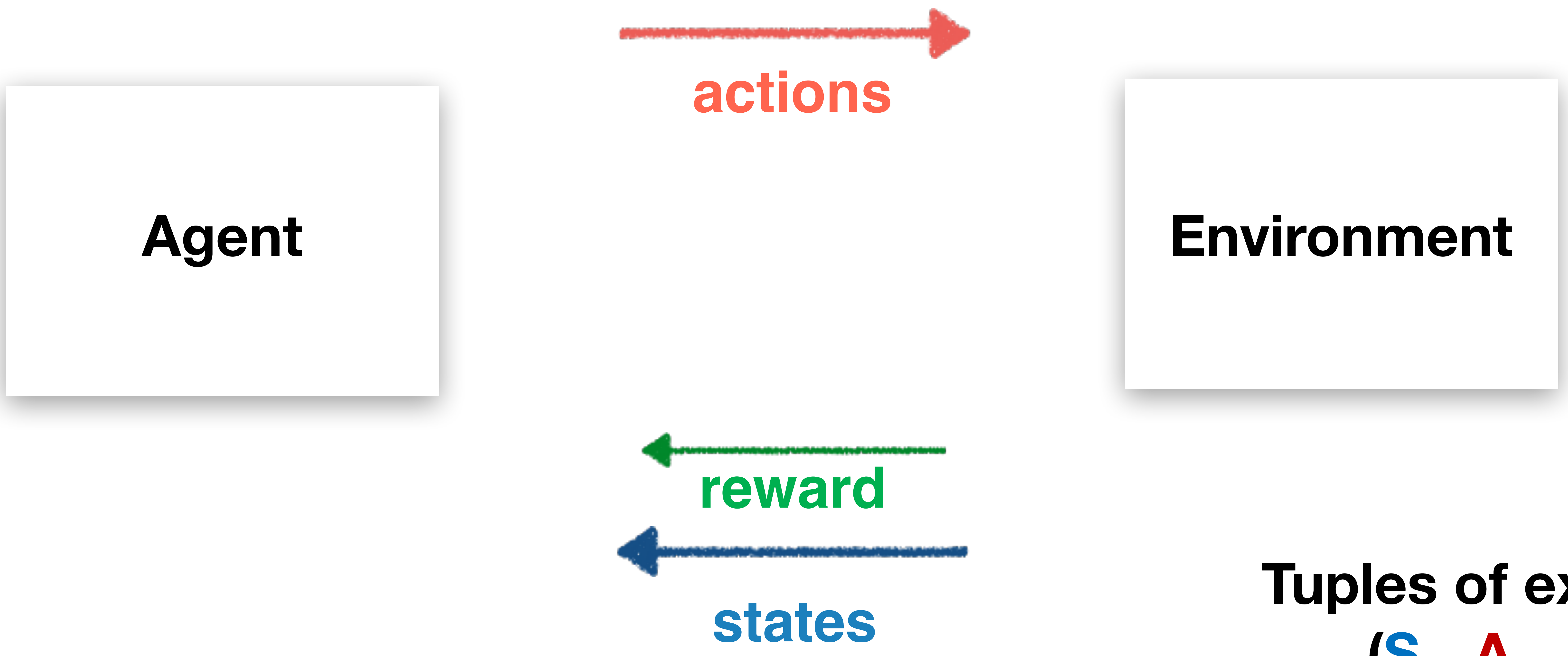
(S_0, A_0, R_1, S_1)

(S_1, A_1, R_2, S_2)

(S_2, A_2, R_3, S_3)

...

Online Reinforcement Learning



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Tuples of experience:

(S_0, A_0, R_1, S_1)

(S_1, A_1, R_2, S_2)

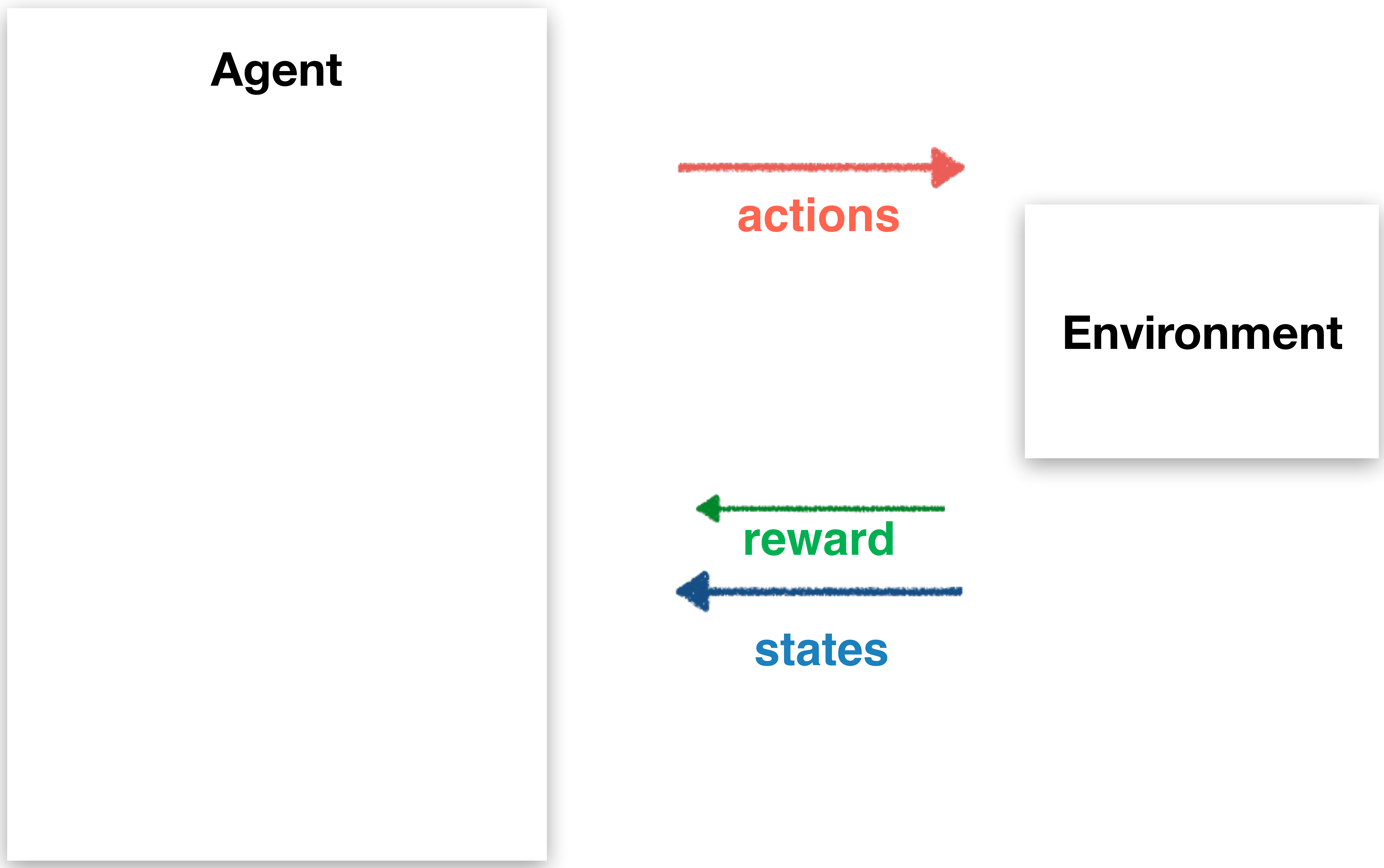
(S_2, A_2, R_3, S_3)

...

Q-learning update:

$$Q(S, A) = Q(S, A) + \alpha\,[\,R + \gamma \max Q(S', A') - Q(S, A)]$$

Online RL without a Model



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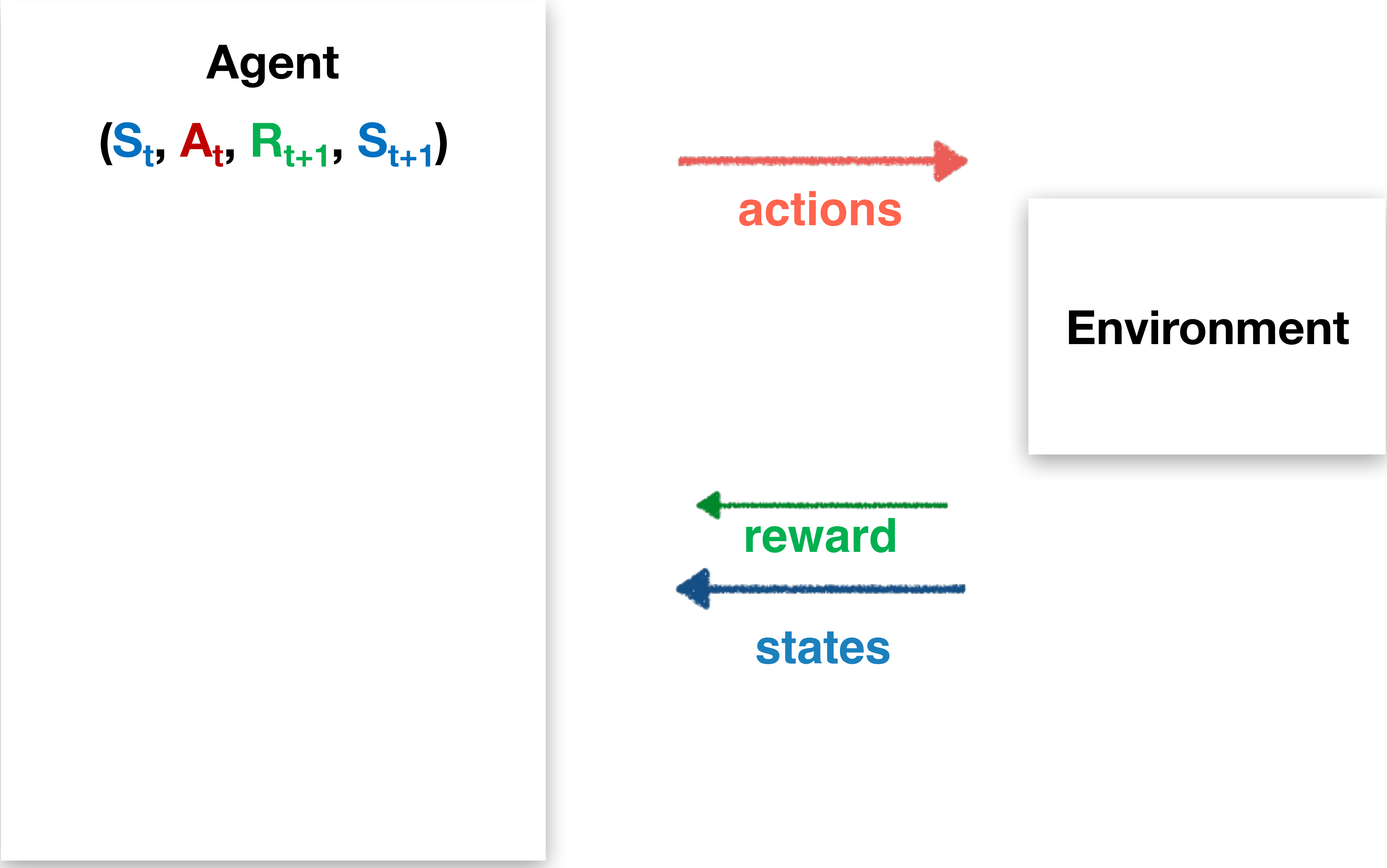

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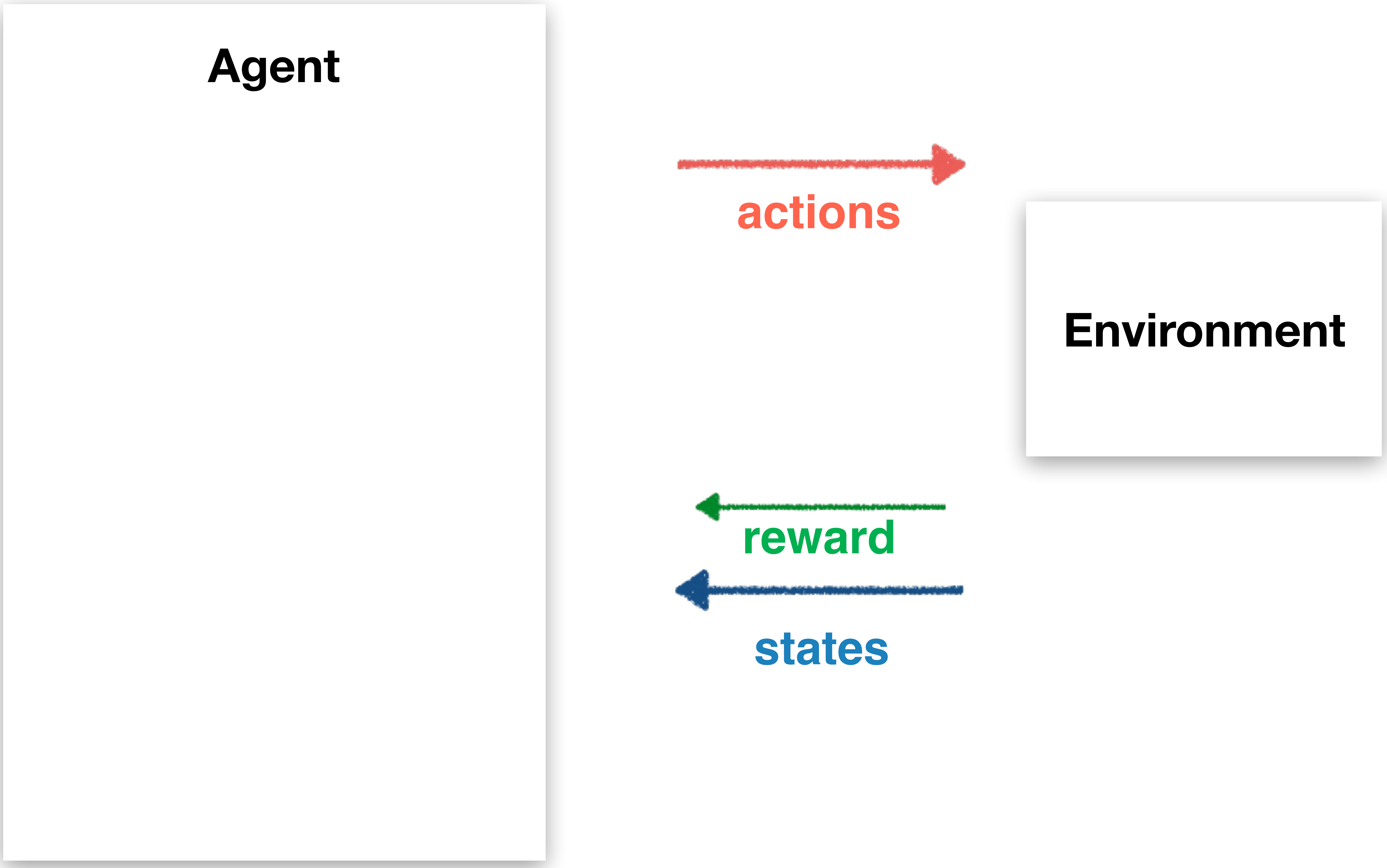

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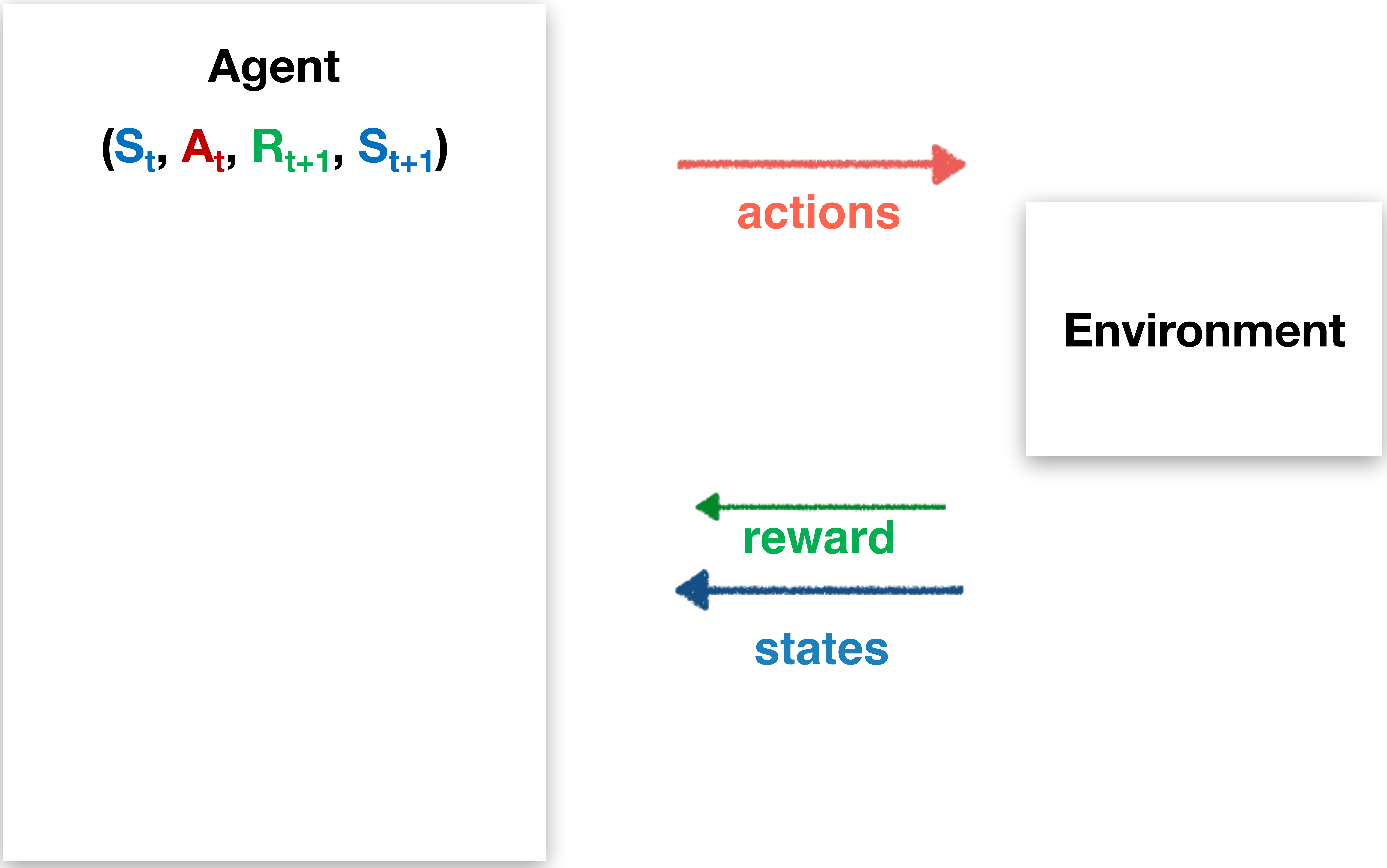

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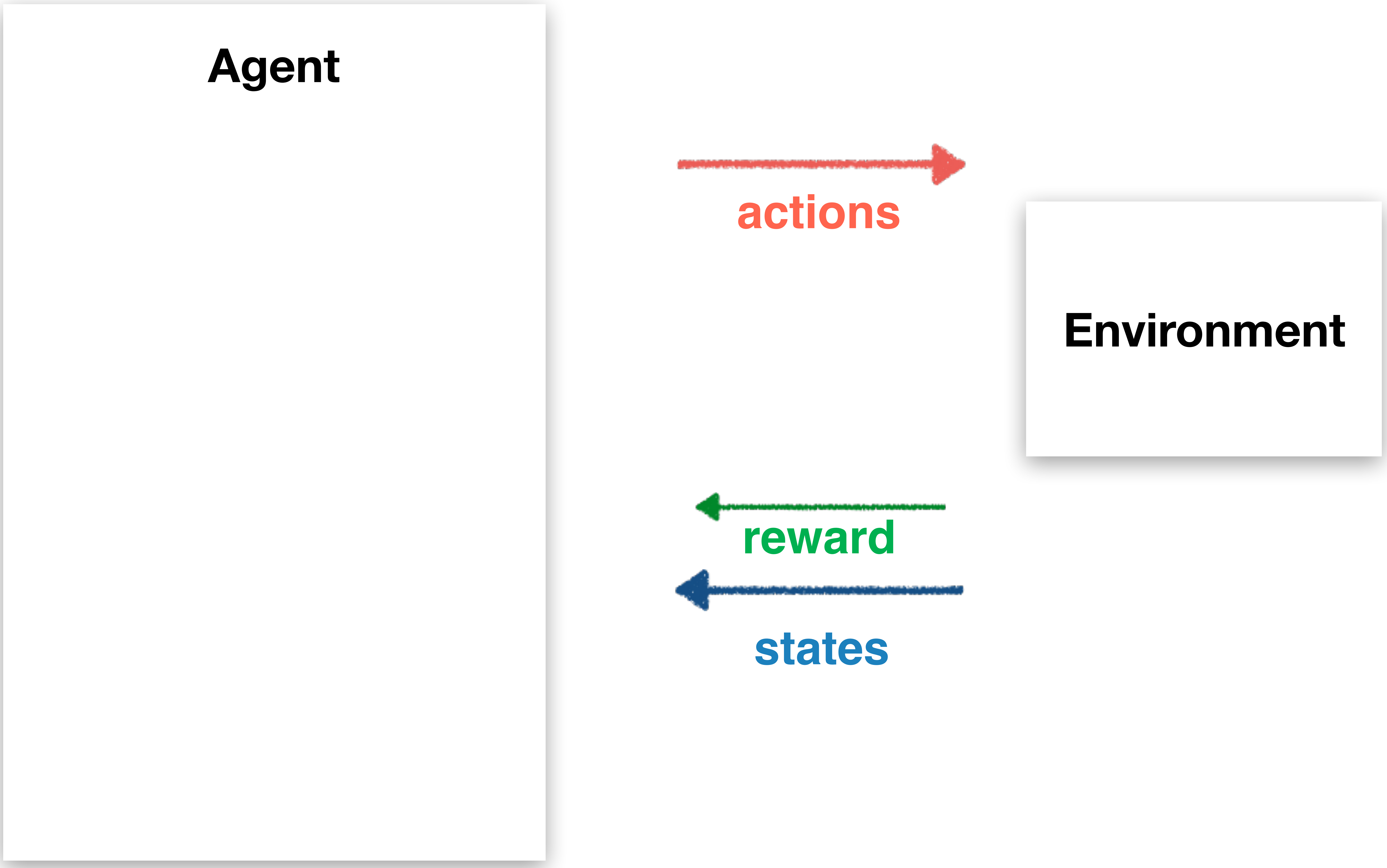
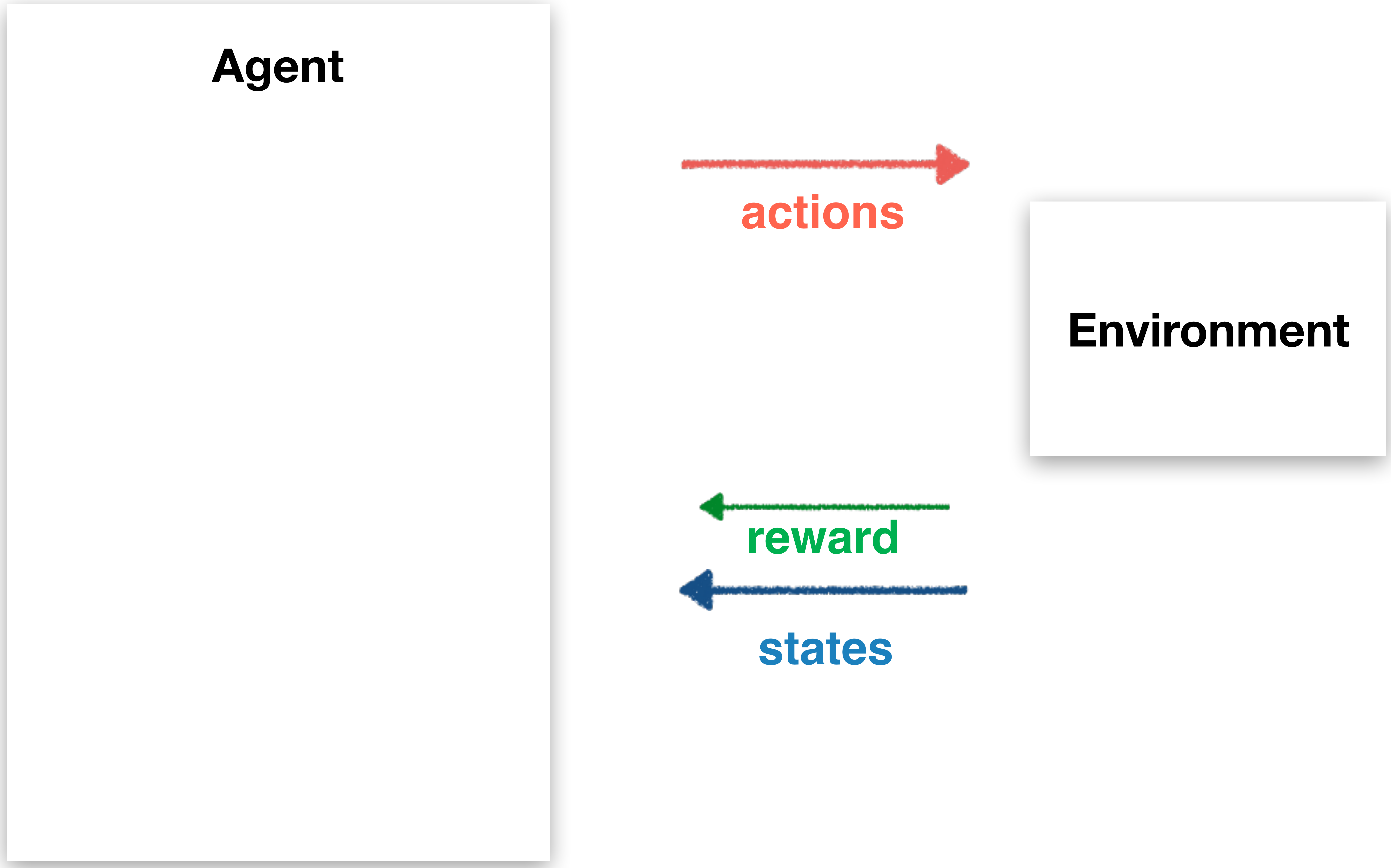


