Sparse Bayesian Harmonic State Estimation

Ye Yuan[1] Wei Zhou[1] Hai-Tao Zhang[1] Zuowei Ping[1]
Omid Ardakanian[2]

[1]Huazhong University of Science and Technology

[2]University of Alberta

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Outline

Outline

Background

The growing adoption of power electronic devices and large non-linear loads has increased *harmonic-related* power quality problems.

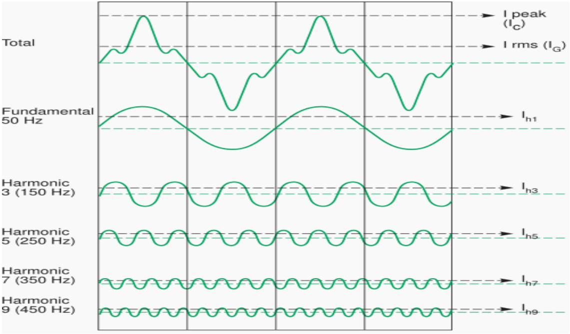
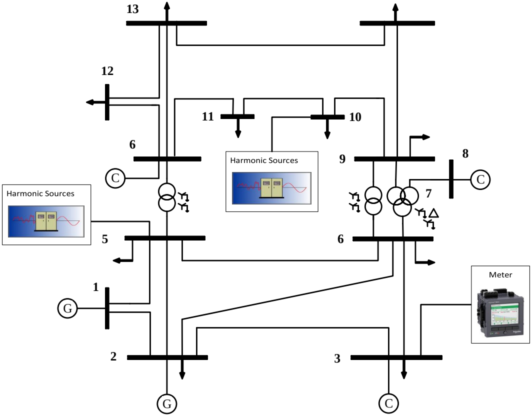


Figure: Harmonic currents.

source: `https://electrical-engineering-portal.com/definition-of-harmonics-and-their-origin`

Harmonic State Estimation (HSE)

- Locate the harmonic sources;
- Estimate harmonic voltage distribution.



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Problem formulation

HSE aims to estimate state variables, x, from harmonic measurements, z, given the measurement noise, ξ:

$$z(h) = \Phi(h)x(h) + \xi, \quad (1)$$

where

$$z(h) = \begin{bmatrix} V_{L(1)}(h) \\ \vdots \\ V_{L(\kappa_1)}(h) \\ I_{L(1)}(h) \\ \vdots \\ I_{L(\kappa_2)}(h) \end{bmatrix}, \Phi(h) = \begin{bmatrix} a_{L(1)1} & \cdots & a_{L(1)\bar{N}} \\ \vdots & \ddots & \vdots \\ a_{L(\kappa_1)1} & \cdots & a_{L(\kappa_1)\bar{N}} \\ b_{L(1)1} & \cdots & b_{L(1)\bar{N}} \\ \vdots & \ddots & \vdots \\ b_{L(\kappa_2)1} & \cdots & b_{L(\kappa_2)\bar{N}} \end{bmatrix},$$

where $\Phi(h)$ is the system matrix with $a_{L(i)j} = [Y^H(h)^{-1}]_{L(i)W(j)}$ and $b_{L(i)j} = [Y^{bf}(h)Y^H(h)^{-1}]_{L(i)W(j)}$.

Problem formulation

Sparsity

The state variable is sparse when there is a small number of sources producing harmonics simultaneously at each harmonic order:

$$||x(h)||_0 \leq s,$$

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ℓ_0 Problem

Taking sparsity of $x(h)$ into account, the HSE problem can be formulated as:

$$\begin{aligned} \min \quad & ||x(h)||_0 \\ \text{s.t.} \quad & ||z(h) - \Phi(h)x(h)||_2 \leq \eta. \end{aligned} \tag{P0}$$

It is not trivial to solve this problem!

An ℓ_1 minimization is solved as a convex relaxation of this problem (P0).

- The ℓ_1 method is incapable of finding the sparse solution when the columns of $\Phi(h)$ are weakly orthogonal.

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Challenges:

- Limited measurements pose an under-determined equation;
- The strong correlation between the columns of the system matrix.

Outline

Our contribution is twofold

- We propose a novel harmonic state estimator based on sparse Bayesian learning (SBL) [1].
- We show through simulations that the proposed state estimator outperforms existing methods in terms of estimation and localization errors.

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The Proposed State Estimator

SBL-based Estimator

$$\hat{x}^{(k)} = \arg\min_{x} \frac{1}{2}||\tilde{z} - \tilde{\Phi}x||_2^2 + \lambda \sum_{i=1}^{2\bar{N}} u_i^{(k)}|x_i|, \tag{2}$$

$$\gamma_i^{(k)} = \hat{x}_i^{(k)}/u_i^{(k)}, \tag{3}$$

$$u_i^{(k+1)} = [\tilde{\Phi}_{\cdot i}^{\top}(\lambda I + \tilde{\Phi}\Gamma^{(k)}\tilde{\Phi}^{\top})^{-1}\tilde{\Phi}_{\cdot i}]^{\frac{1}{2}}, \tag{4}$$

The re-weighting parameter u_i promotes the sparsity of x, and the weight parameter λ trades off sparsity and estimation error.

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Observability analysis

Definition

s-Observability [2]: A power system is s-observable if the state variables satisfying the sparsity condition $||x||_0 \leq s$ can be determined uniquely given harmonic measurements z.

Lemma

Sufficient Condition [3]: A power system is s-observable if $Spark(\Phi) > 2s$.

