Primal problem:

$$\min_{\beta_0,\beta} \frac{1}{2}||\omega||^2 + C\sum_{i=1}^{N}\xi_i \tag{1}$$

$$\text{s.t} \quad \xi_i \geq 0, \quad y_i\left(x_i^T\omega + b\right) \geq 1 - \xi_i \quad \forall i.$$

Lagrangian function:

$$L(\alpha,\eta) = \frac{1}{2}||\omega||^2 + C\sum_{i=1}^{N}\xi_i - \sum_{i=1}^{N}\alpha_i\Big\{y_i\left(x_i^T\omega + b\right) - 1 + \eta_i\Big\} - \sum_{i=1}^{N}\eta_i\xi_i \tag{2}$$

Apply KKT,

$$\frac{\partial L(\omega,b,\eta)}{\partial \omega_i} = 0 \quad \longrightarrow \quad \omega = \sum_{i=1}^{N}\alpha_i y_i x_i \tag{3}$$

$$\frac{\partial L(\omega,b,\eta)}{\partial b} = 0 \quad \longrightarrow \quad \sum_{i=1}^{N}\alpha_i y_i = 0 \tag{4}$$

$$\frac{\partial L(\omega,b,\eta)}{\partial \eta_i} = 0 \quad \longrightarrow \quad C - \alpha_i - \eta_i = 0 \tag{5}$$

$$\alpha_i \geq 0, \eta_i \geq 0 \tag{6}$$

$$\alpha_i\Big\{y_i\left(x_i^T\omega + b\right) - 1 + \eta_i\Big\} = 0 \tag{7}$$

$$\eta_i\xi_i = 0 \tag{8}$$

Dual problem: By substituting (3) into (2), we have

$$L(\omega,\eta) = -\frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N}\alpha_i\alpha_j y_i y_j x_i^T x_j + \sum_{i=1}^{N}\omega_i \tag{9}$$

$$\text{s.t} \quad \alpha_i \geq 0, \quad \sum_{i=1}^{N}\alpha_i y_i = 0 \tag{10}$$

$$\eta_i \geq 0, \tag{11}$$

$$C = \omega_i - \eta_i \tag{12}$$

Constraints (11) and (12) can be reduced into one constraint:

$$\alpha_i \leq C \tag{13}$$

Thus the dual problem is

$$L_D(\alpha,\eta) = -\frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N}\alpha_i\alpha_j y_i y_j x_i^T x_j + \sum_{i=1}^{N}\alpha_i \tag{14}$$

$$\text{s.t} \qquad 0 \leq \alpha_i \leq C, \quad i = 1,\cdots,n \tag{15}$$

$$\sum_{i=1}^{N}\alpha_i y_i = 0 \tag{16}$$