Online RL with a Model



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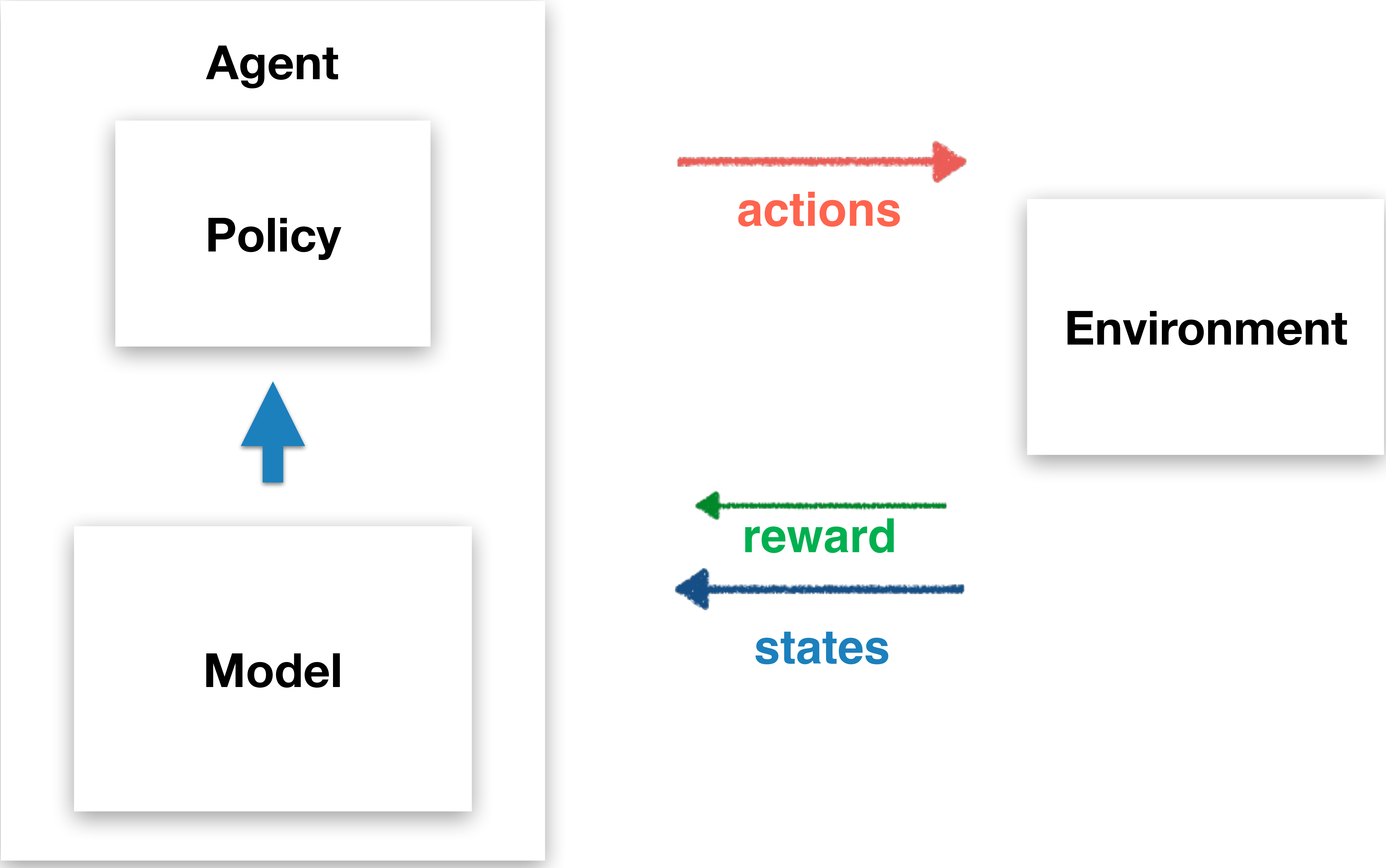

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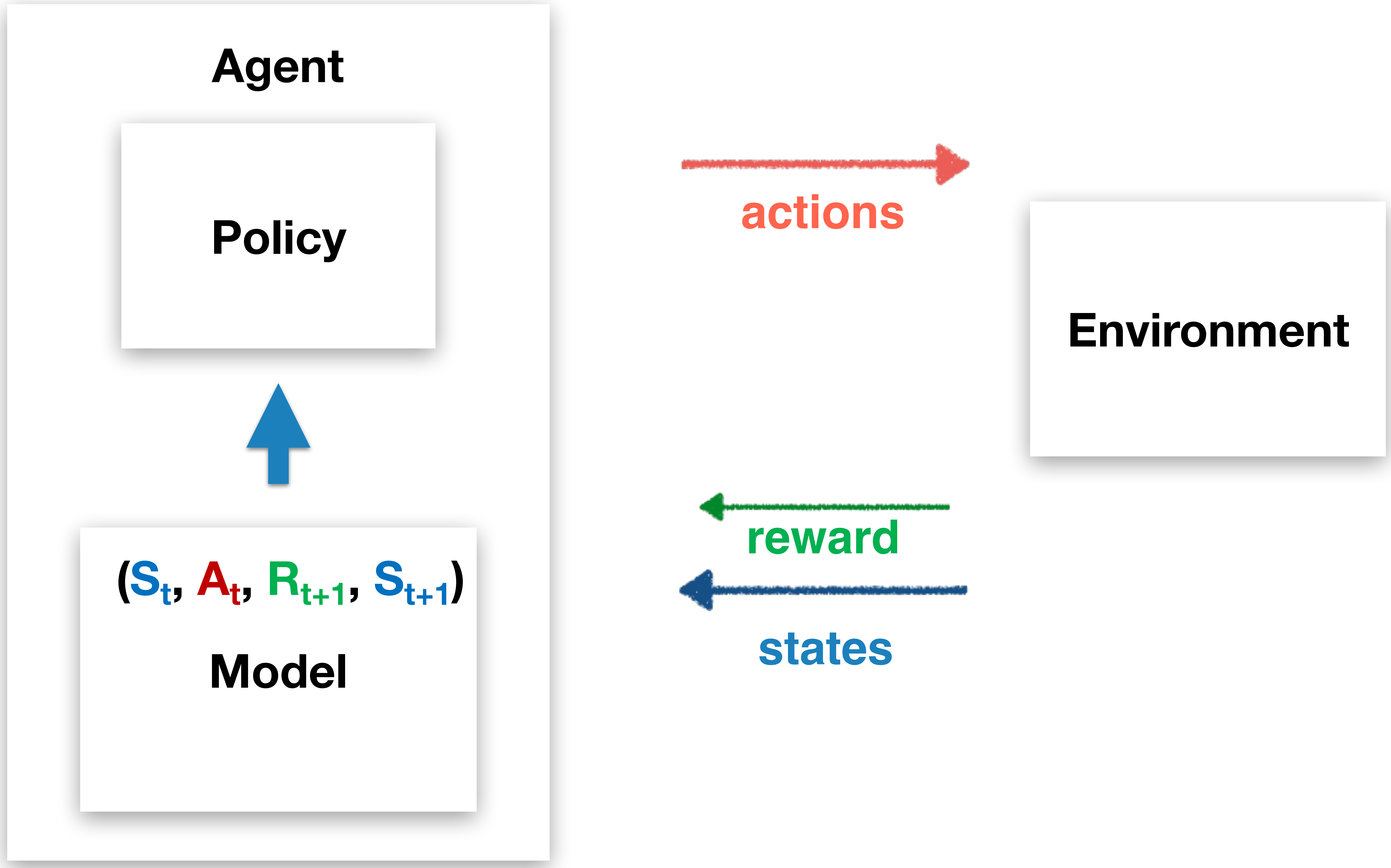

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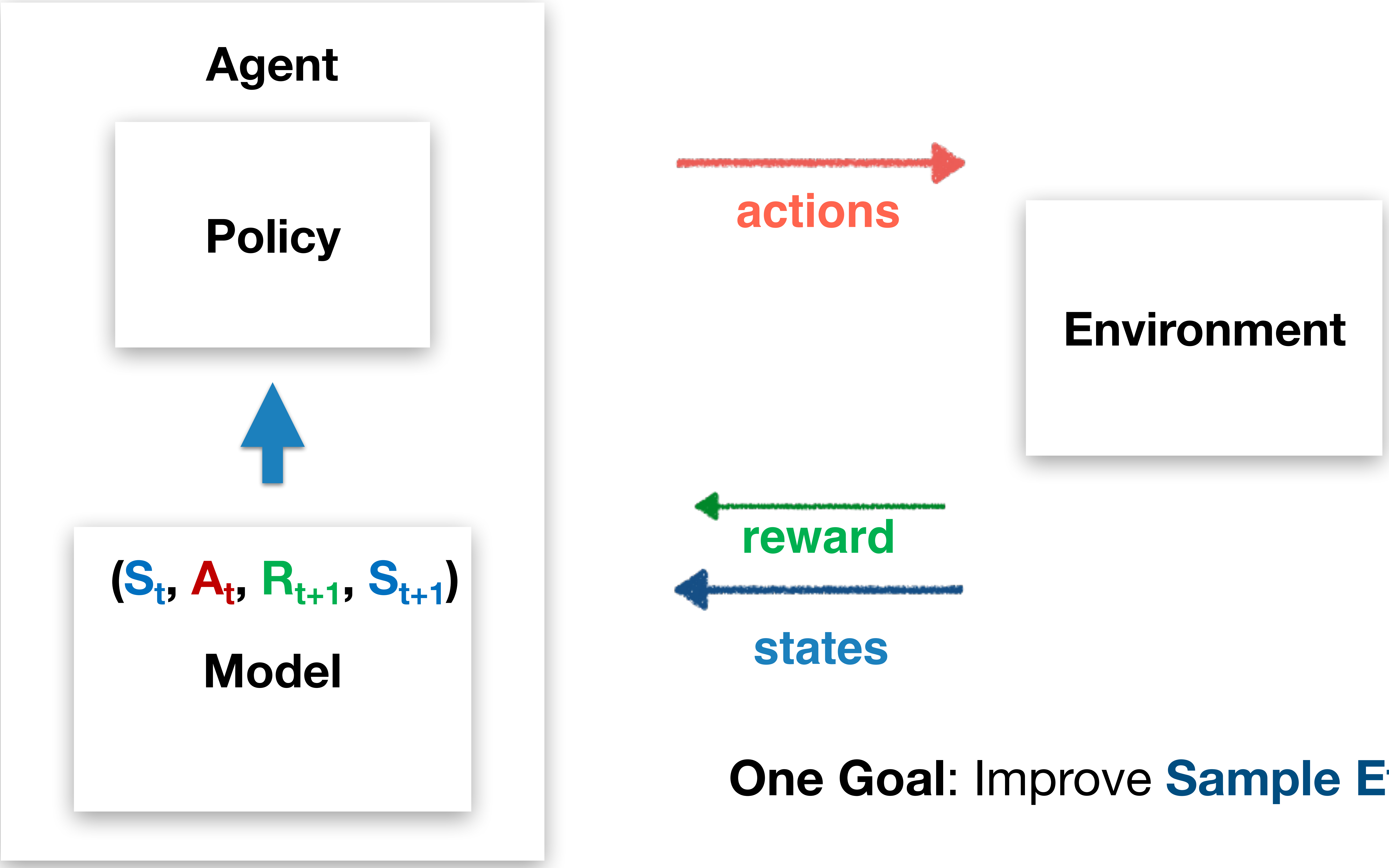


One Goal: Improve **Sample Efficiency**

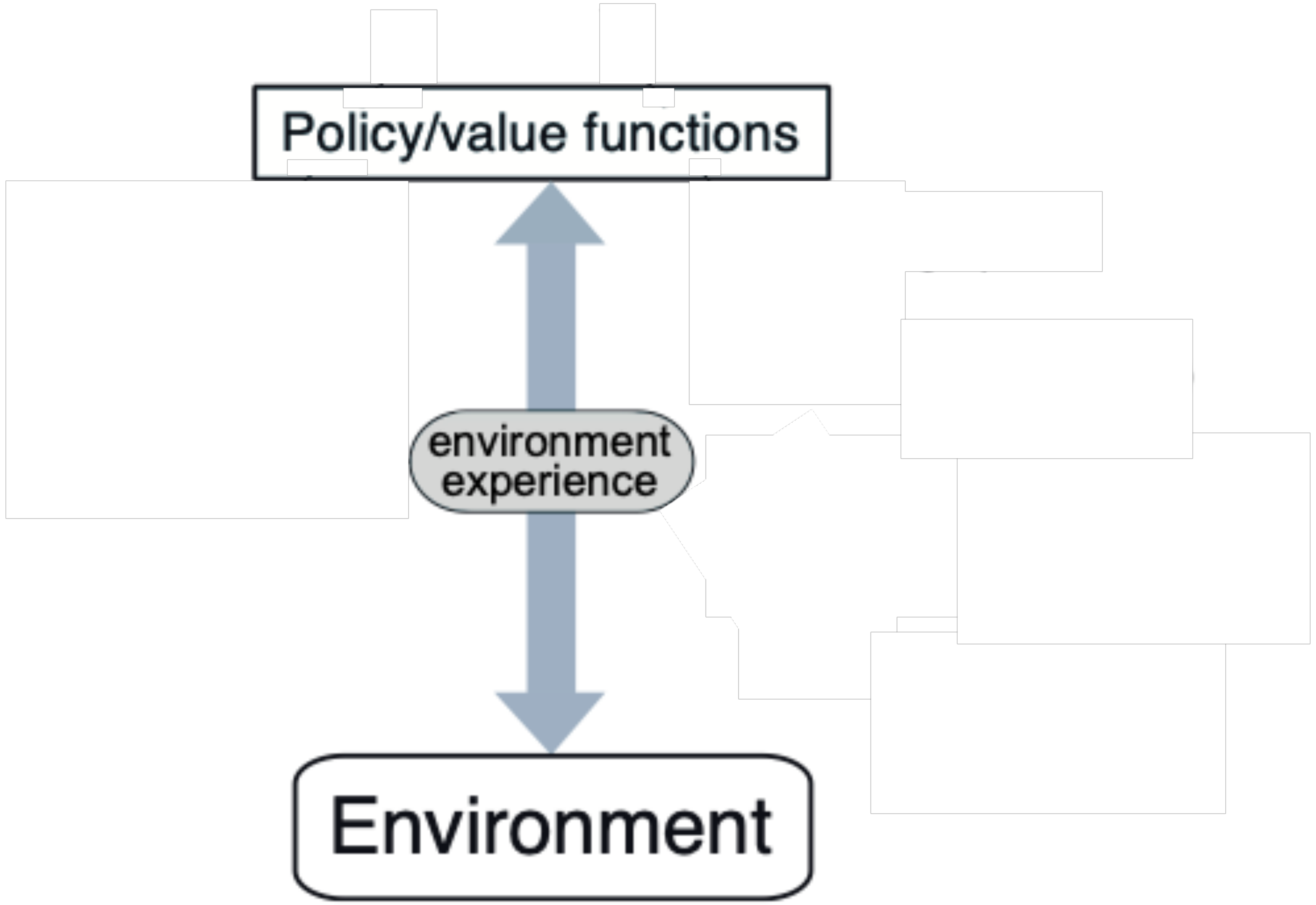
What are possible learned models?

- **Most obvious answer**: $\hat{p}(s', r \mid s, a)$
- **Realistically**: models with state abstraction and temporal abstraction
- **For now**: let's assume we learn approximation $\hat{p}(s', r \mid s, a)$

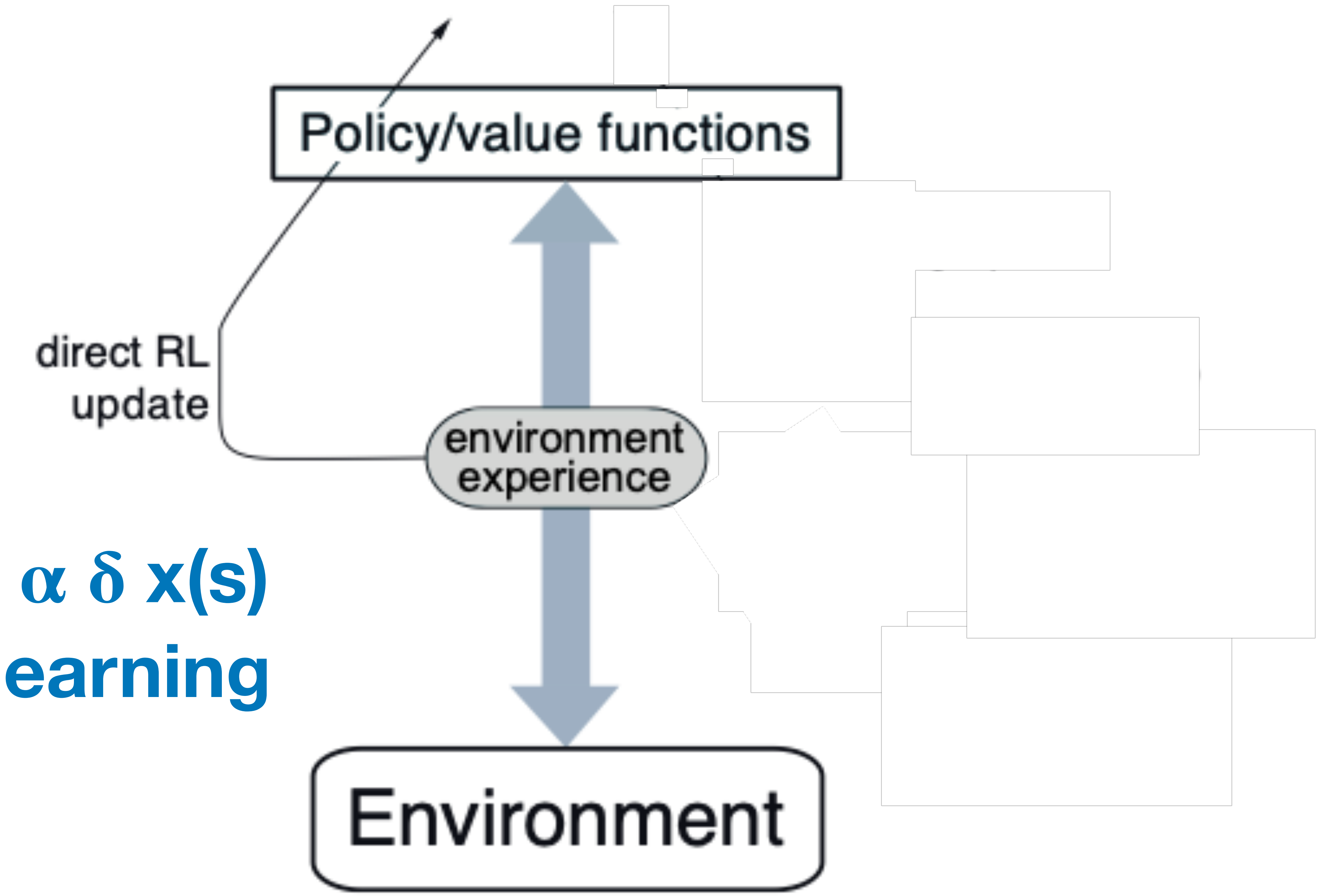
Outline for Part 2:
Moving to Learned Models

- Introduce a **planning** framework called **Dyna**
  - Explain how Experience Replay is a simple instance of Dyna
- Discuss two key choices in Dyna: **Model** and **Search Control**
- Discuss different choices for the **Model**
- Discuss different choices for **Search Control**

What is Dyna?


Policy/value functions
environment experience
Environment

What is Dyna?