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IEEE 14-bus test system with three evaluation metrics

- Three metrics are considered for evaluation, *i.e.*, the identification error, the localization success rate (LSR), and the root-mean-square error (RMSE).

$$\epsilon_x(h,i) := \left|x_i^{es}(h) - x_i^{tr}(h)\right| , \tag{5}$$

$$LSR := \frac{M_c}{M} \times 100\%, \tag{6}$$

$$\text{RMSE}_{\text{IM}} := \sqrt{\frac{\sum_{i=1}^{\bar{N}} |Mag(x_i^{tr}) - Mag(x_i^{es})|^2}{\bar{N}}}, \tag{7}$$

$$\text{RMSE}_{\text{VM}} := \sqrt{\frac{\sum_{i=1}^{N} |Mag(V_i^{tr}) - Mag(V_i^{es})|^2}{N}}, \tag{8}$$

Outline

Noise-Free Case

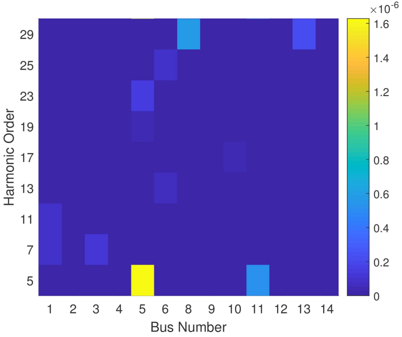

Figure: The identification error of injected current magnitudes for the meter configuration $\mathbb{M}_a$ [2] in the noise-free scenario.

$\mathbb{M}_a$: 9 harmonic meters installed on transmission lines measuring current phasors.

Noise-Free Case

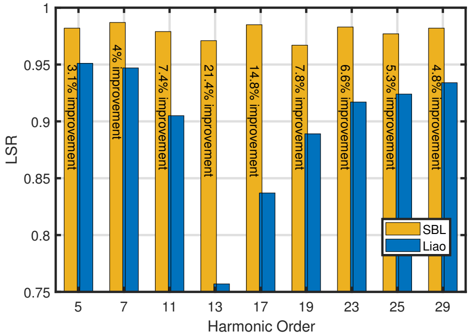

Figure: Comparing LSR of Lasso and SBL for different harmonic order under $\mathbb{M}_a$. The LSR increased by 8.3% on average.

Noise-Free Case

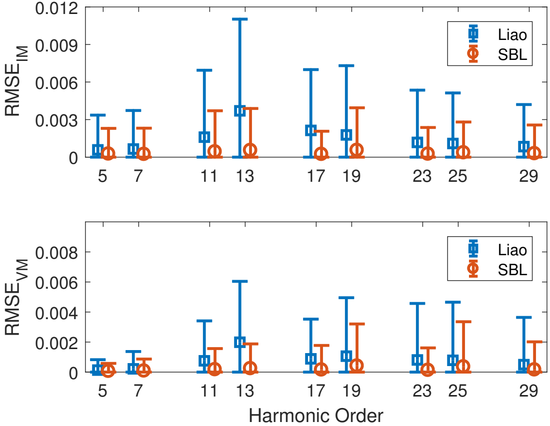

Figure: Comparing RMSE of Lasso with that of SBL.

Noise-Free Case

Summary of results:

- the proposed state estimator achieves an identification error of less than 1.6×10^{-6} and can locate harmonic sources with an average success rate of 97.92%.
- Our method outperforms Lasso in terms of estimation and localization error.

Outline

Weak orthogonality of the system matrix

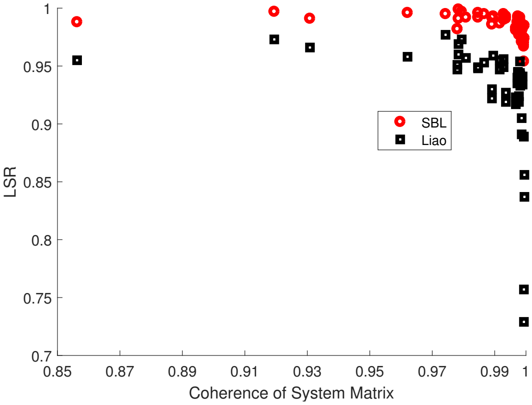

Figure: LSR of the proposed state estimator versus Lasso with 9 harmonic meters when the system matrix has weak orthogonality.

Weak Orthogonality of the System Matrix

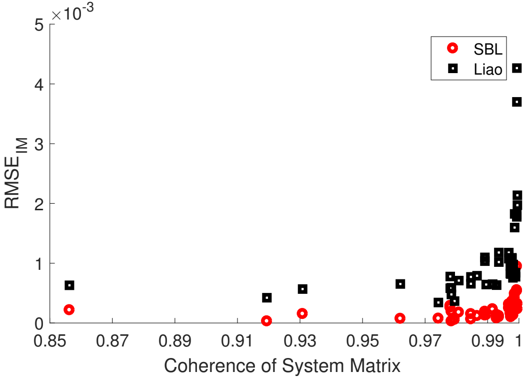

Figure: RMSE of the proposed state estimator versus Lasso with 9 harmonic meters when the system matrix has weak orthogonality.

Weak Orthogonality of the System Matrix

Summary of results:

- The proposed SBL-based state estimator converges to the sparest solution even when the system matrix has weak orthogonality.

Summary

Takeaways

- The proposed SBL-based harmonic state estimator has superior performance despite the strong correlation between columns of the system matrix; this eliminates the need to check the RIP or coherence condition.
- The proposed state estimator outperforms the state-of-the-art in terms of estimation and localization errors using limited measurements.

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Takeaways

- The proposed SBL-based harmonic state estimator has superior performance despite the strong correlation between columns of the system matrix; this eliminates the need to check the RIP or coherence condition.
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Future work

- We will explore the optimal placement of harmonic meters.
- We intend to extend the HSE framework to distribution systems.

References

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