$w = w + \alpha\ \delta\ x(s)$
e.g., Q-learning

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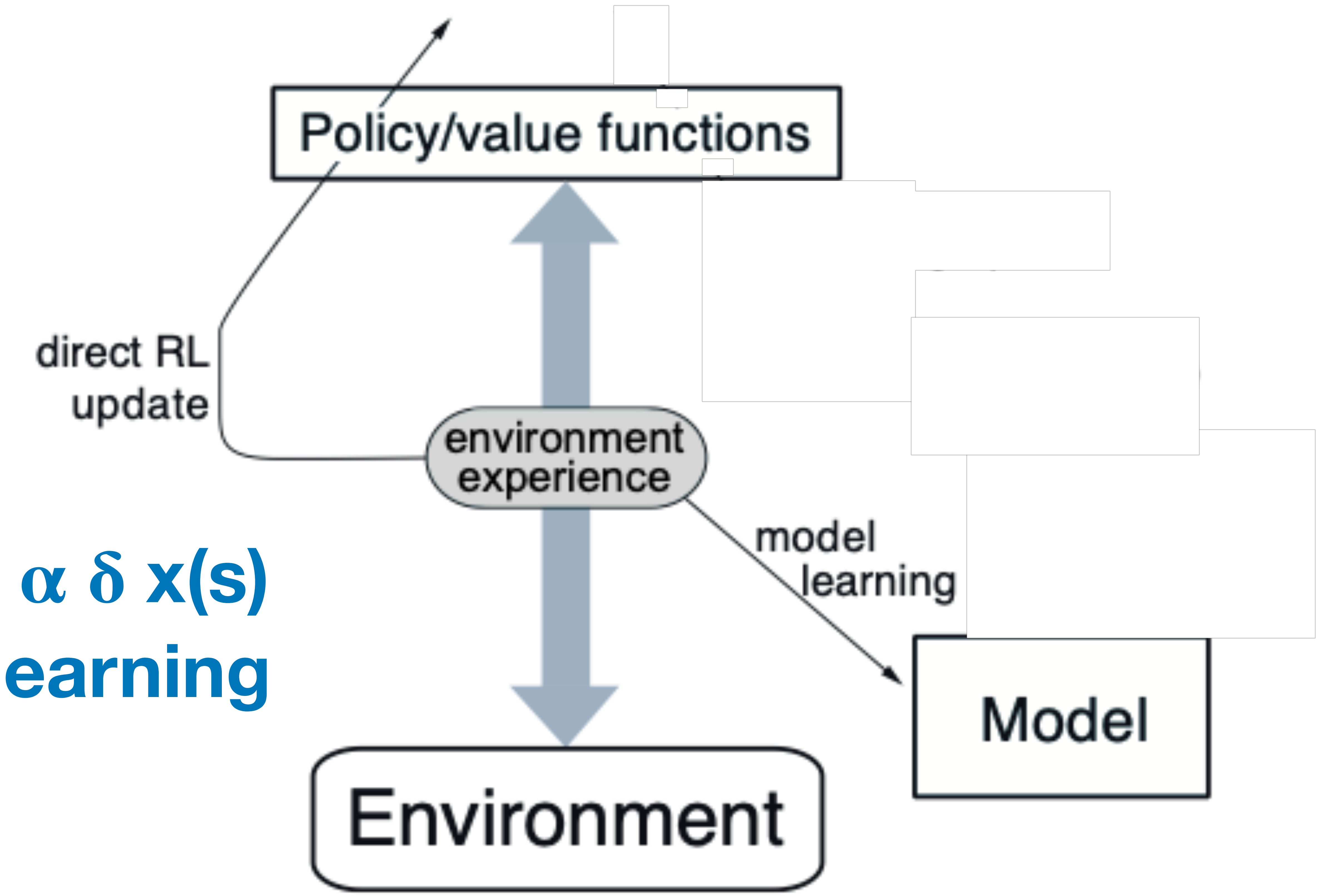

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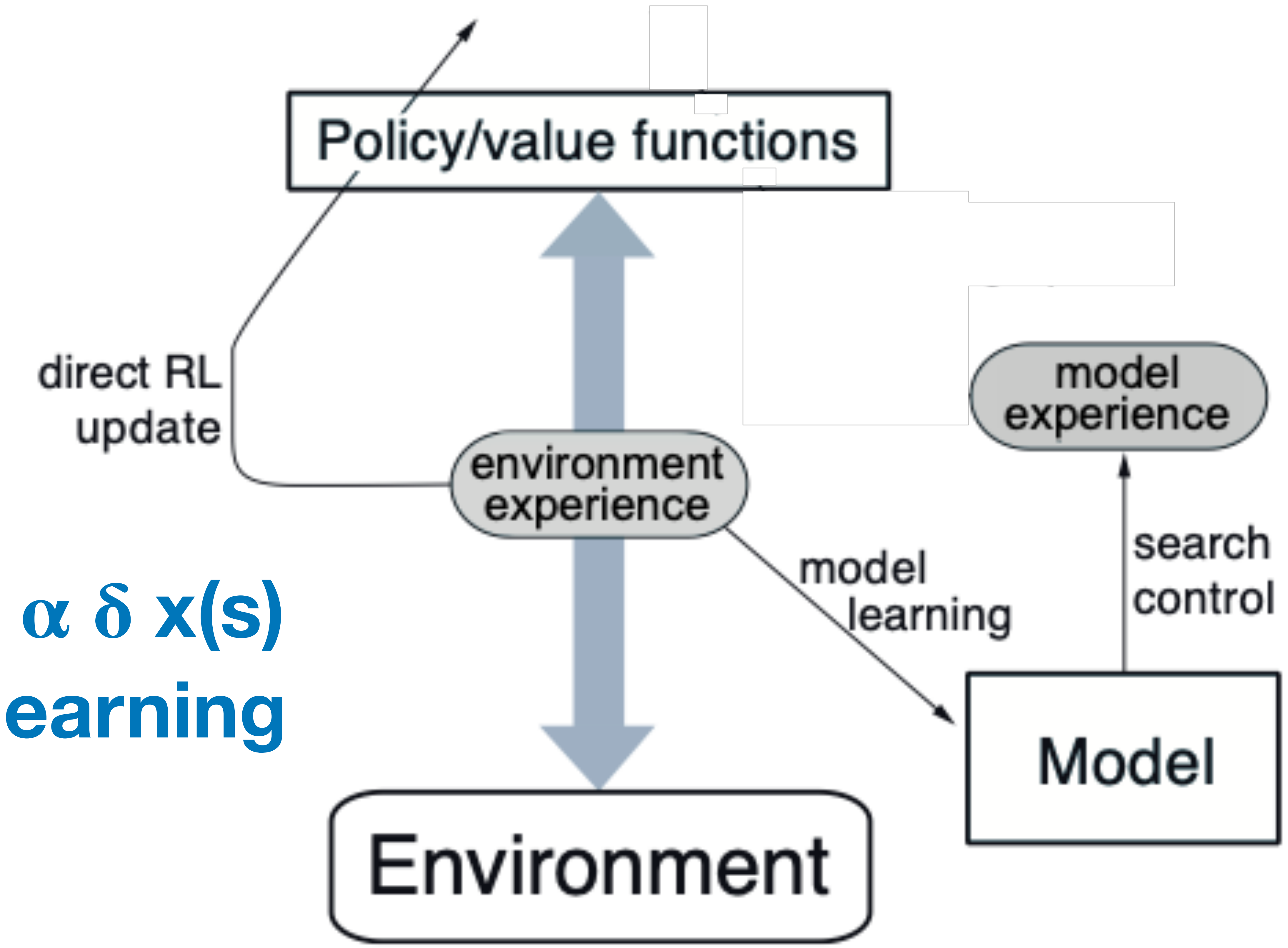

$w = w + \alpha\, \delta\, x(s)$
e.g., Q-learning

s’, r ~ M(s,a)

(s,a)

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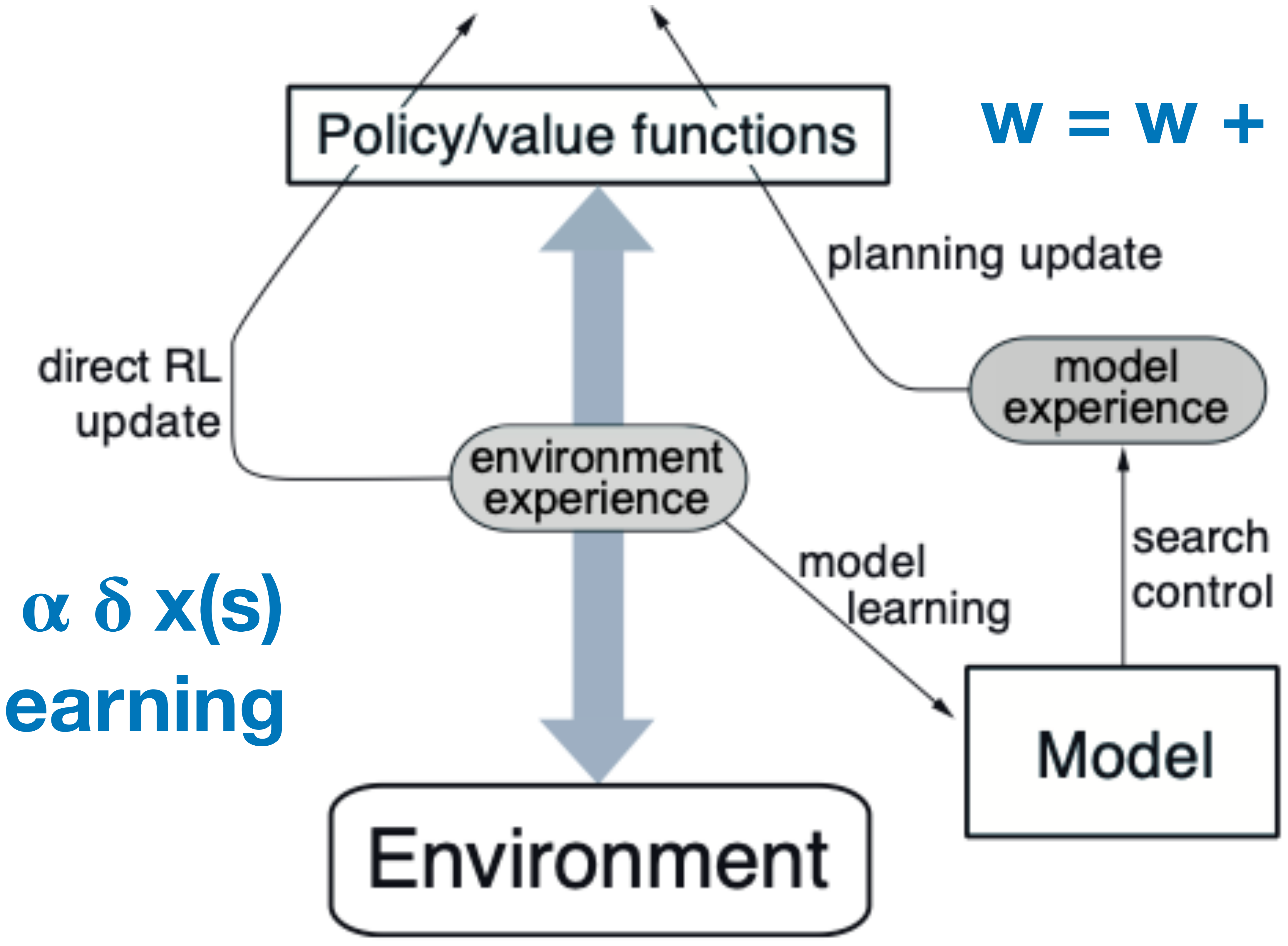

$w = w + \alpha \delta x(s)$

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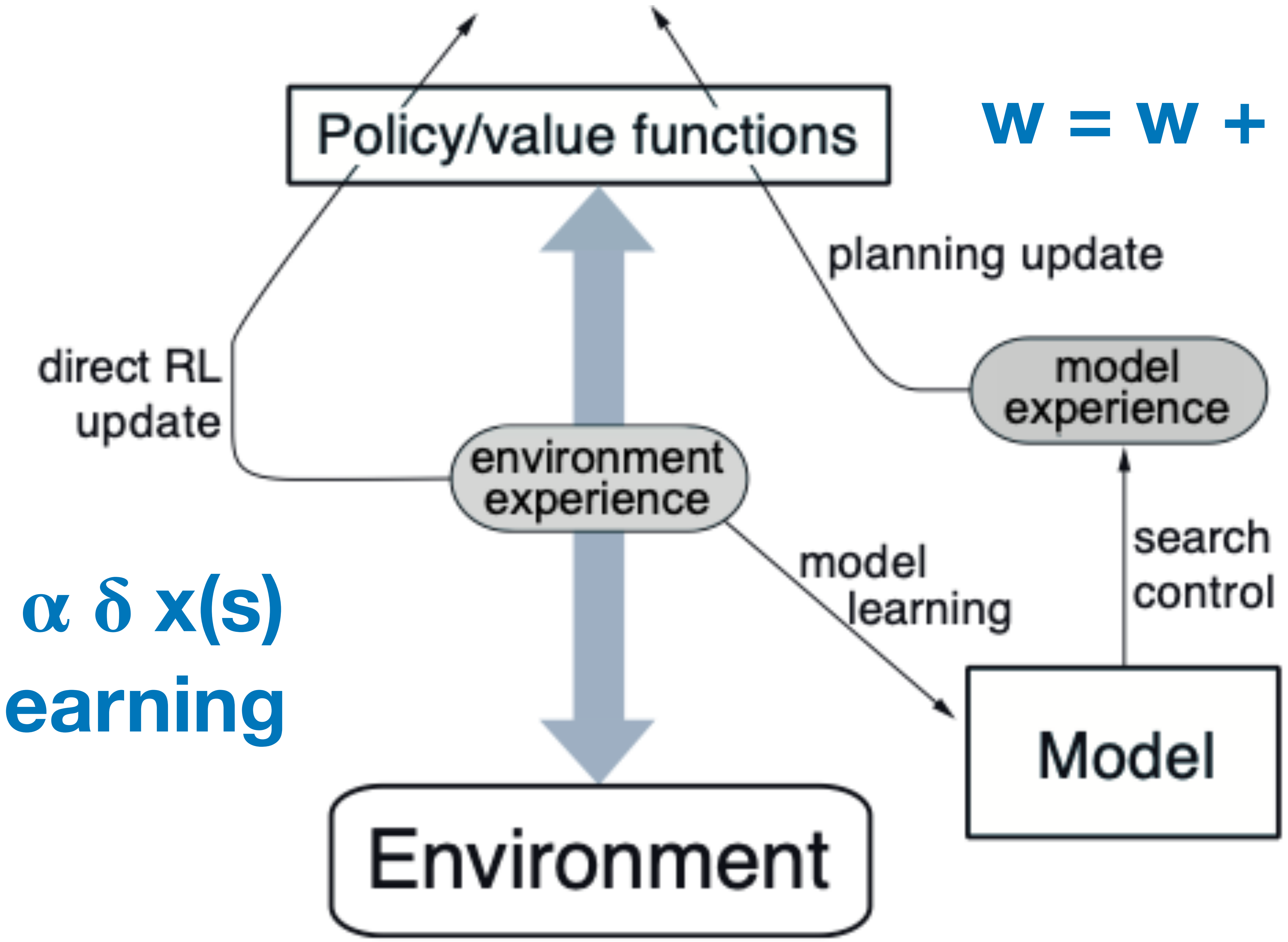

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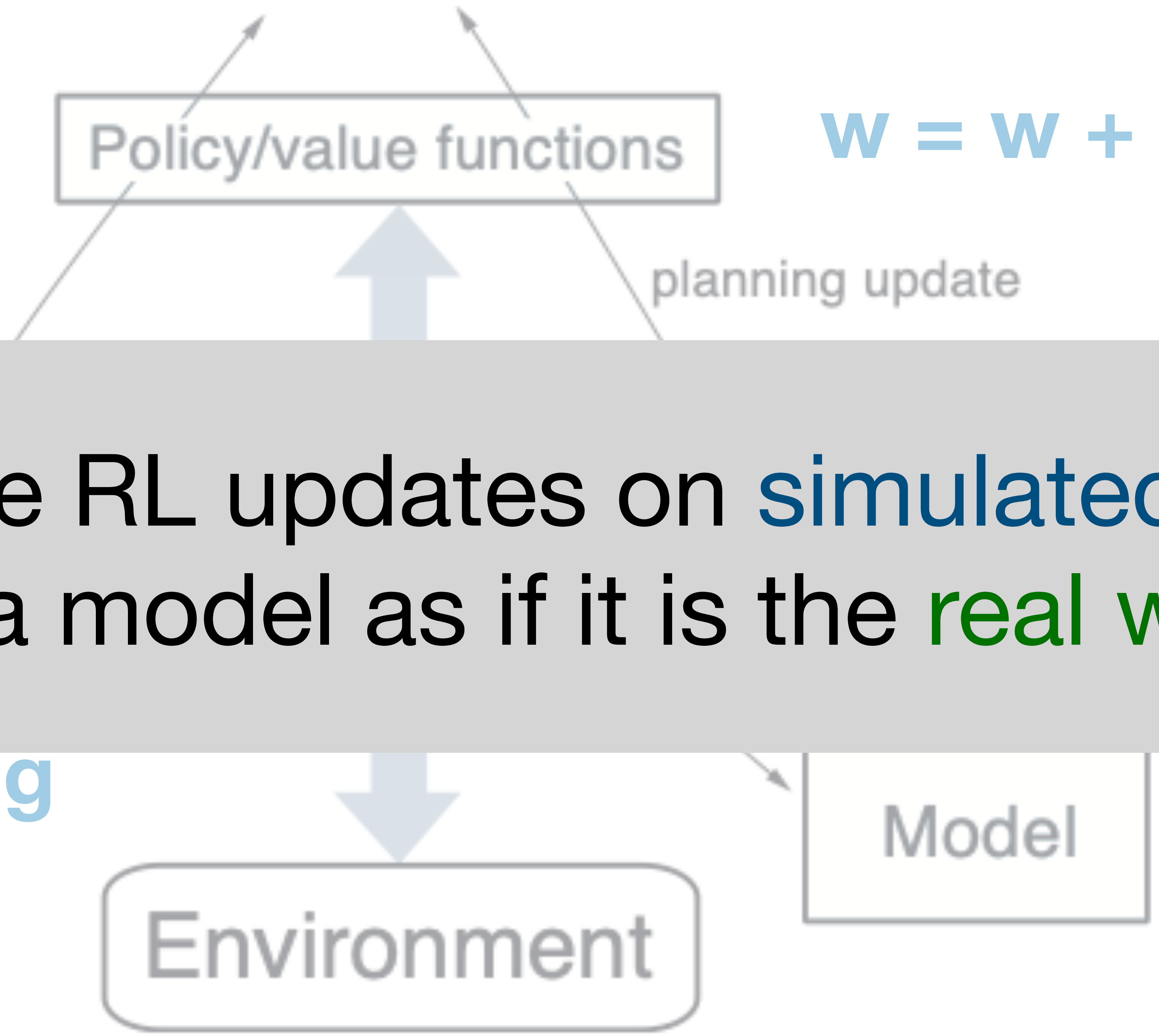

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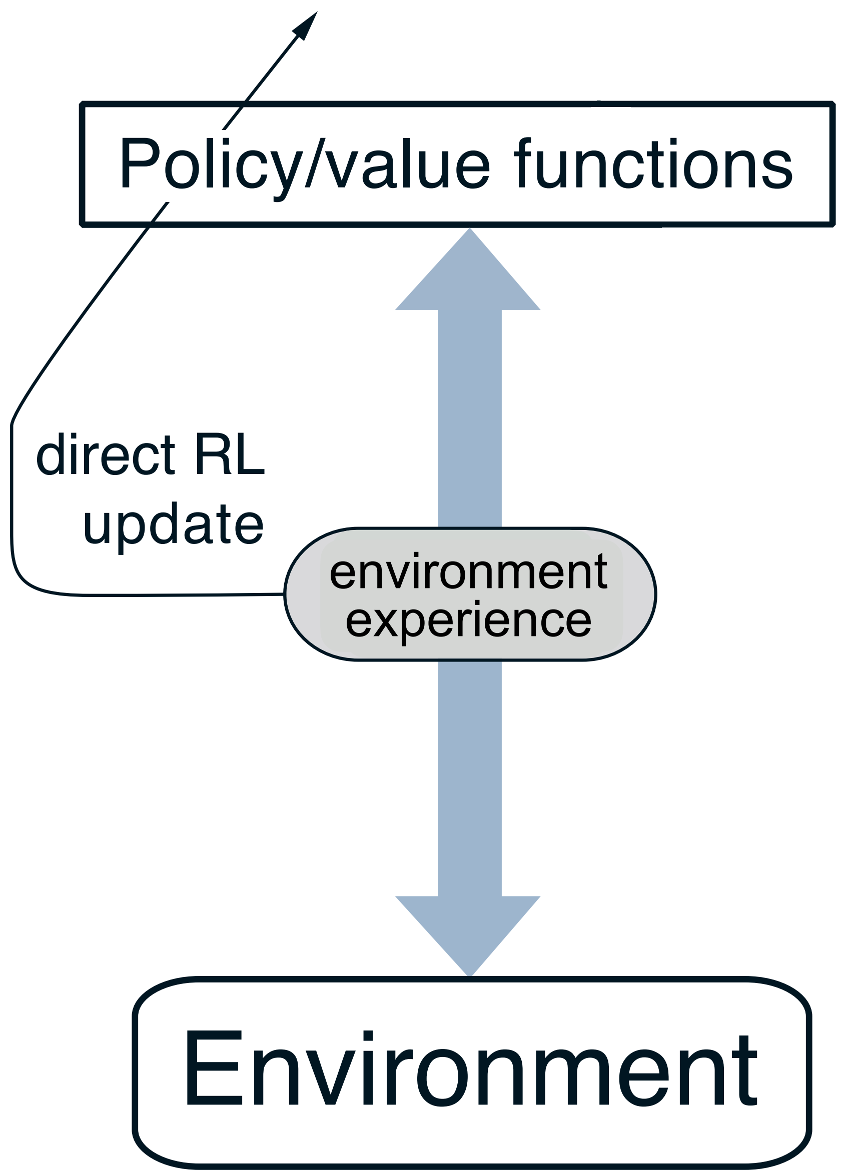
Key Idea: Use RL updates on simulated experience from a model as if it is the real world

Pseudocode

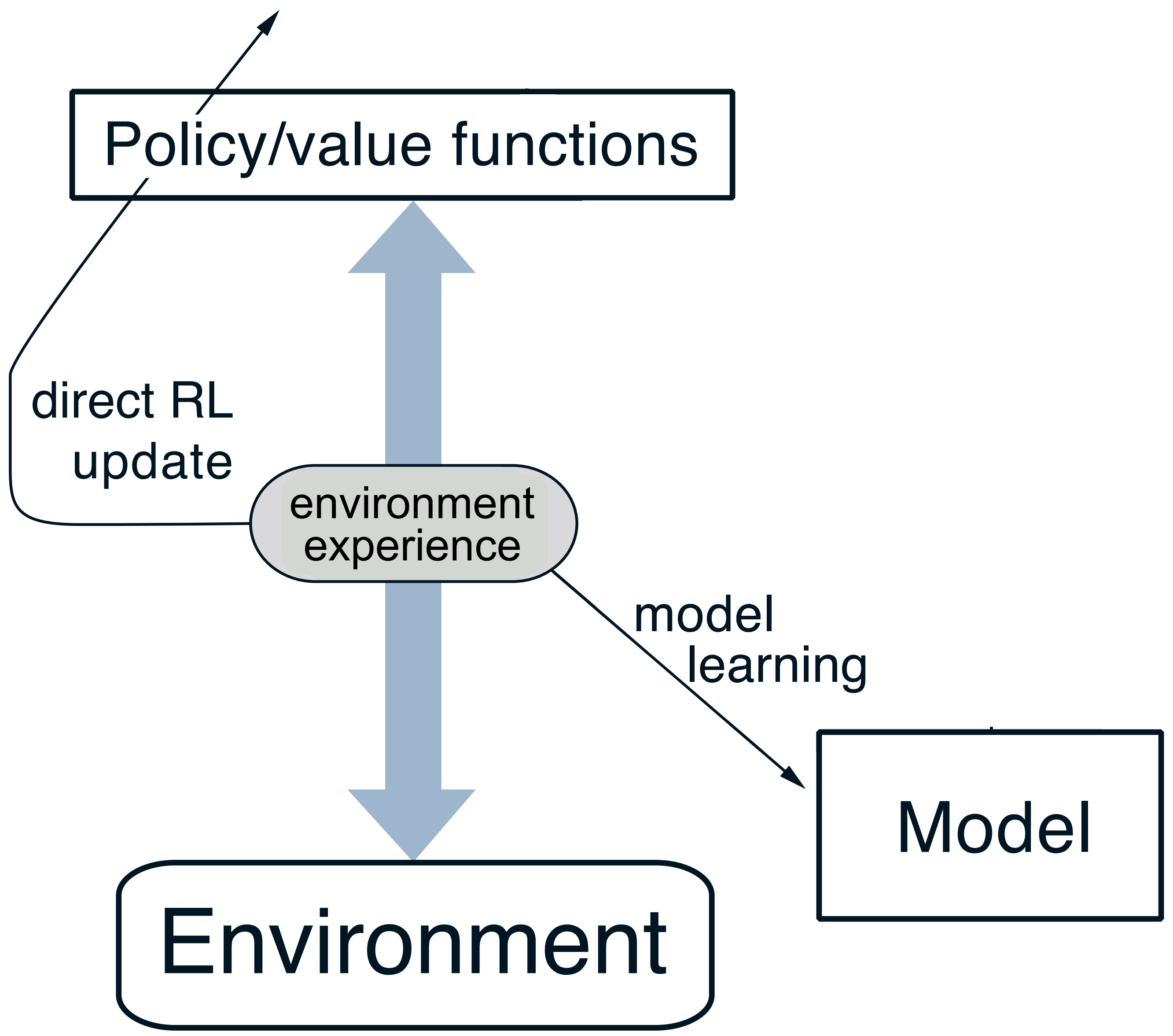
Dyna-Q



Pseudocode

Dyna-Q

Initialize $Q(s,a)$ and $Model(s,a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Loop forever:
(a) $S \leftarrow$ current (nonterminal) state
(b) $A \leftarrow \varepsilon$-greedy(S, Q)
(c) Take action A; observe resultant reward, R, and state, S'
(d) $Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]$



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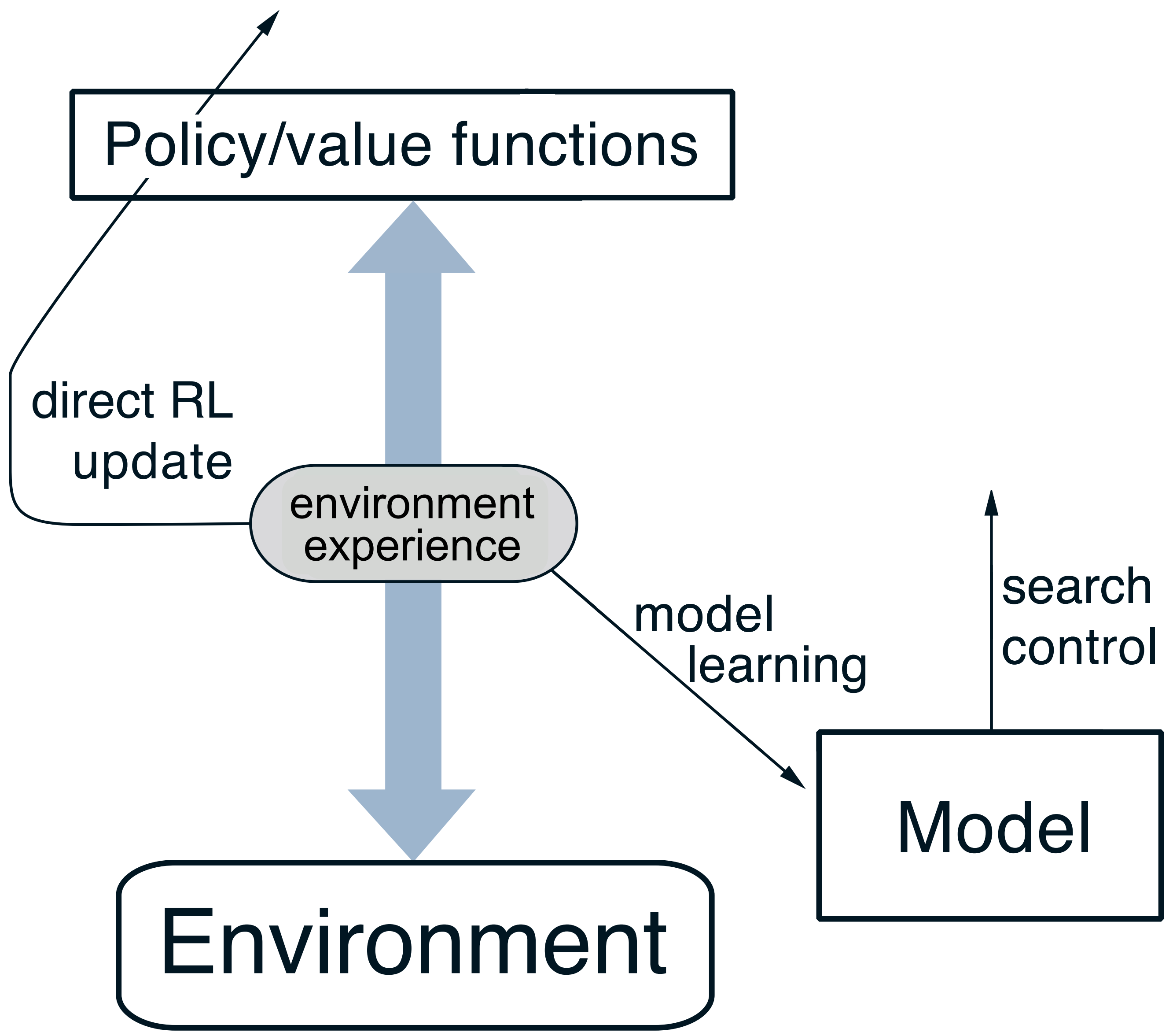

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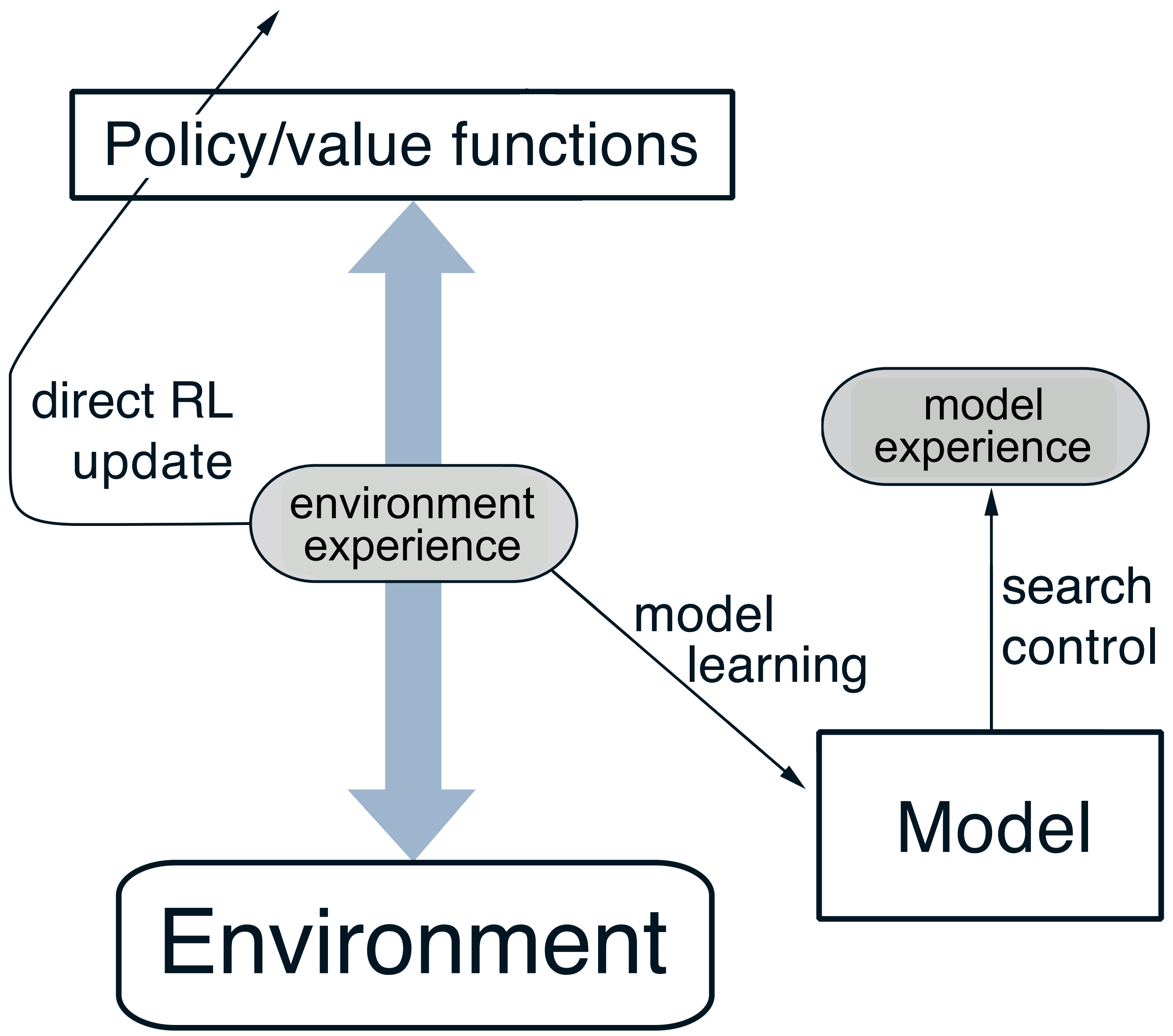
$S \leftarrow$ random previously observed state
$A \leftarrow$ random action previously taken in S



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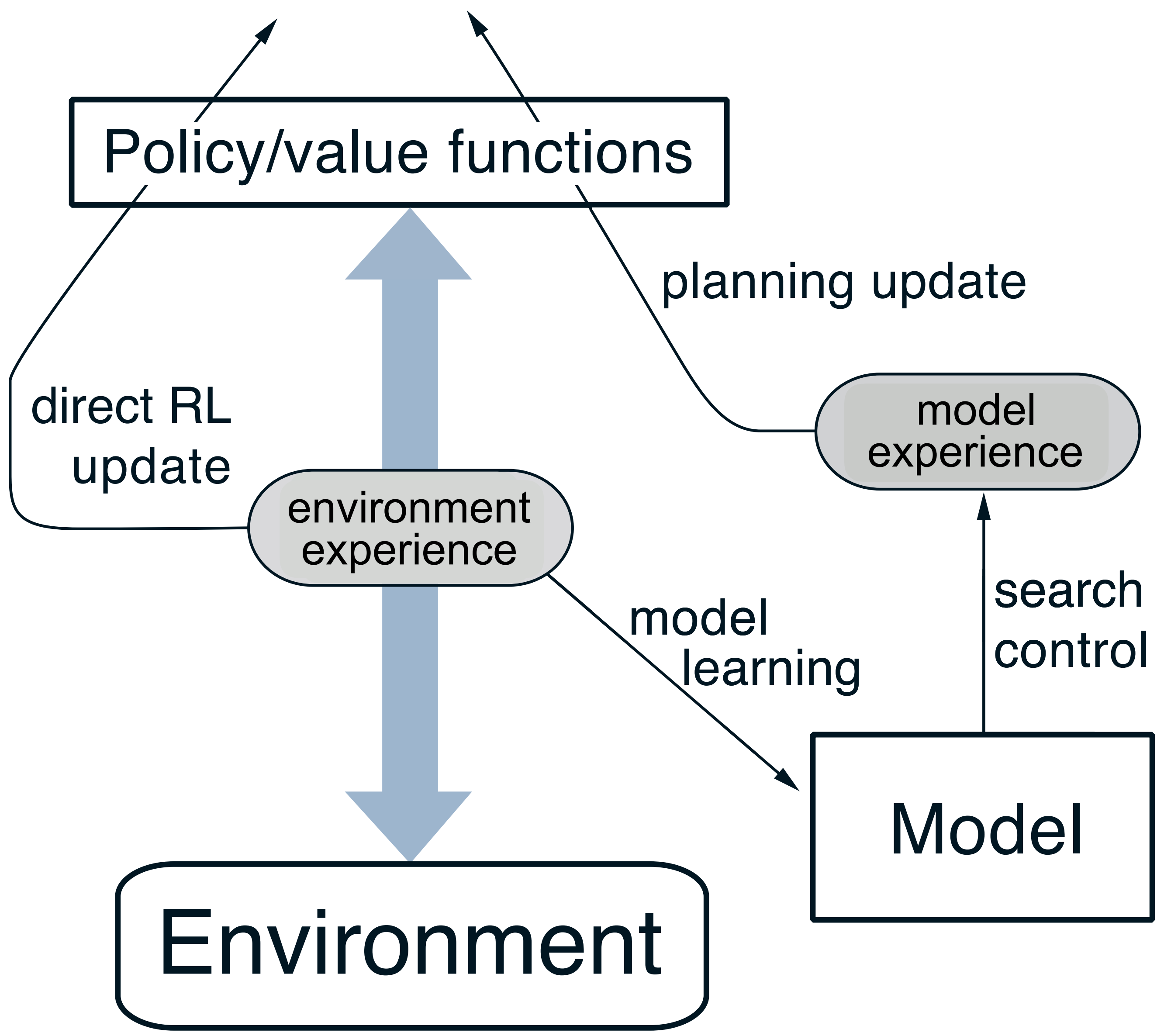
$R, S' \leftarrow Model(S,A)$



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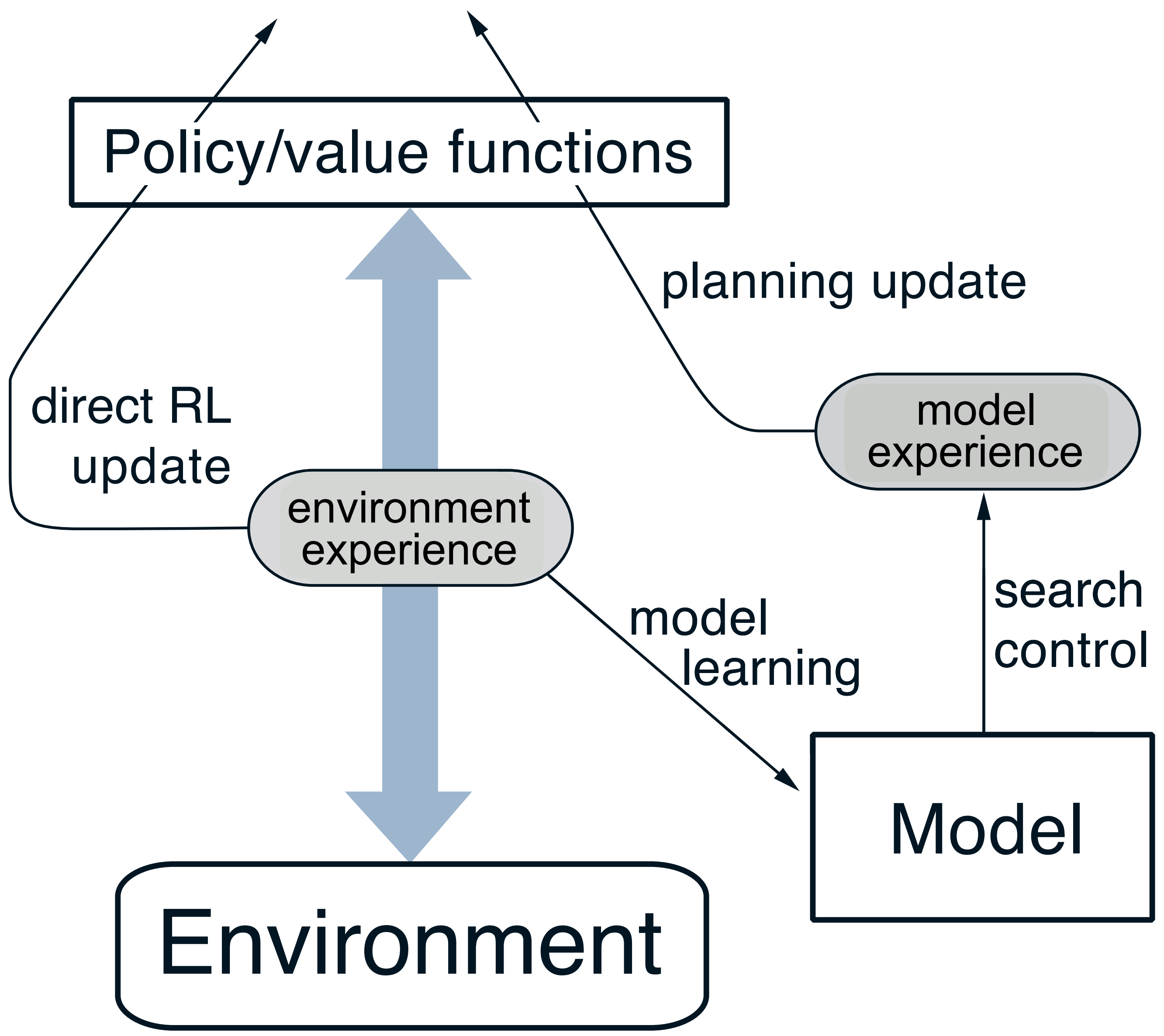

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(e) $Model(S, A) \leftarrow R, S'$

(f) Loop repeat n times:

$S \leftarrow$ random previously observed state

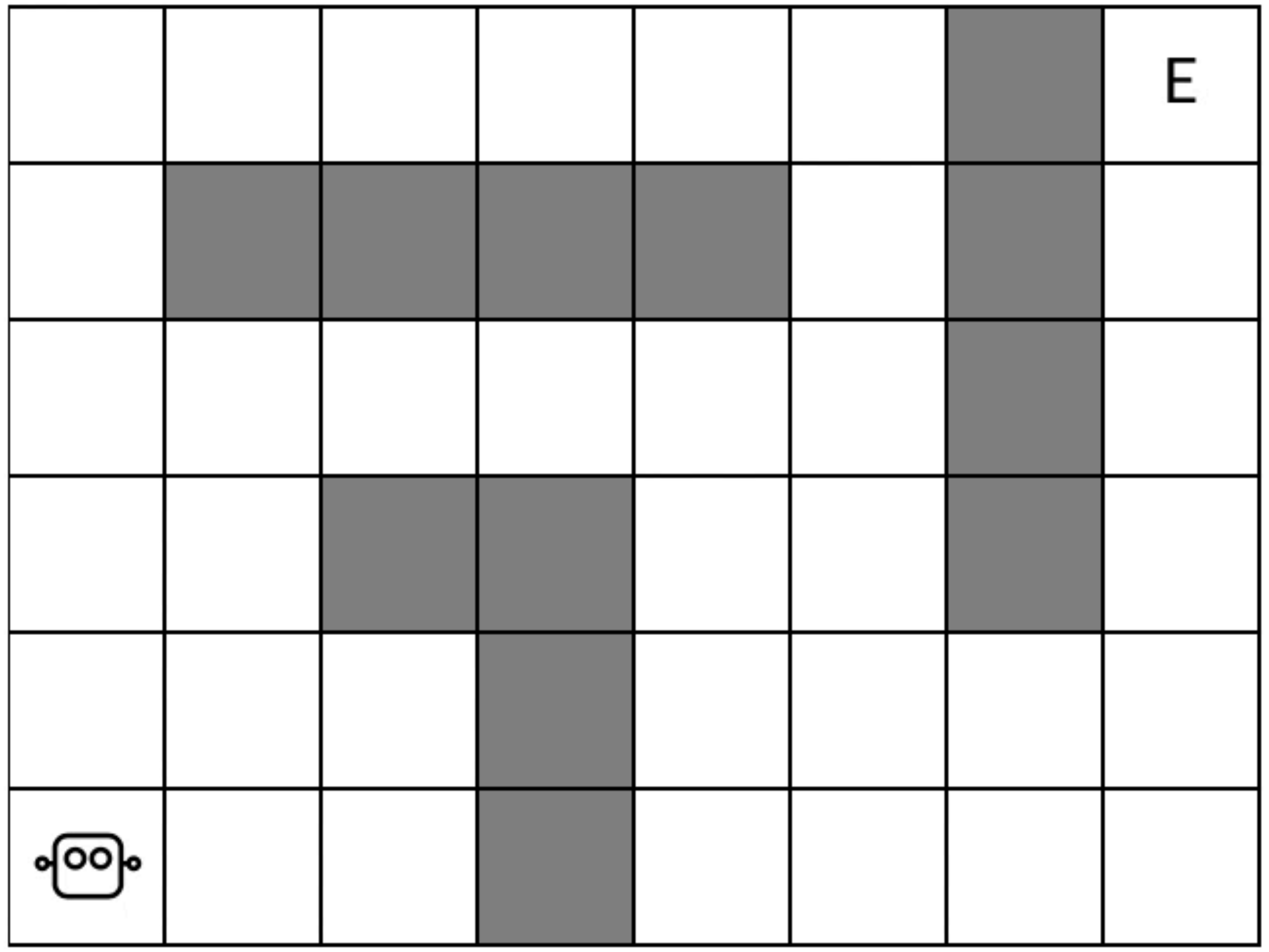
$A \leftarrow$ random action previously taken in S

$R, S' \leftarrow Model(S, A)$

$Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$

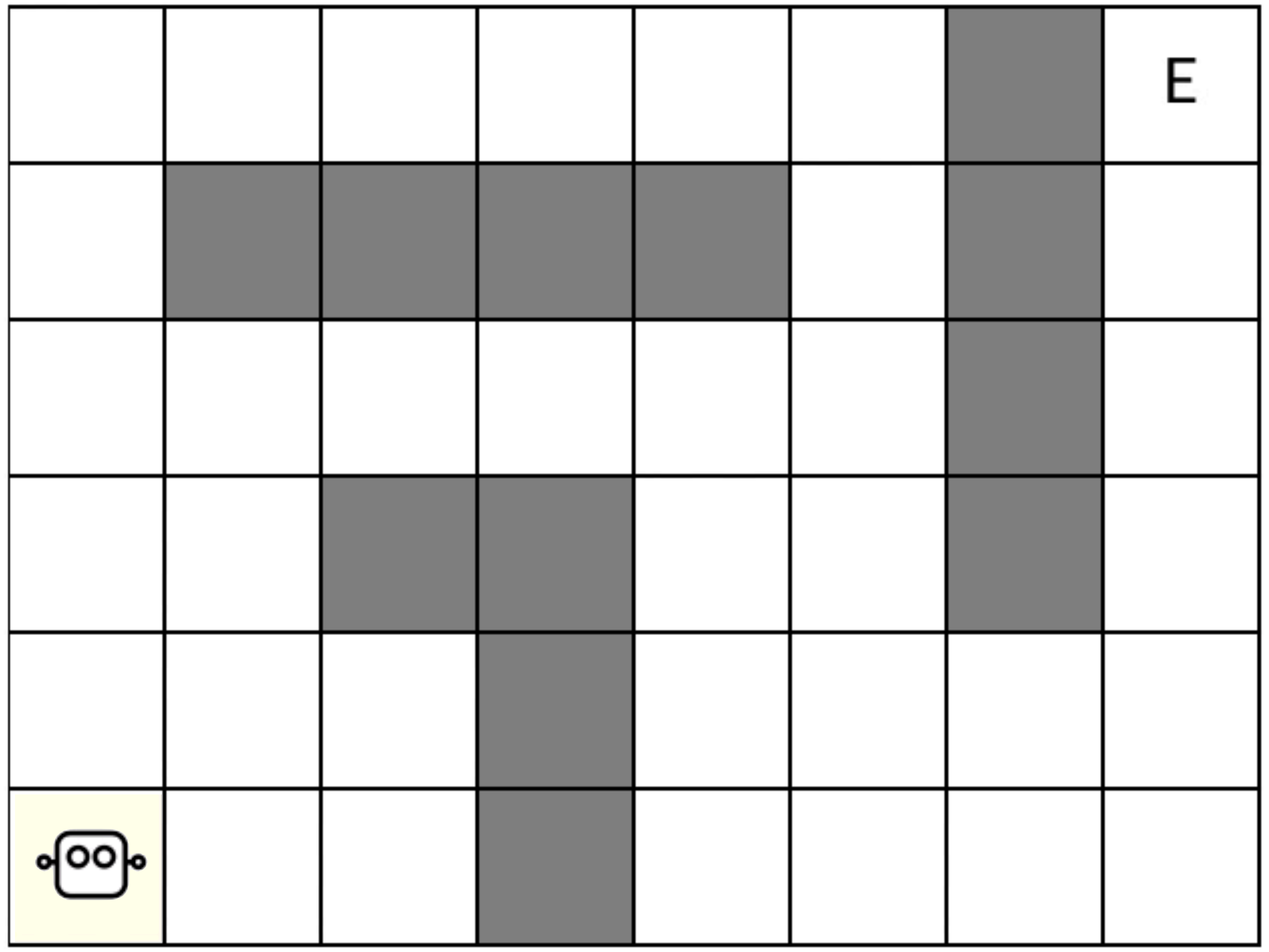
Let's see how much **better** an agent can do with **Dyna**

Agent in the first episode

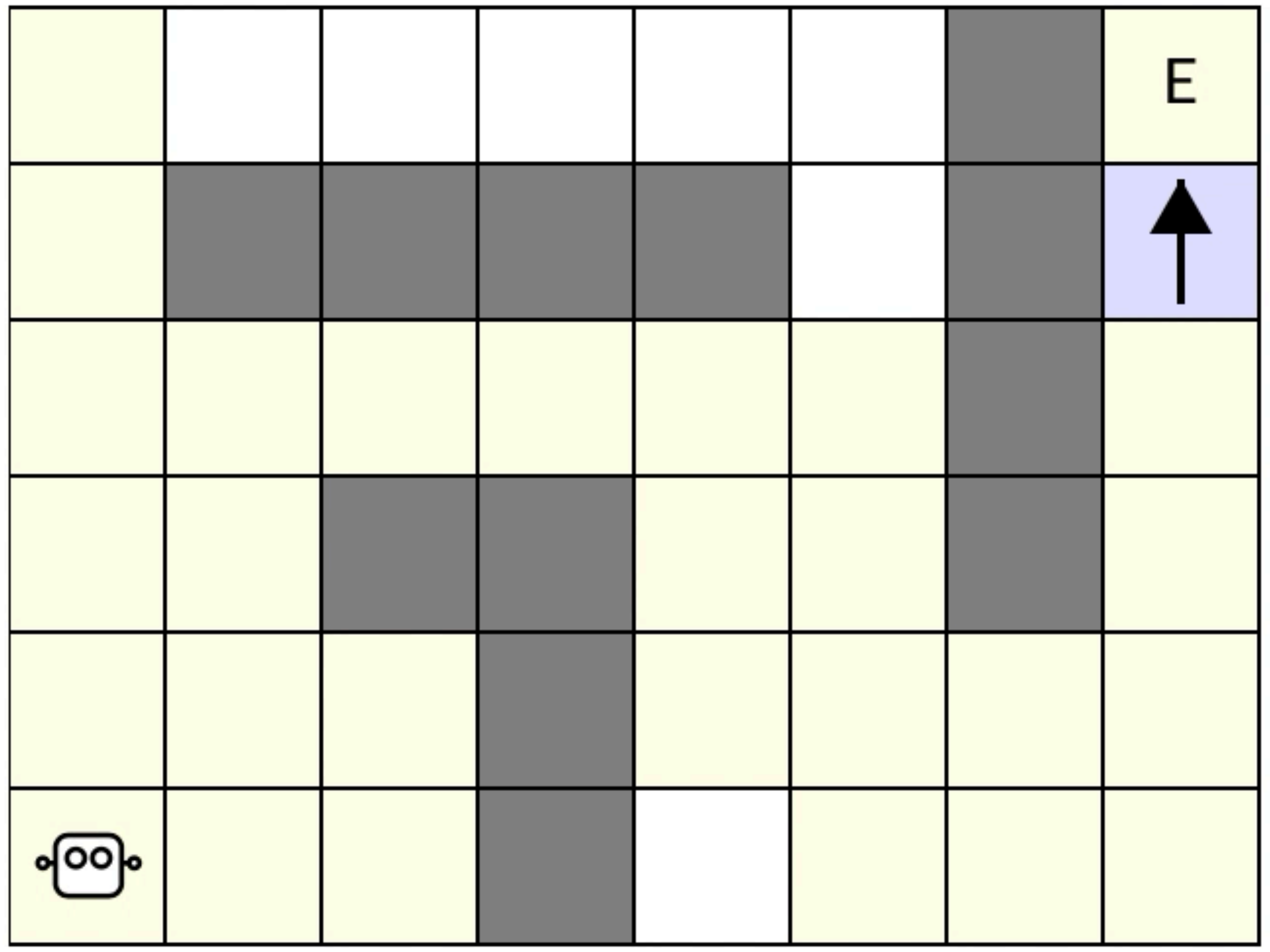


Agent in the first episode

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left(r + \gamma \cdot max_{a'} Q(s',a') - Q(s,a) \right)$$



Agent's knowledge after the first episode



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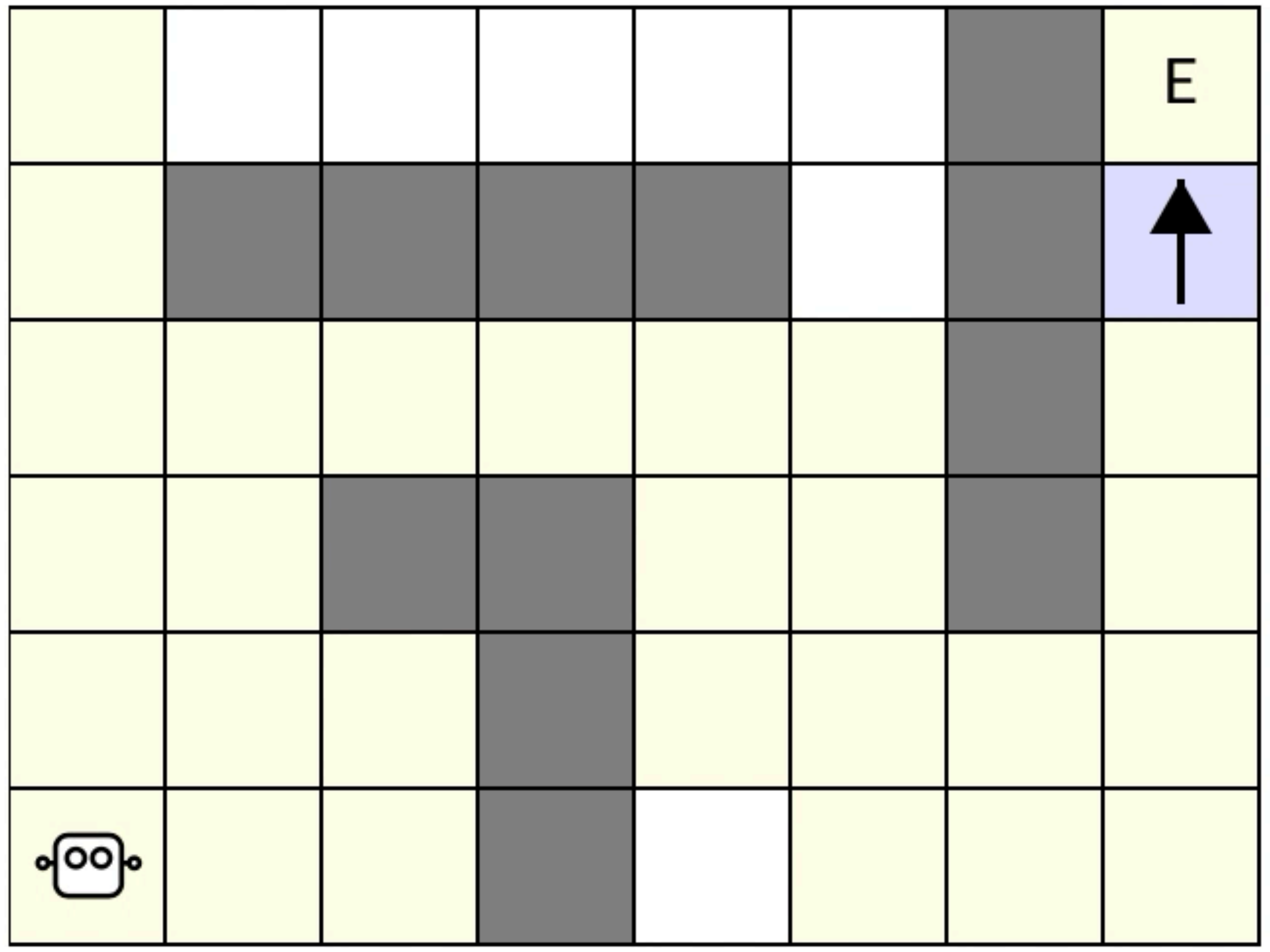
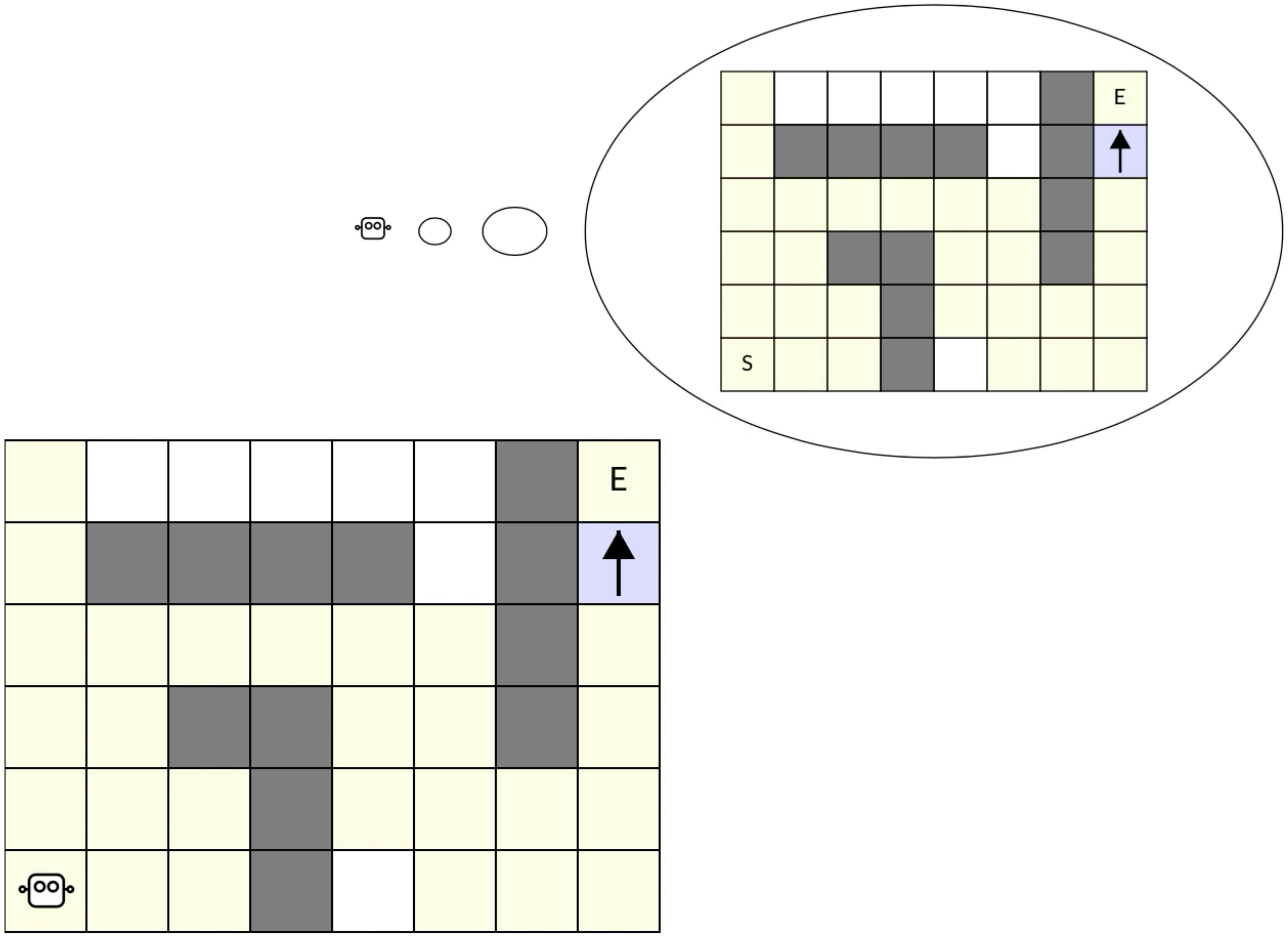


Agent using many planning steps in Dyna



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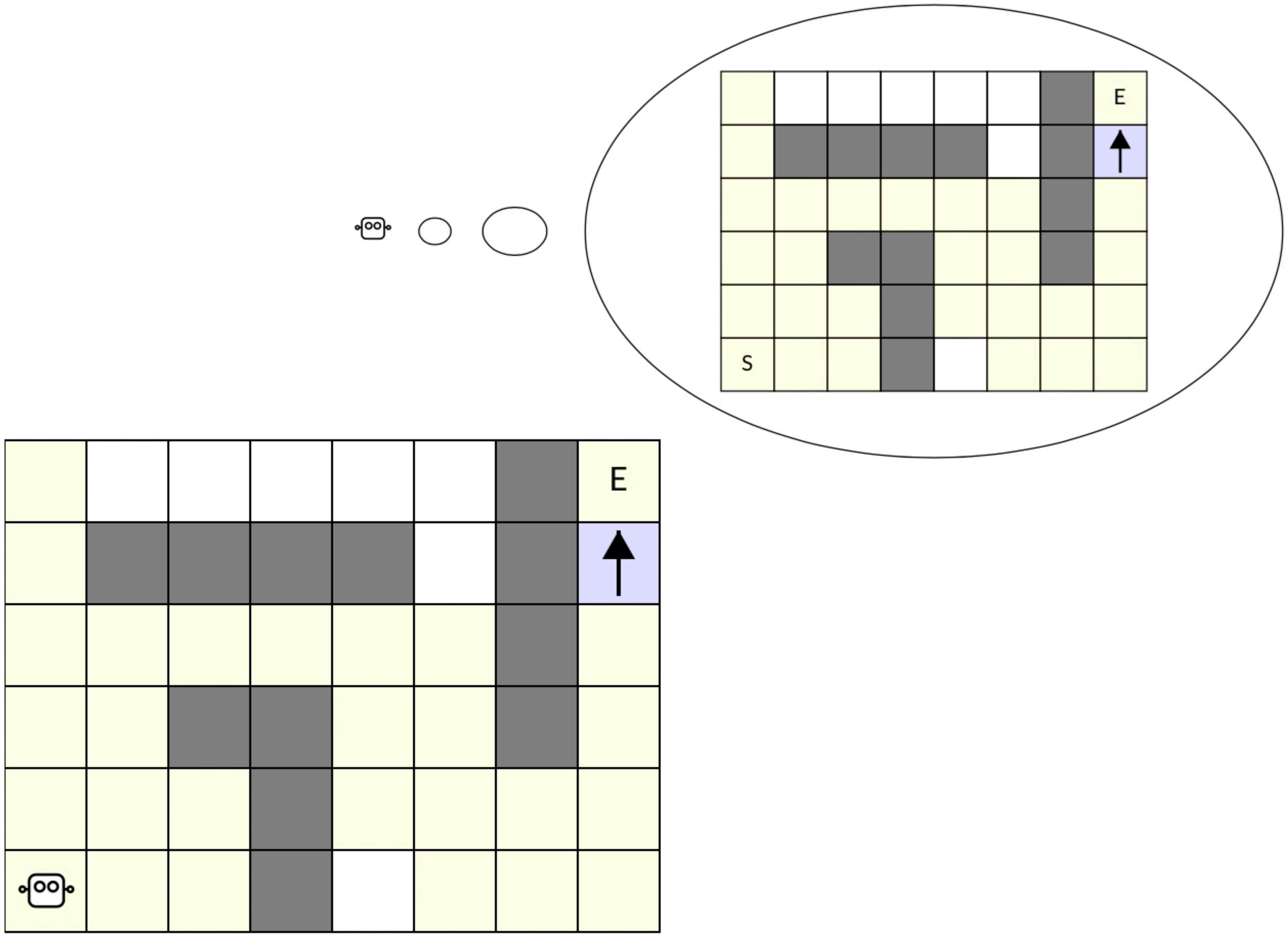
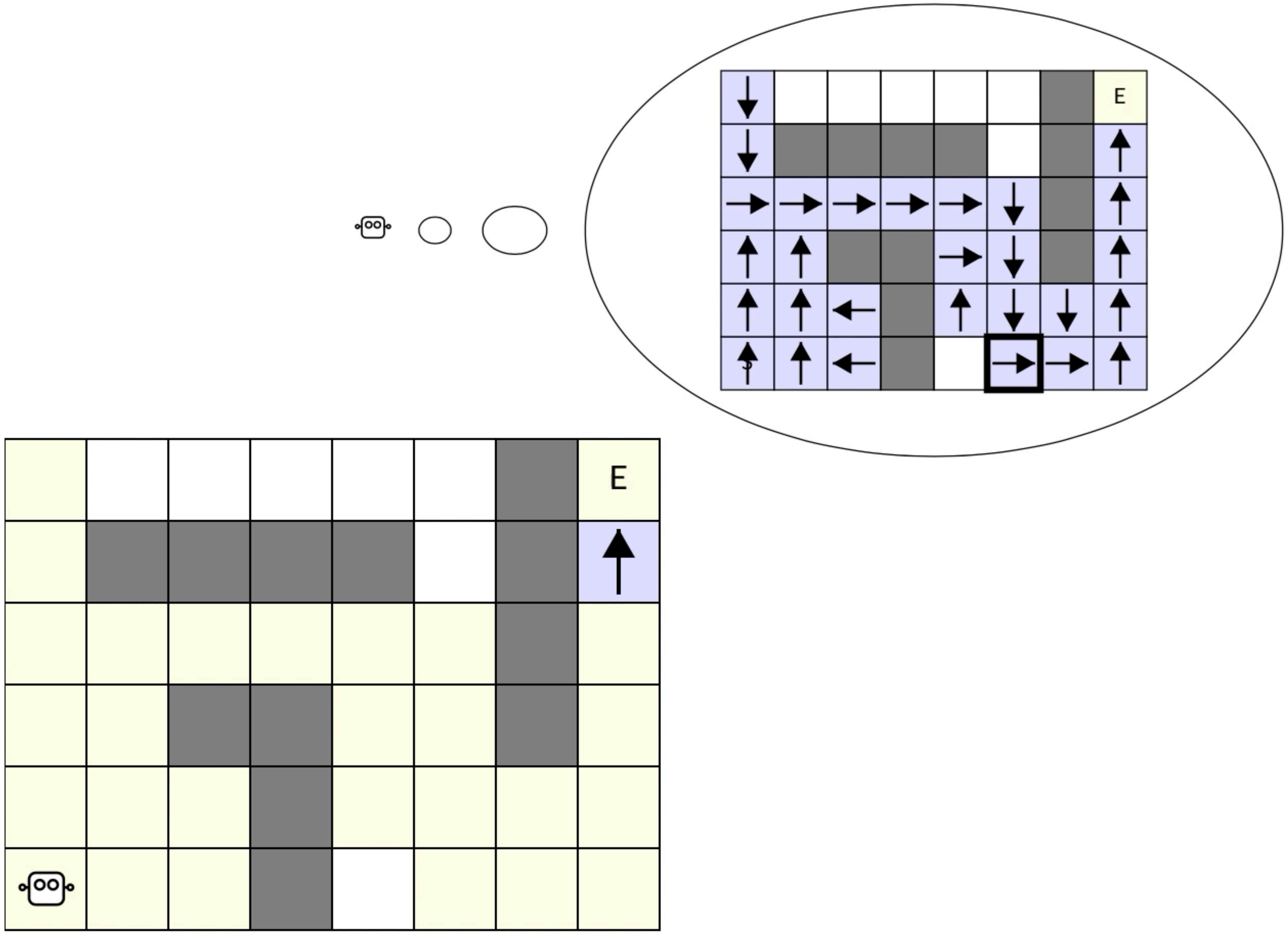


Agent has the optimal policy after just one episode



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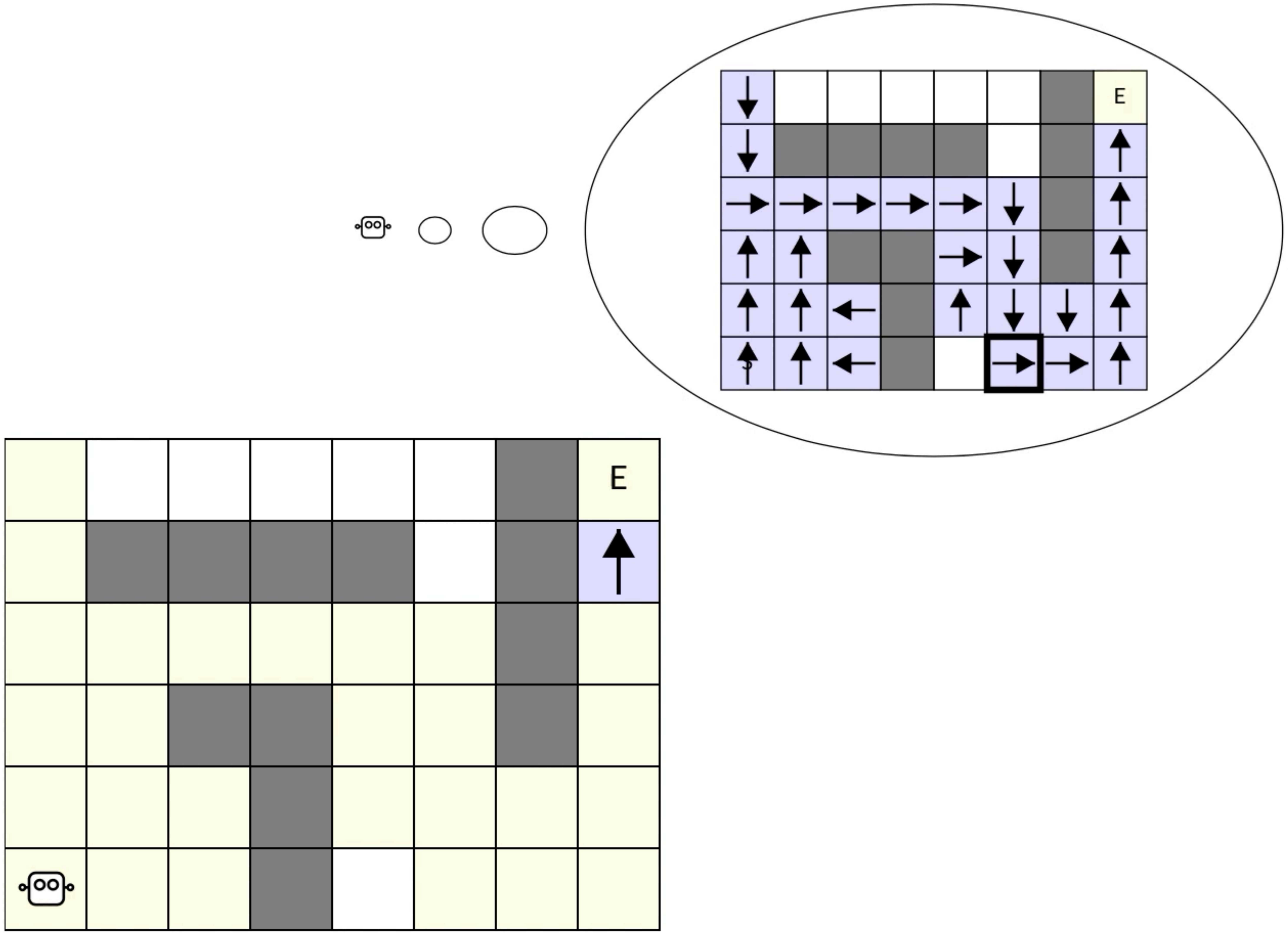
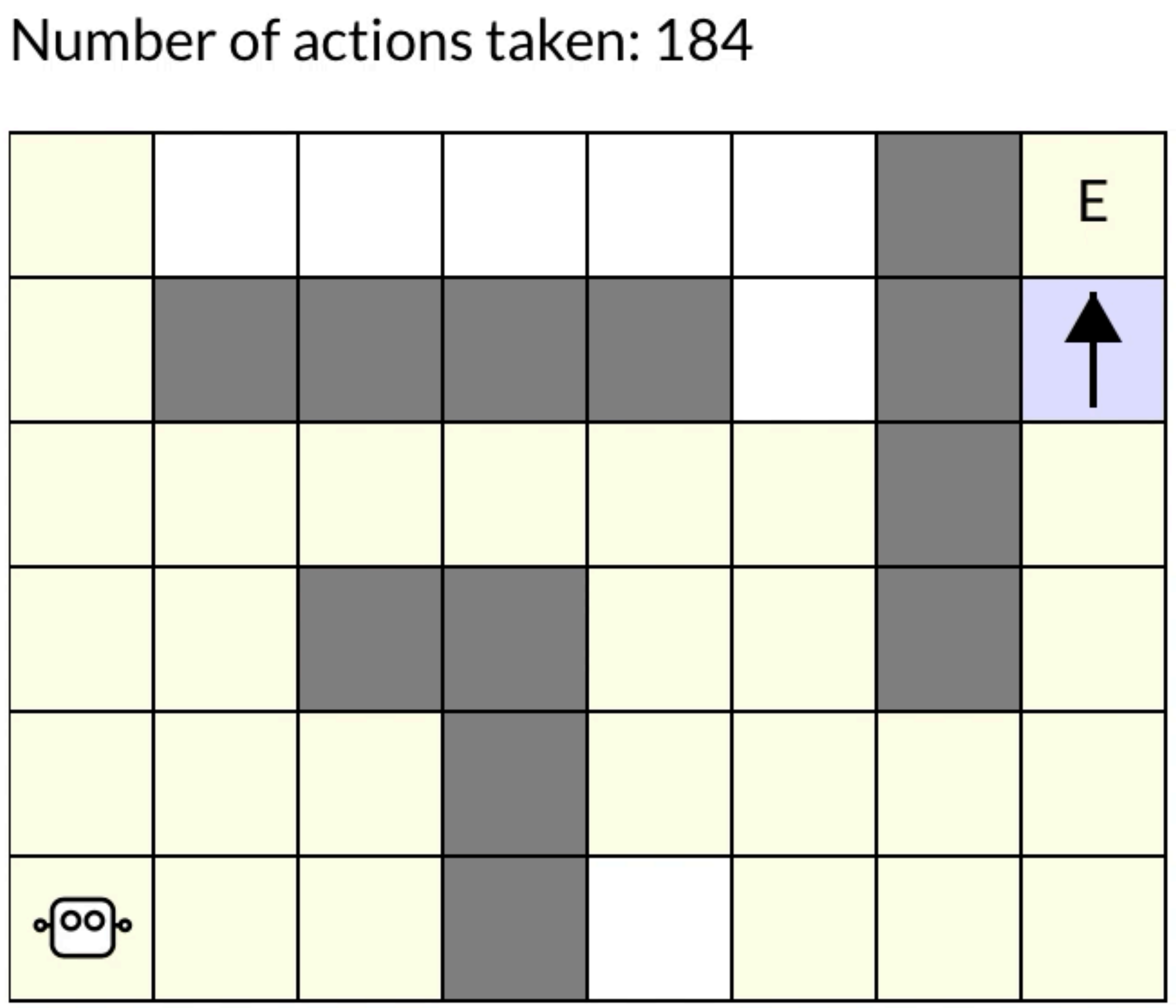
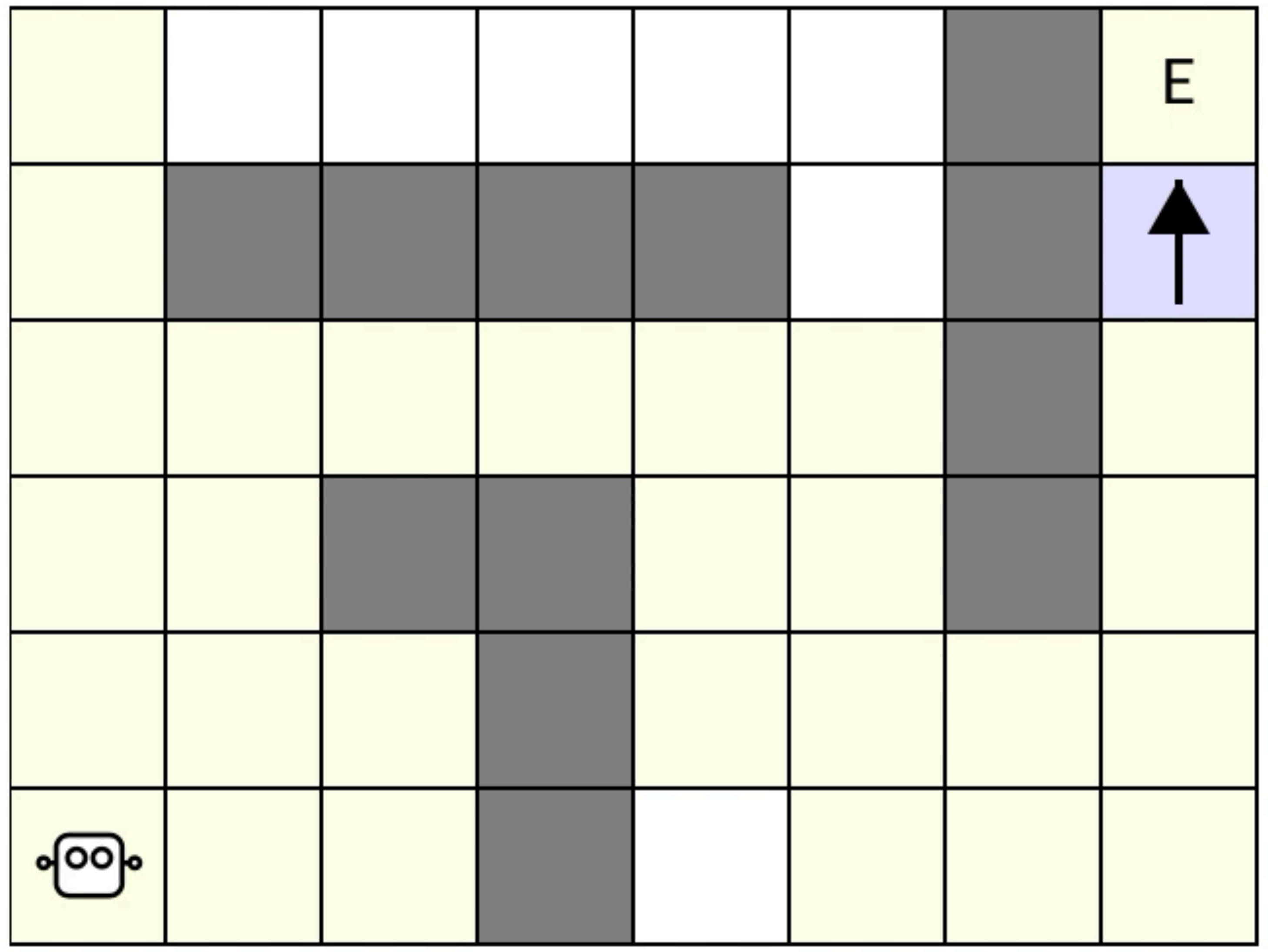


Interleaving planning and acting



Interleaving planning and acting

Number of actions taken: 184



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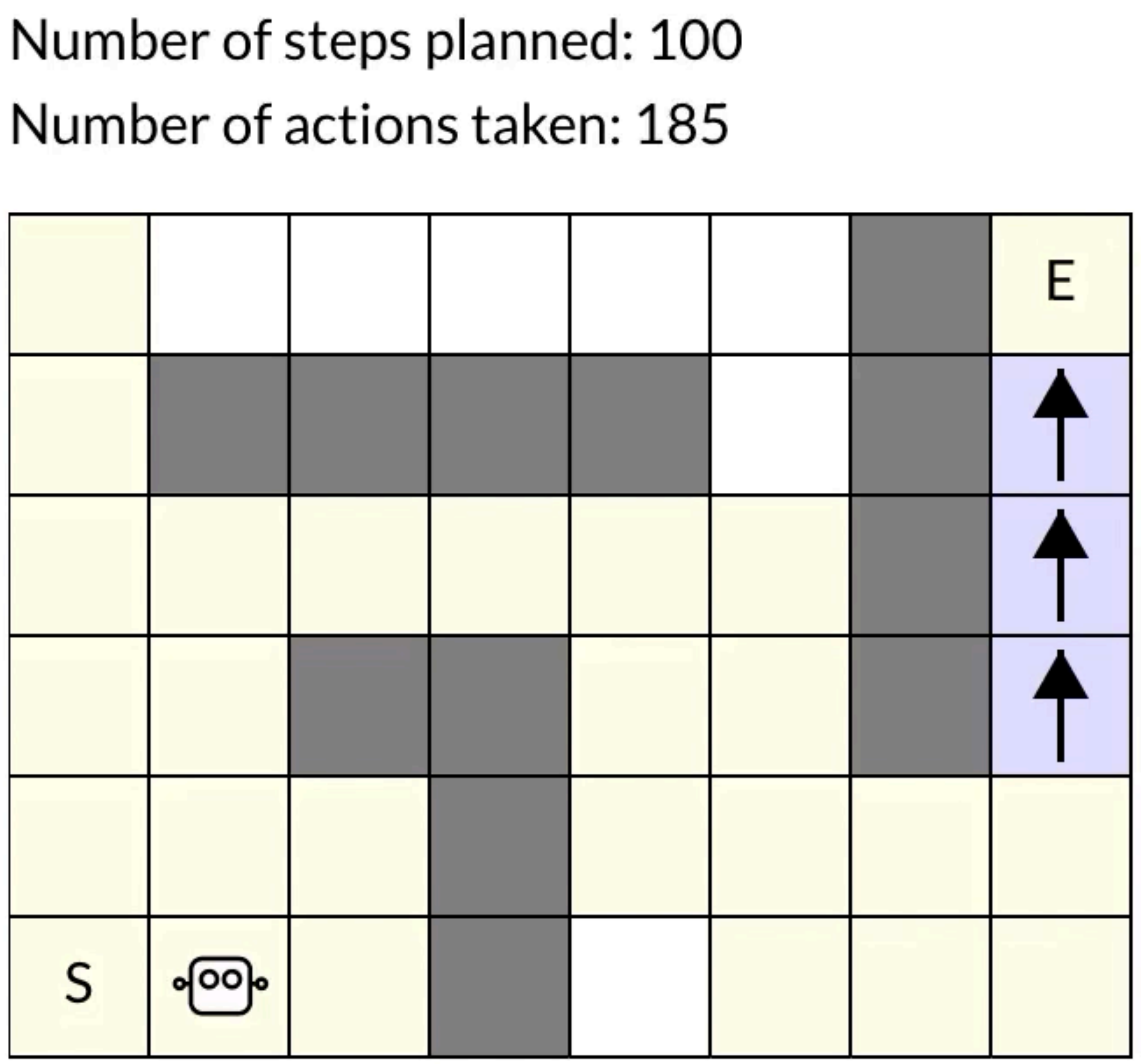
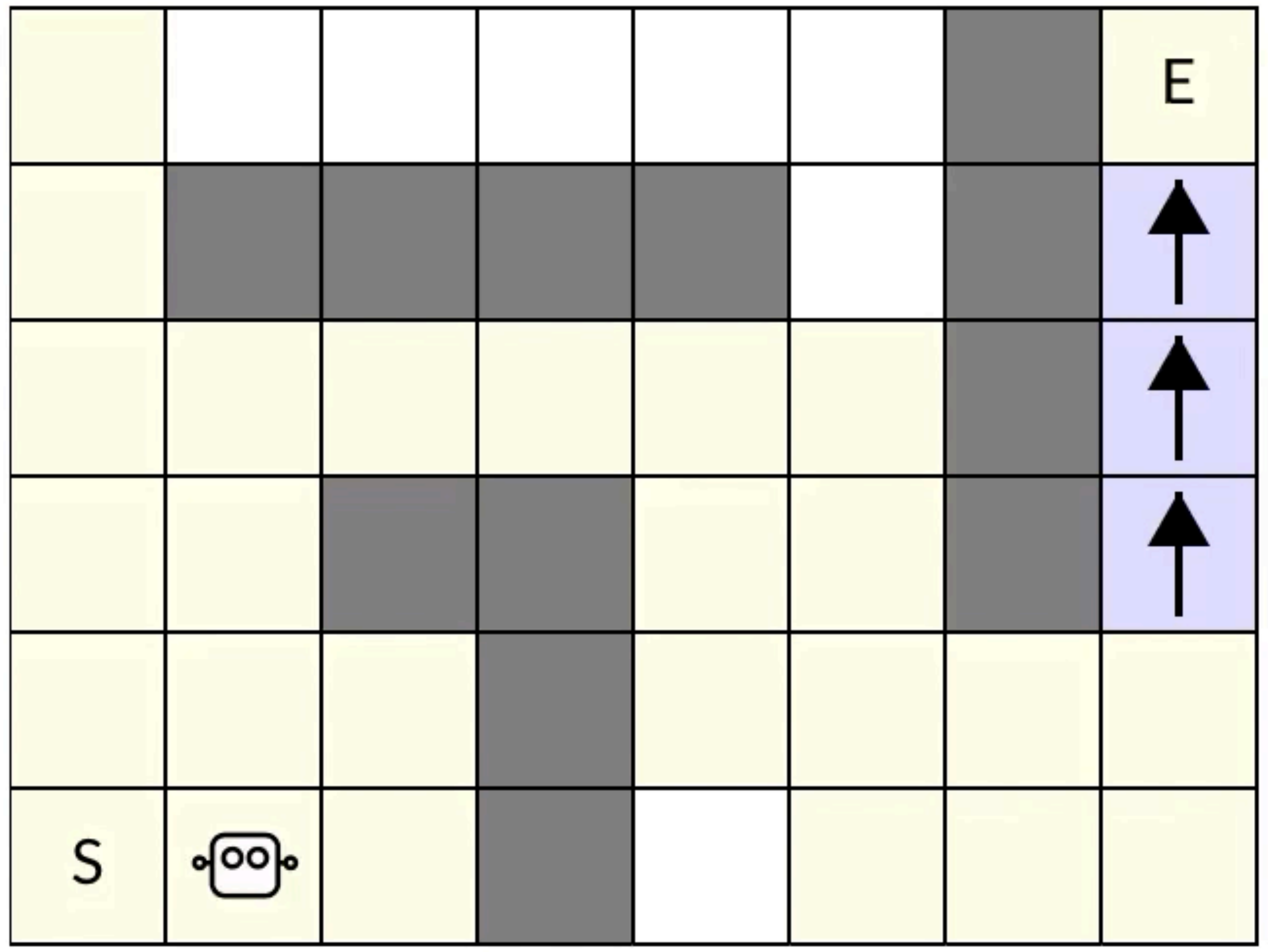


Interleaving planning and acting

Number of steps planned: 100

Number of actions taken: 185



Dyna = Background Planning

- Given **unlimited computation**, each planning update in the background could essentially solve the Bellman equation for the current model
  - Loop over all states and actions many times
  - At extreme of computation, behaves like Dynamic Programming
- In practice, have **limited** computation
- Use any **extra computation** for **background planning**: do as many updates to the value function or policy as computation allows

Advantages of Dyna

- **Anytime** planning (asynchronous, occurs in the background)
  - contrasts Decision-time planning
- Can take advantage of **parallelism**
- Naturally enables **partial models**
- Can still do **long-term planning** use temporal abstraction, but avoids multi-step rollouts

Now let’s dive into specific instances of Dyna

Important choices

- The type of model
- Search-control

Dyna-Q

Initialize $Q(s, a)$ and $Model(s, a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Loop forever:
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Important choices: Model

- The type of model
- Search-control

Dyna-Q

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Let’s look at a **simple** example of **Dyna**:

Experience Replay

Experience Replay

- Essentially using a batch method in an online setting
- Store buffer of recent transitions (s, a, s’, r)
  - e.g., sliding window buffer
- Sample mini-batch updates from the buffer, for updates to the value function or policy

Exercise: How can ER be seen as an instance of Dyna?
What is the choice for the **Model** and for **Search Control**?

Experience Replay Pseudocode

Initialize buffer B and Q(s,a) for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$

Loop forever:

(a) $S \leftarrow$ current (nonterminal) state

(b) $A \leftarrow \varepsilon$-greedy(S, Q)

(c) Take action A; observe resultant reward, R, and state, S'

(d) $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

(e) Add (S, A) to buffer B, drop oldest sample

(f) Loop repeat n times:

Grab random (S, A, S', R) from buffer B

$Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$

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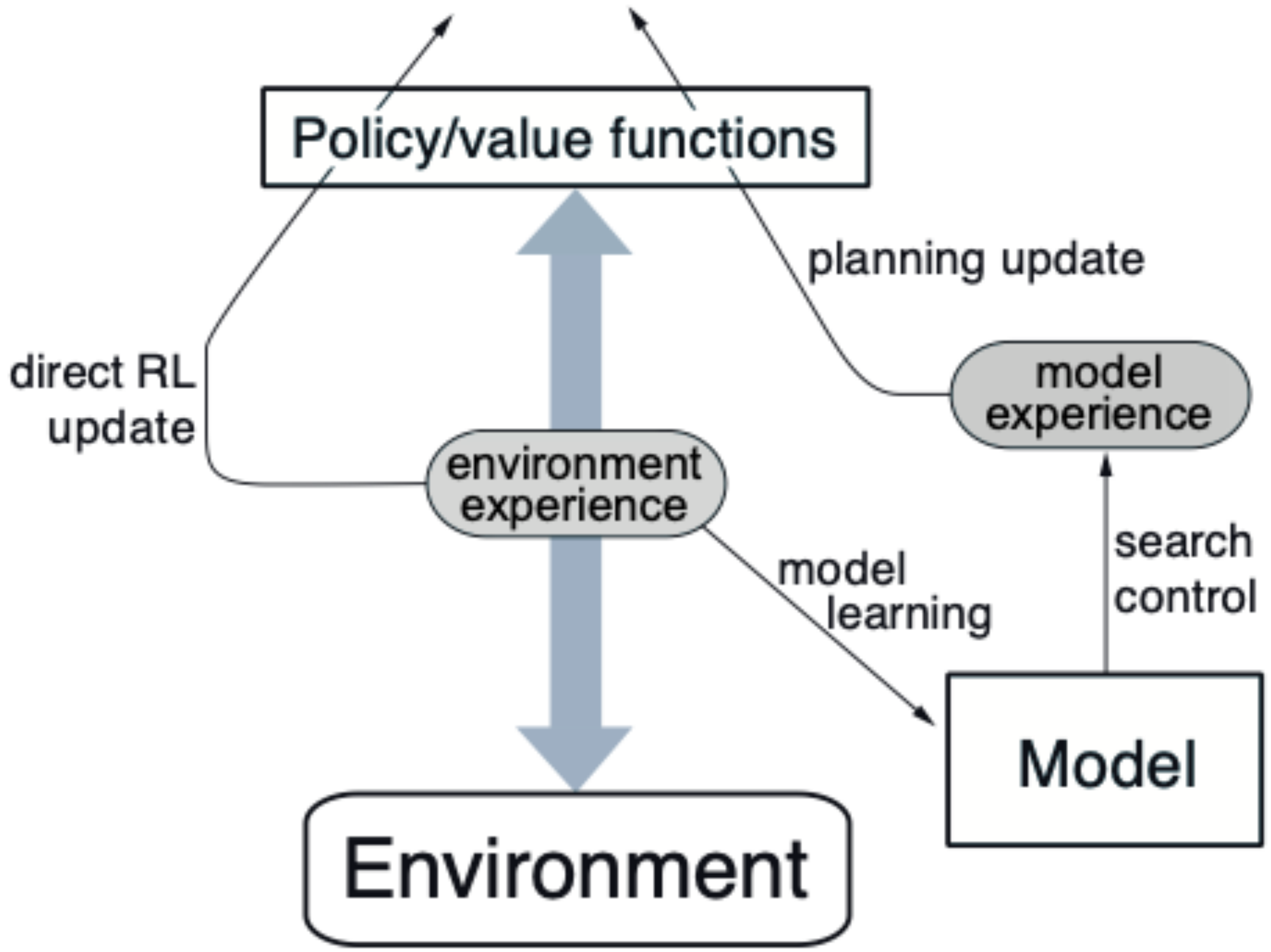
Experience Replay:
A Simple Example of Dyna



s', r ~ M(s,a)

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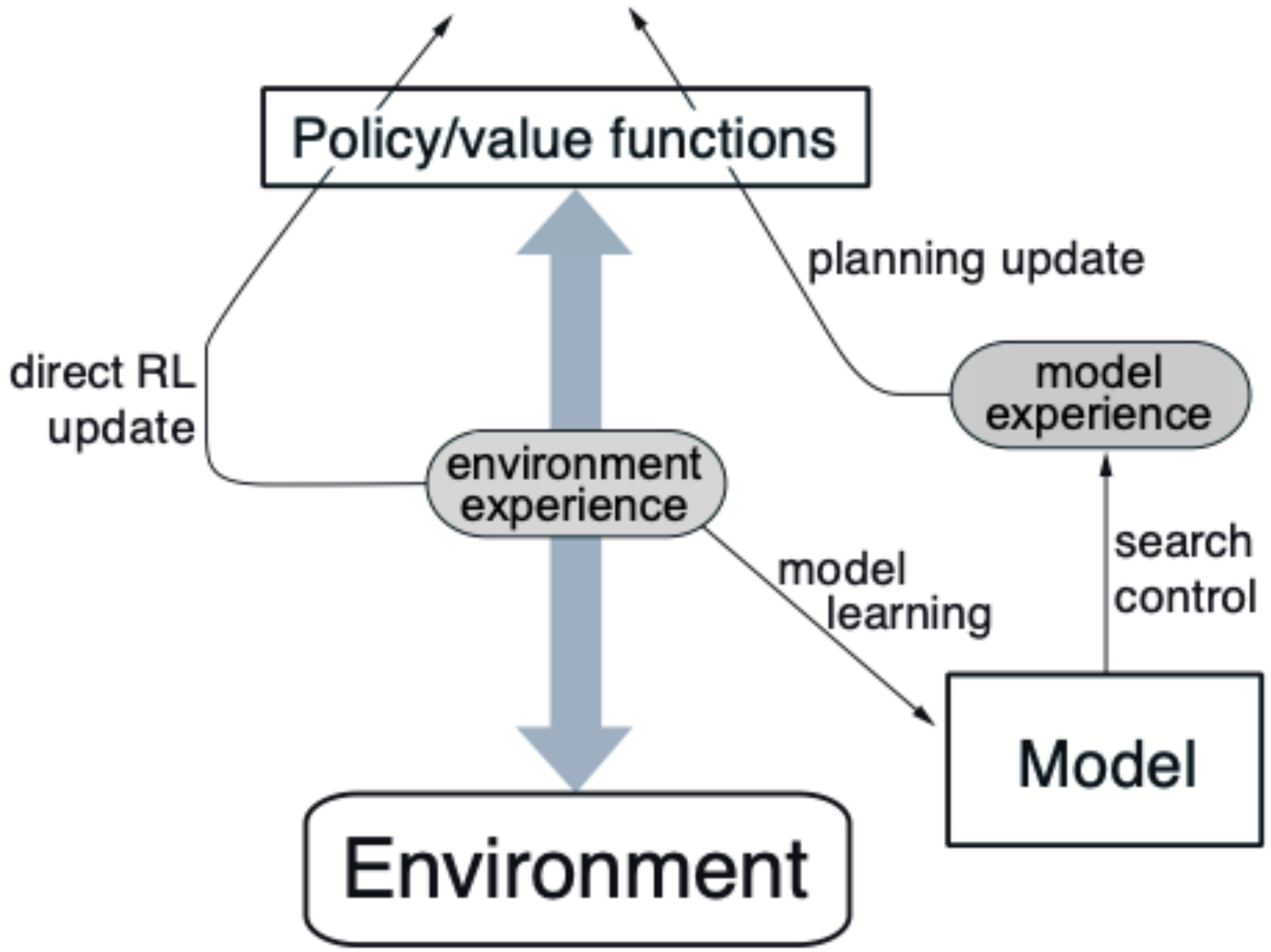

$s', r \sim M(s,a)$

(s,a)

Model is tuples of experience:

(s_0, a_0, r_1, s_1)
(s_1, a_1, r_2, s_2)
(s_2, a_2, r_3, s_3)
…

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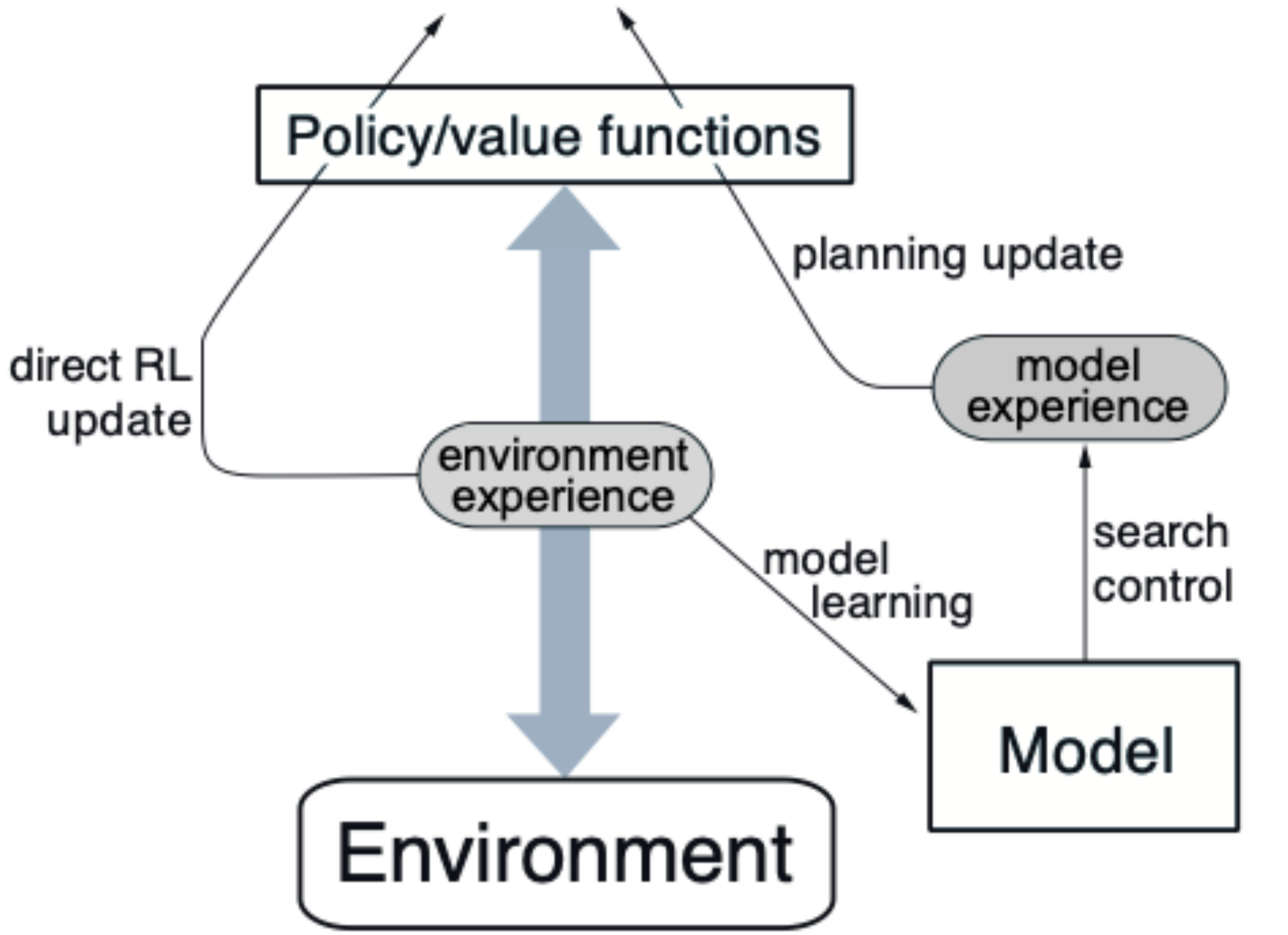

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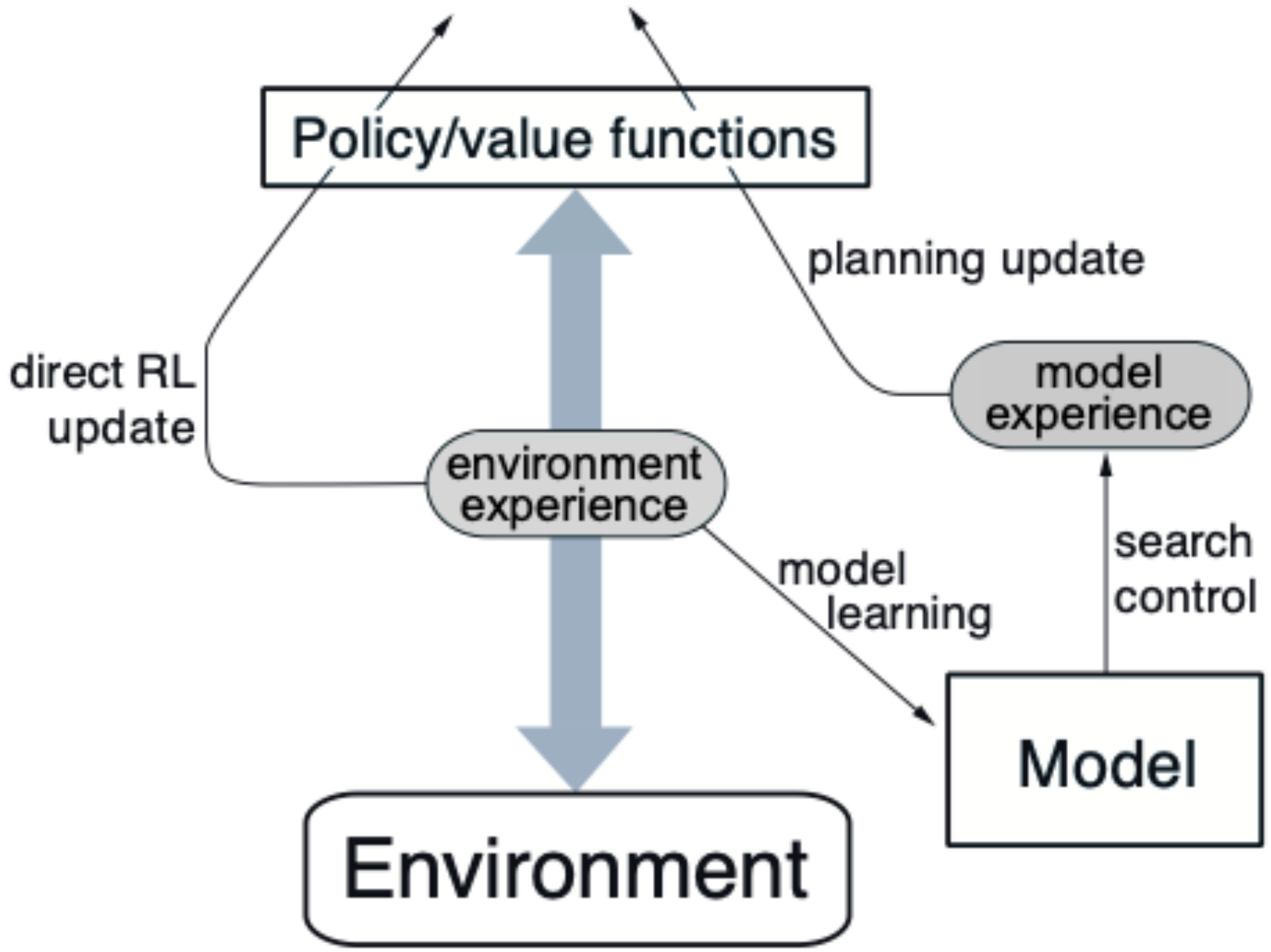

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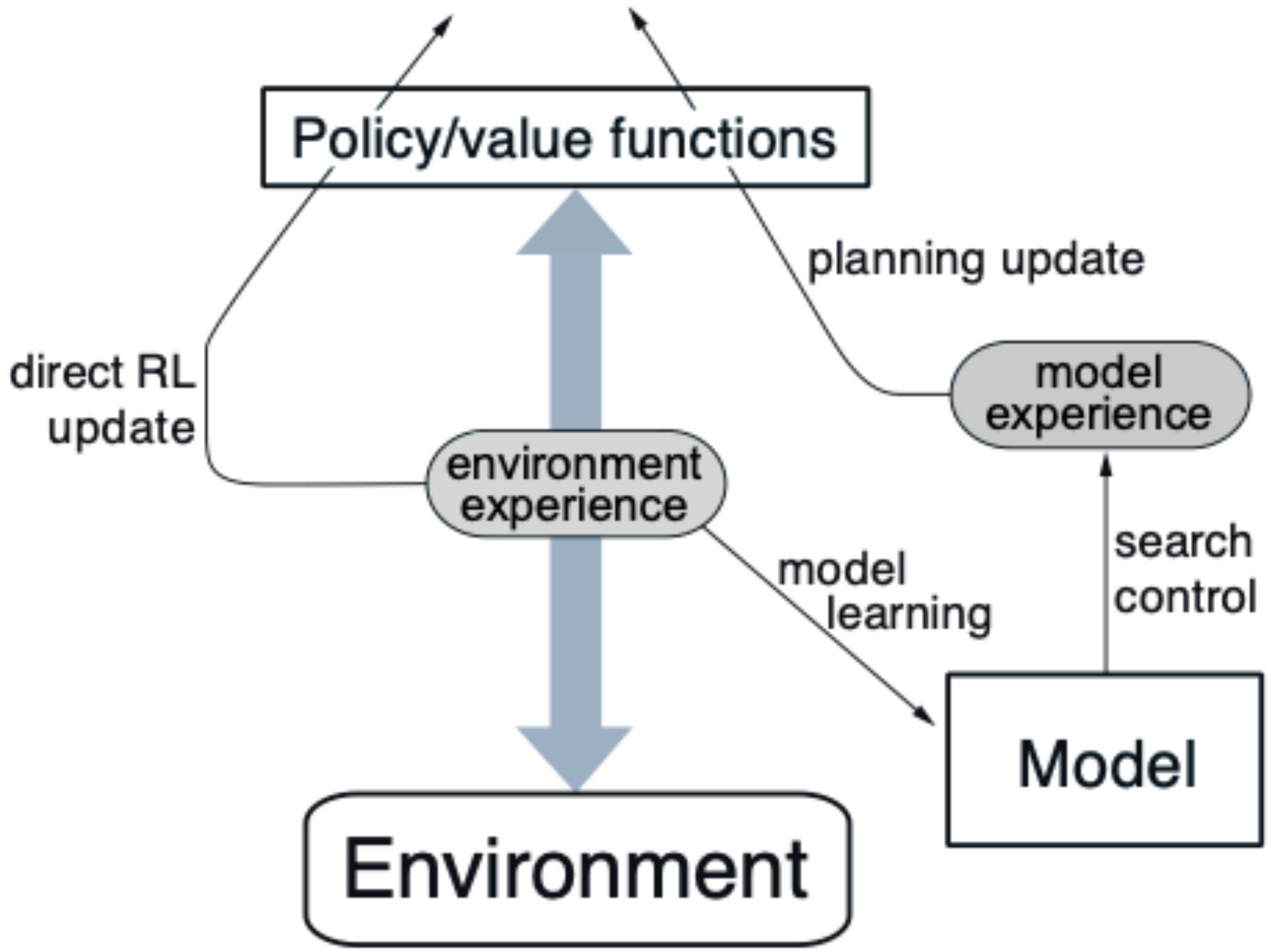

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We should be able to get a better **Model**
and smarter **Search Control**

Advantages of a Learned Model over a Transition Buffer

- **Compactness**: summarizes experience
- **Coverage**: cannot store all experience, so in ER common to use most recent experience (does not cover space)
- **Querying**: can query a model from a particular (s,a)

Important choices: Model

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What are possible learned models?

$$\hat{p}(s', r \mid s, a)$$

Increasing
Abstraction
and/or
Simplicity

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- Most obvious answer: $\hat{p}(s', r \mid s, a)$

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  - agent state = constructed vector summarizing key information

$\downarrow$ **Increasing Abstraction and/or Simplicity**

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- Predictions about **(cumulative) rewards** in the future
- …or even **Q(s,a)**?

Increasing Abstraction and/or Simplicity

So is Sarsa a model-based RL algorithm?

- This question only arises due to being imprecise
- Let’s try to be more precise

What does the model do?

- The **agent** uses knowledge/predictions about the world (a **model**) to
  - improve estimates of the **optimal** value function/policy
  - learn about **new** things **faster**
    - e.g., learn new option policies (new skills)
    - e.g., help agent re-visit parts of the space in non-stationary problems

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Notice now that Q(s,a) does not really count as a model

But the model does not have to be the transition dynamics

Important aspects of the model

- **State-to-State vs Observation-to-Observation**

Models on Agent State

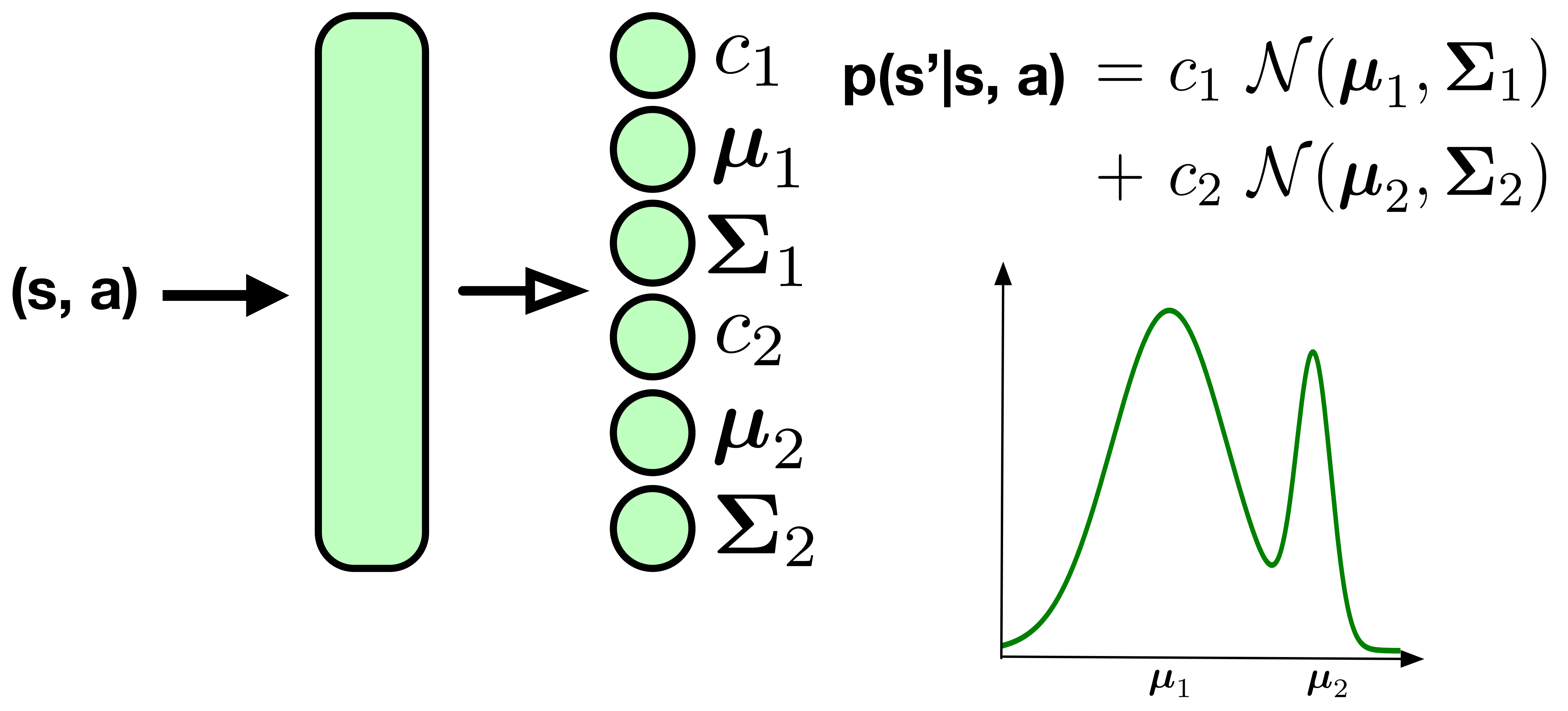
- Construct agent state $\hat{s}$
  - e.g., recurrent neural network to summarize history (POMDPs)
  - e.g., remove unnecessary detail from an image, only keep key info in the agent state needed to make predictions
- Learn one-step model for agent state $\hat{p}(\hat{s}', r \mid \hat{s}, a)$
- Only model what the agent thinks is important, avoid pixel-to-pixel models

Important aspects of the model

- State-to-State vs Observation-to-Observation
- **Expectation vs Sample Models**

Sample Models

- Given (s,a), obtain a **sample** of s' and r
- Examples:
  - Conditional Gaussian distribution
  - Conditional Mixture Model
  - Mixture Density Network



Expectation Model

- Given (s,a), output **expected** next state and reward
- **Exercise**: Imagine you train a feedforward NN with input-output pairs ((s,a), (s', r)), with a squared error
- Would this result in a Sample Model or Expectation Model
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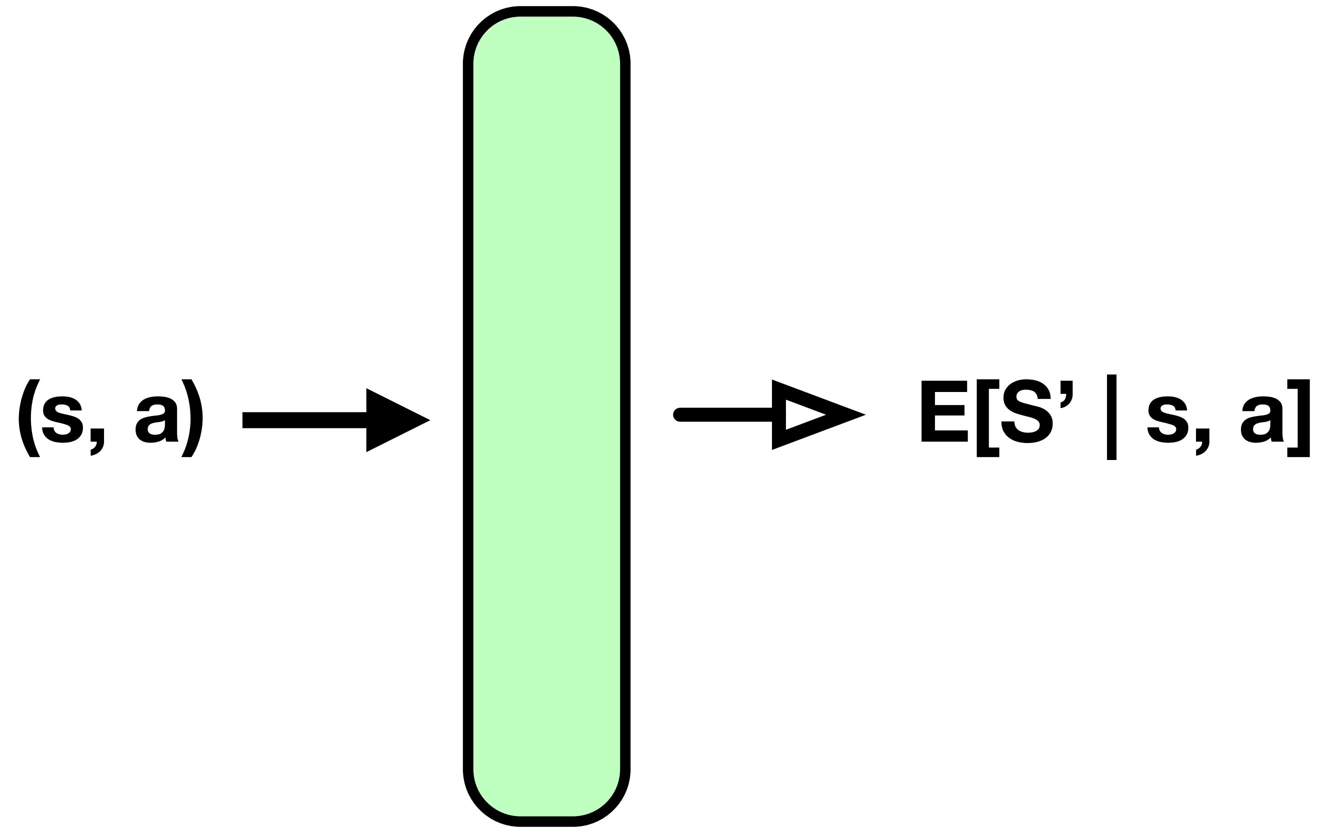
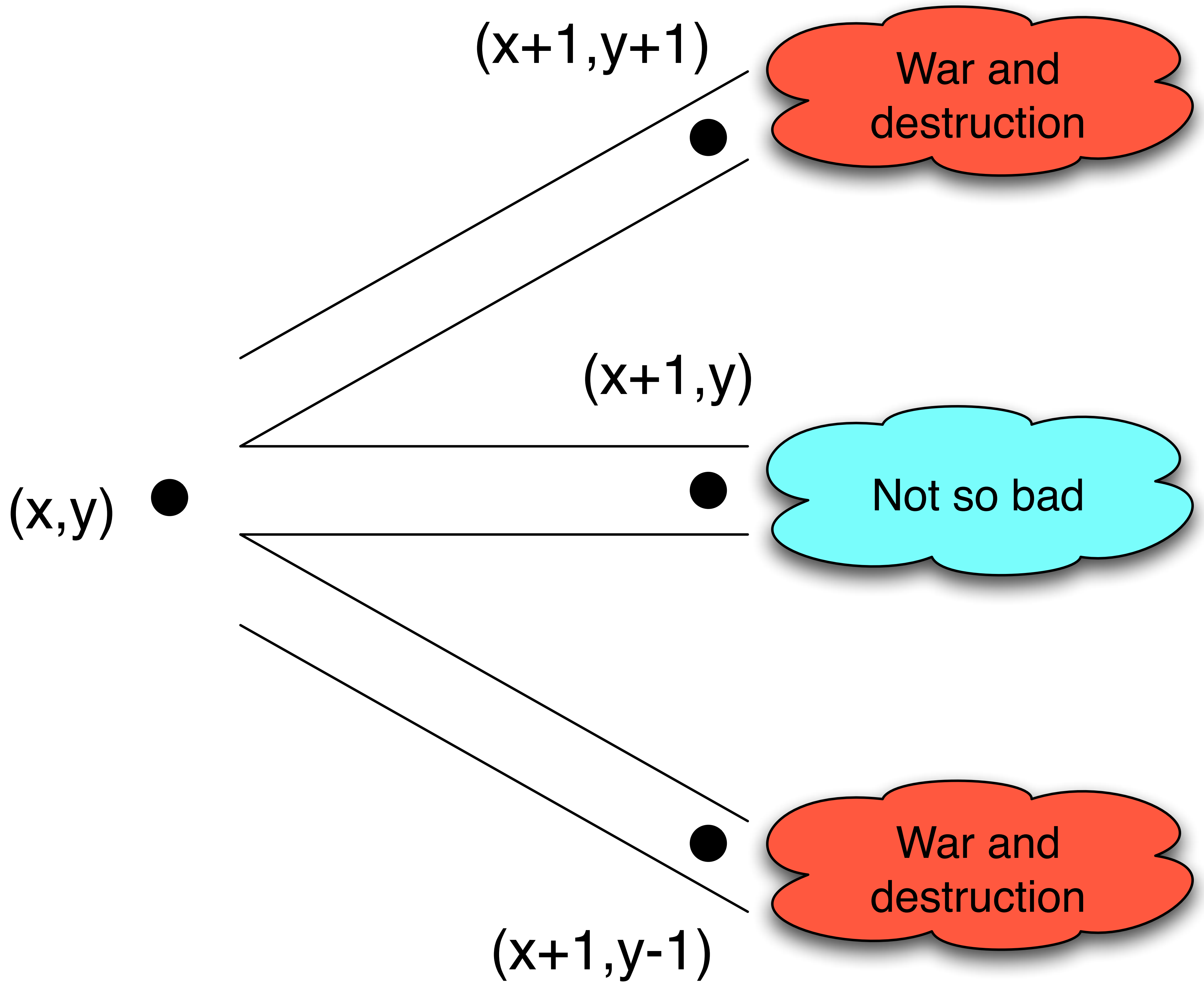
  - or something else?
- **Answer**: Expectation Model

Expectation Model

- Given (s,a), output **expected** next state and reward
- Examples:
    - Linear function of (features of) (s,a)
    - Neural Network



Potential Issues with an Expectation Model



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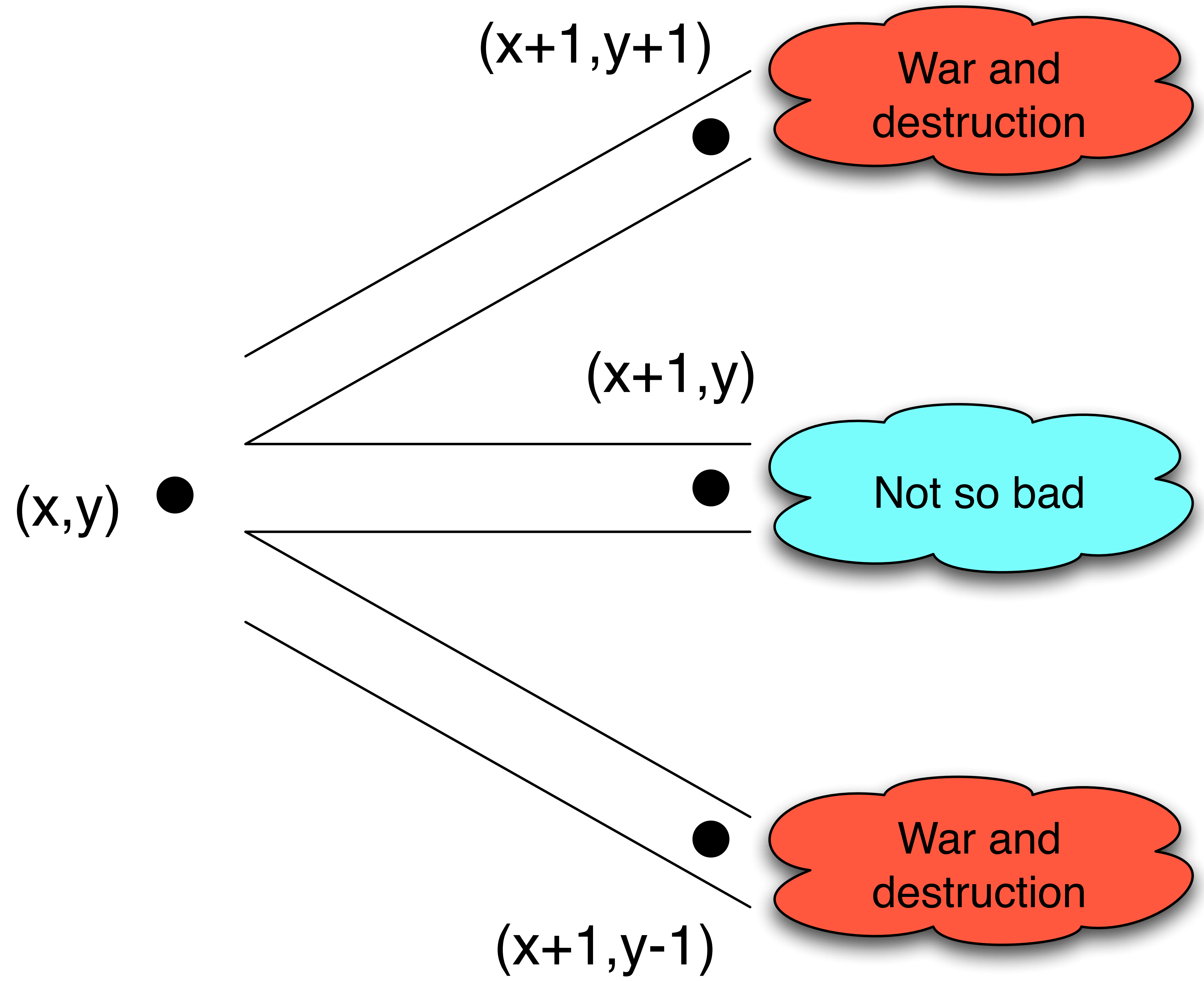

$$
\begin{aligned}
&\mathbb{E}[R_{t+1} + \gamma v(S_{t+1}) | S_t = s] \\
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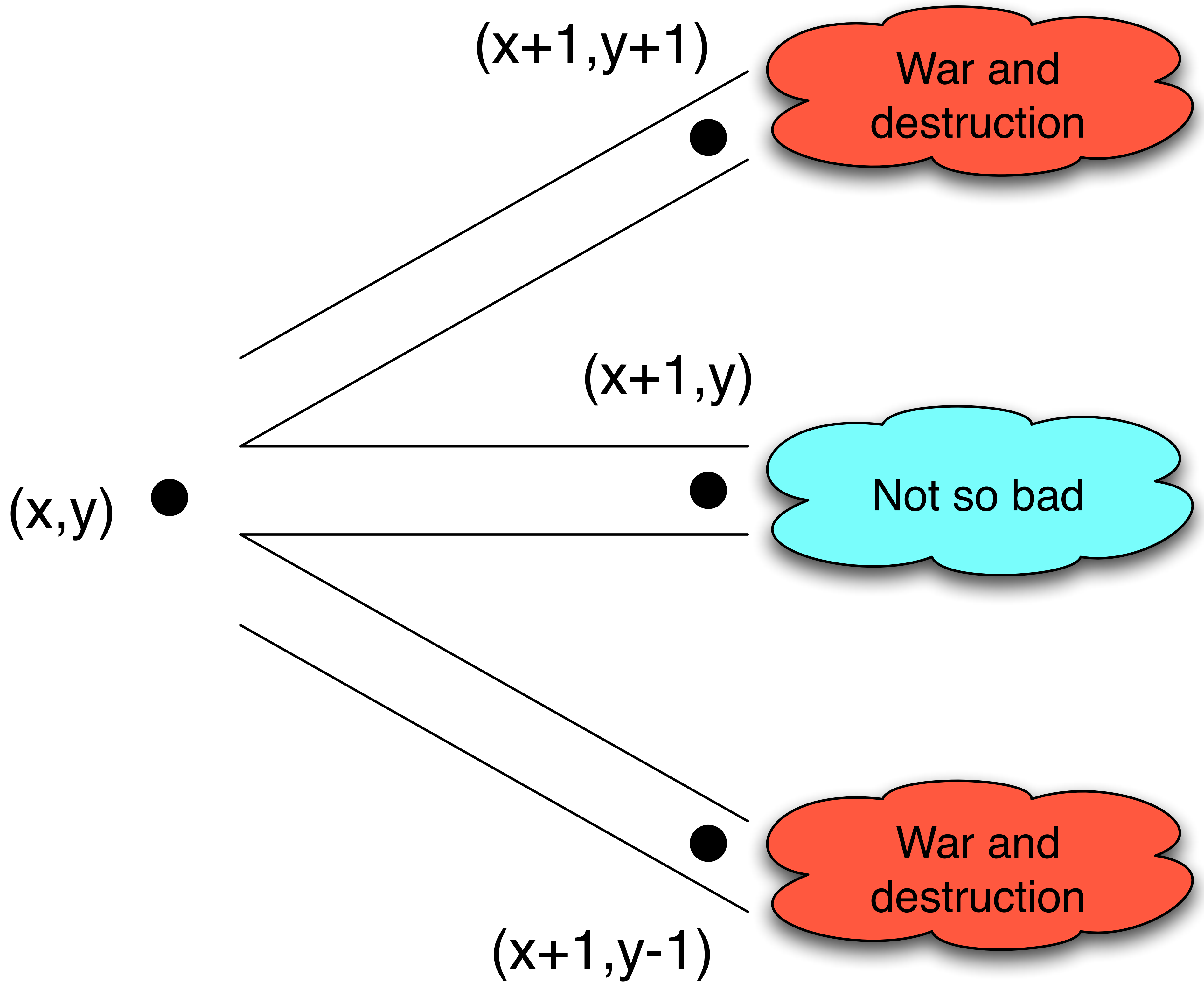

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Some Benefits of an Expectation Model

- Likely simpler to learn
  - Modeling entire distributions more difficult than just statistics like the mean
- If the world is deterministic, then expectation model = sample model
  - …maybe this is not so unreasonable
- If the value function is linear in agent state, then there is no disadvantage to using an expectation model
  - See “Planning with Expectation Models”, Wan et al, 2019

Important aspects of the model

- State-to-State vs Observation-to-Observation
- Expectation vs Sample Models
- **Rollouts vs Temporal Abstraction**

Issues with Rollouts

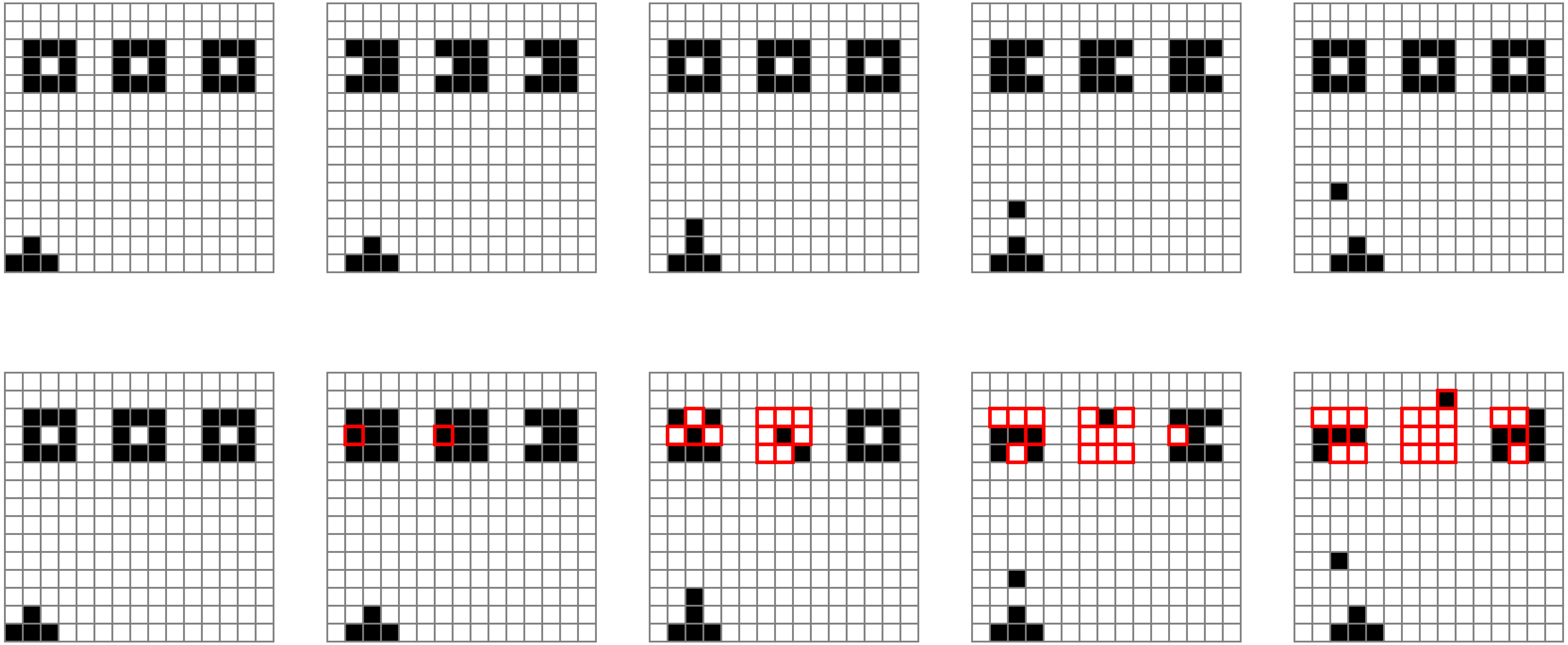

*image from Erin Talvitie, “Self-Correcting Models for Model-Based Reinforcement Learning”, 2017

Temporal Abstraction

- Use options to define **macro-actions**
  - e.g., Imagine a navigation robot. It could have a policy that tells it how to get to the door (policy defined by option, Andre will talk about this more)
- Agent can plan over options, $\hat{p}(s', r \mid s, \pi)$
  - e.g., can ask: "What is the resulting agent-state and (accumulated) reward from a given agent-state when following the option policy?"
- **Advantage**: Can reason about longer horizons (multiple steps into future)
  - Without rolling out the model many steps

Important aspects of the model

- State-to-State vs Observation-to-Observation
- Expectation vs Sample models
- Rollouts vs Temporal abstraction
- Full transition dynamics or a subset of predictions about the future
- Whether model outputs certainty estimates
- Sample efficiency in learning the model
- Computational efficiency for querying/sampling from the model

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Any other suggestions?

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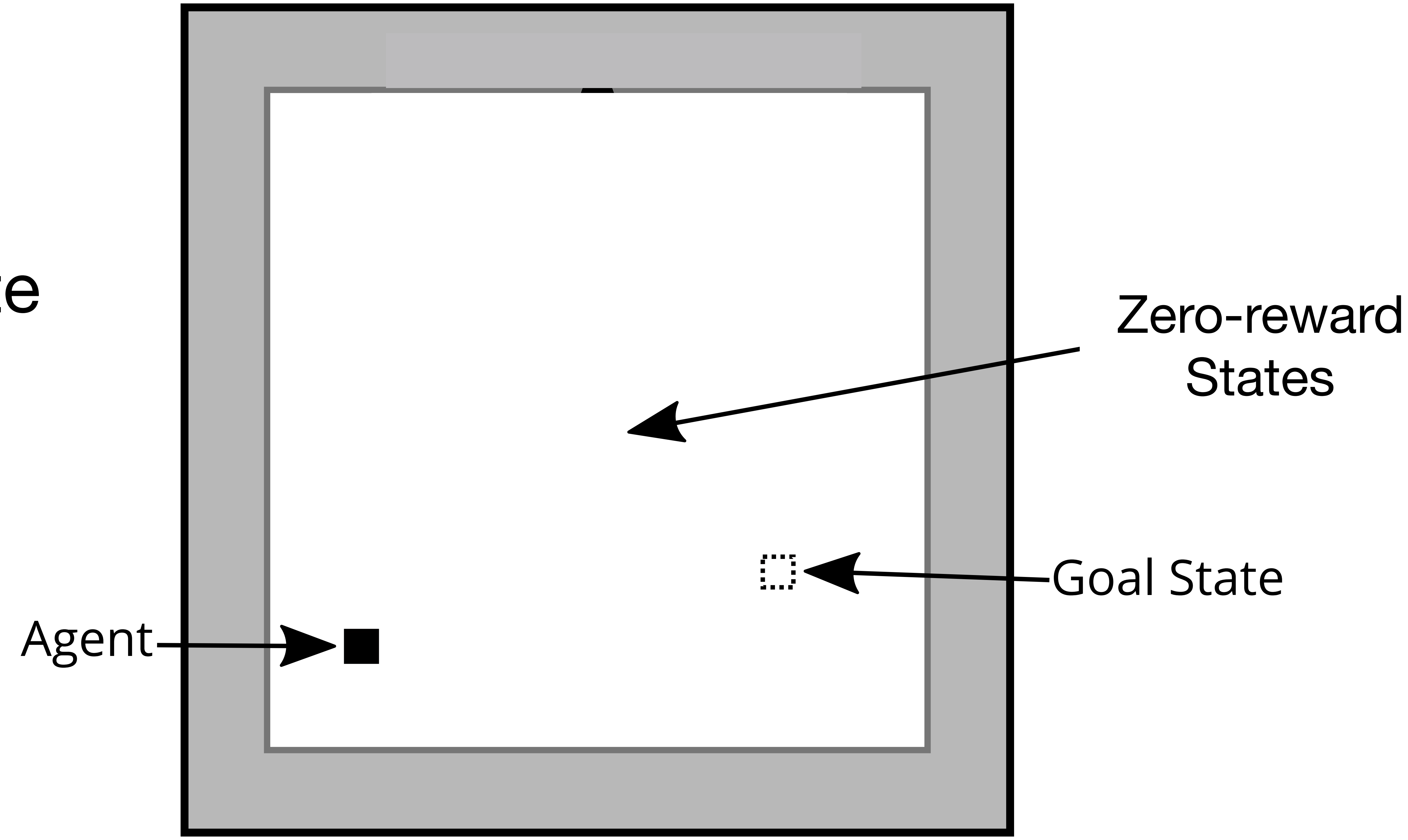
The utility of updating with predecessor states

- Imagine agent initializes values to zero
- Updates in the center are all zero!
- Then imagine it reaches the Goal State and transitions back to the Start
- What happens if the agent updates around the start state now?
- What happens if the agent updates predecessors around the goal?



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- See “Organizing experience: a deeper look at replay mechanisms for sample-based planning in continuous state domains”, Pan et al, 2018

Search Control strategies: How to pick (s,a)

- Prioritize samples with high error
- Update backwards from “important” states
- Update with on-policy transitions
- **Update current state or nearby region around current state**
  - improve values right before they are used
  - see Dyna-2 (Silver et al., 2016), “Hill Climbing on Value Estimates for Search-control in Dyna”, Pan et al., 2019

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- **Two key take-aways:**
- How we use the model (search-control) can have a huge impact on how useful it is (can have very little impact, just a waste of computation)

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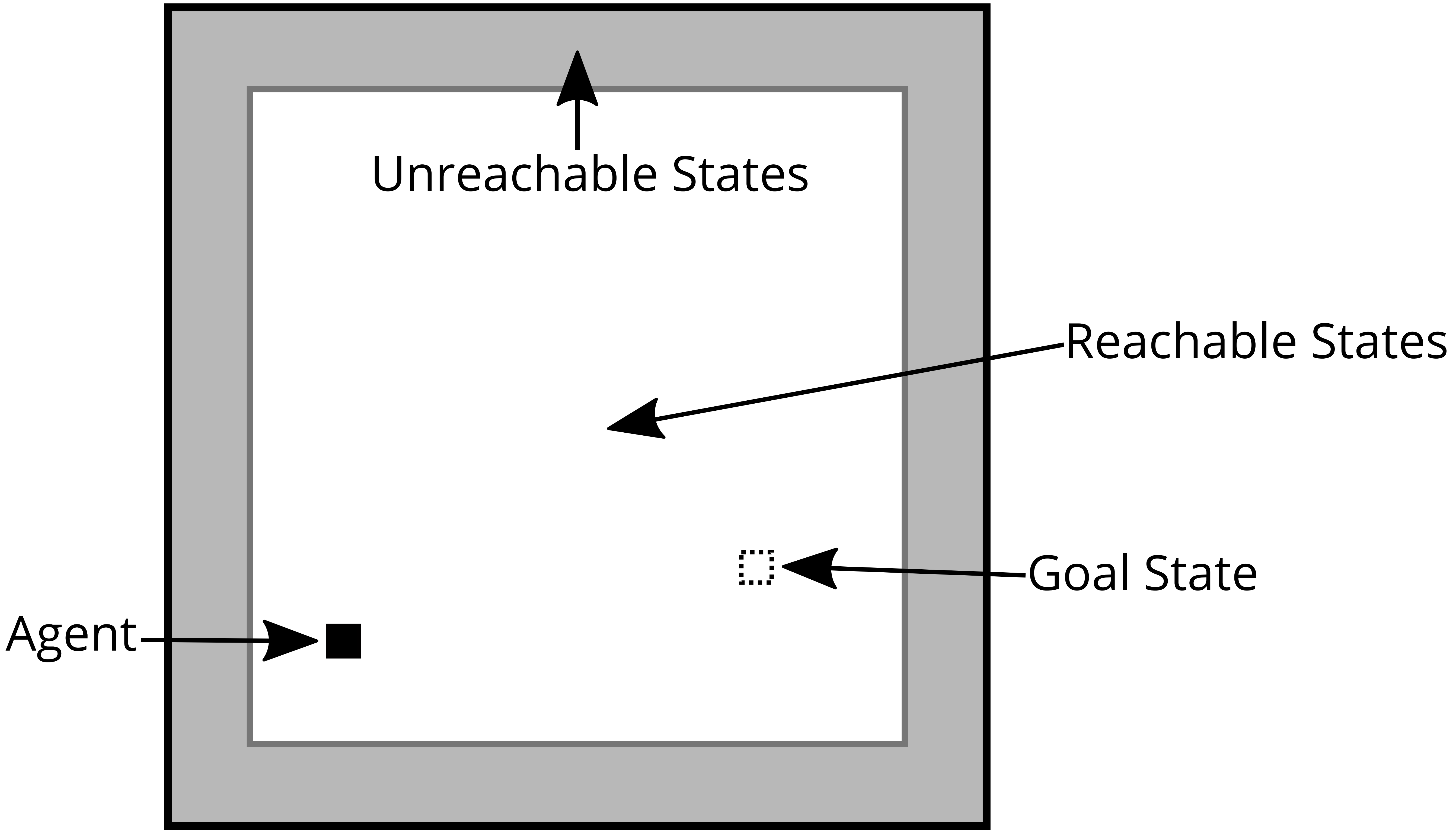
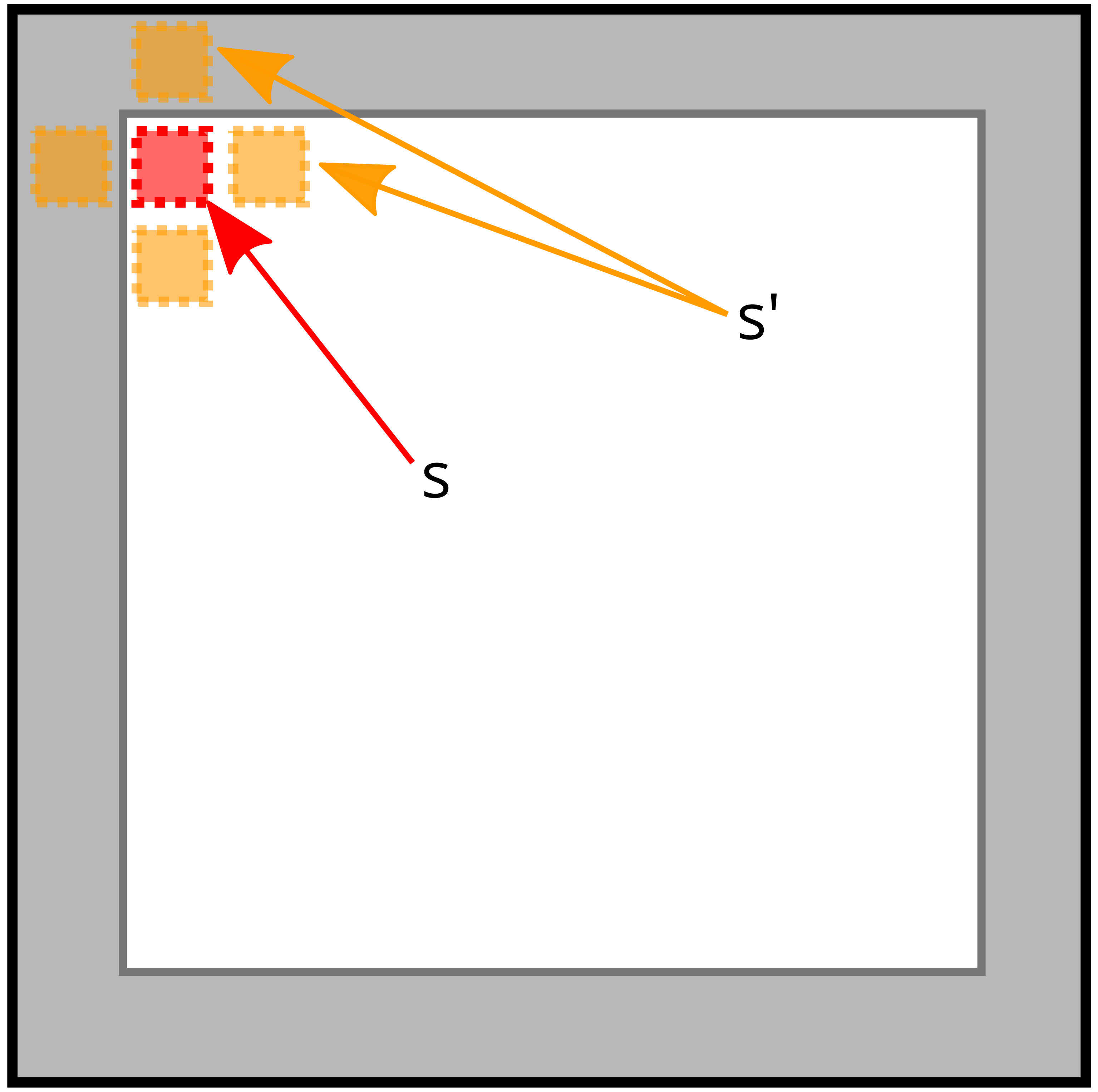
- Can we just learn an accurate model with a deep NN and use Dyna?
- **Two key take-aways:**
- How we use the model (search-control) can have a huge impact on how useful it is (can have very little impact, just a waste of computation)
- Small errors in the model can result in big errors in the policy

Examples of Bad Errors





- Imagine we initialize optimistically
- Imagine we do search-control from observed states

*** credit to Taher Jafferjee**

High-level Outline

- Part 1: Learning the optimal policy given the model (offline)
- Part 2: Moving to learned models (online)
- **Part 3: A brief discussion about other ways to use models**

Models are useful. They have been used in a variety of ways in RL.

- **Most related**: Learn a model and then use dynamic programming on this learned model to obtain approximate values
  - e.g., KBRL, KBSF, Compressed CME, Pseudo-MDPs
- Decision-time Planning
  - Model Predictive Control (see work from Byron Boots), MCTS
- Use model to improve exploration
- Use model to obtain better estimates of policy gradients (PILCO)
- Use model as inductive bias on value function (e.g., Predictron)

KBRL, KBSF, and CCME

- Learn values only for a representative set of points
- Define smaller (pseudo)-MDP only on these states
- Use value iteration (dynamic programming) on this smaller MDP, which is reasonably efficient
- Value function for whole state-space a simple weighting of the values for these representative states

Exploration with models

- Huge research area using learned models for sound exploration
  - Often consider an optimistic model in the set/distribution of models
  - Most algorithms though are very computationally expensive
- Reward bonuses: accuracy of learned models to incentivize exploration
  - Only indirectly using model, no planning

Implicit Planning

- Optimize model and planner based on the reward the agent receives, using end-to-end learning
  - Contrasts learning the model using a separate objective and updating using explicit planning steps
- Can be seen as an inductive bias on value function architecture
- Examples:
  - Predictron (DeepMind)
  - TreeQN and ATreeC (Whiteson and others)

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Questions?