On the Use of Teletraffic Theory in Power Distribution Systems

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Suppose $n$ independent ON-OFF traffic sources share a link.

The probability that a source is in state ON is $p$.

The traffic bit rate in state ON is $r$.

What is the probability that the bit rate of the aggregate traffic is greater than $kr$?

$$\sum_{i=k+1}^{n} \binom{n}{i} p^i (1 - p)^{n-i}$$
Teletraffic theory – a simple example

• Suppose \( n \) independent ON-OFF traffic sources share a link
• The probability that a source is in state ON is \( p \)
• The traffic bit rate in state ON is \( r \)

What is the probability that the bit rate of the aggregate traffic is greater than \( kr \)?

What is the minimum value of \( k \) for which this probability is less than \( \varepsilon \)?

How can we approximate this probability for large values of \( n \) and \( k \)?
Teletraffic theory – another example

- Suppose we have $n$ independent ON-OFF traffic sources
- The probability of being in state ON is $p$
- The traffic bit rate in state ON is $r$
- A router serves the aggregate traffic at rate $C$
- This router has a buffer with capacity $B$

What is the probability that an arriving bit/packet finds the buffer full?
Suppose we have $n$ independent ON-OFF traffic sources.

- The probability of being in state ON is $p$.
- The traffic bit rate in state ON is $r$.
- A router serves the aggregate traffic at rate $C$.
- This router has a buffer with capacity $B$.

What $(C,B)$ pairs can be chosen such that this probability is less than $\varepsilon$?
Teletraffic theory allows us to dimension a telecommunication network with:

- Heterogeneous traffic sources
- Shared transmission facilities
- Specific quality of service requirements
A power distribution network consists of:

- Stochastic electricity demand
- Shared lines, transformers, and storage

A certain level of “reliability” is guaranteed
Reliability of the grid

- Loss of load probability is one measure of reliability
- Loss of load *may* happen when a transformer is *overloaded*
Goal:

To size transformers, storage, and renewable energy generators in power distribution networks using teletraffic theory originally developed to size links, routers, and buffers in telecommunication networks
Sizing for the peak – the current practice

• Demand uncertainty is low
• The peak demand can be forecasted with high accuracy
• Optimal transformer sizing can be found using the load profile of the peak days of year
A new sizing approach is needed

- The demand uncertainty will increase
- Storage will be installed in the distribution network to smooth out variations in demand
- Transformers can be sized closer to the average demand
Contributions

• Modelling the distribution network as a fluid queueing system
• Applying teletraffic theory to size distribution transformers
• Validation of the proposed sizing approach using actual and synthetic load traces
Queueing Models & teletraffic-based sizing

The buffer/bandwidth trade-off
Observations:

- Storage is a buffer

- A transformer charges storage as traffic sources fill the buffer

- Loads discharge storage as a router empties the buffer

- The loss of load probability is similar to the packet loss probability
A fluid queueing model can be associated to a radial power distribution network

- A fluid queue with constant arrival rate and arbitrary service rate

- We want to quantify the buffer (storage) underflow probability in this model

- Unfortunately teletraffic analysis does not deal with this question
A dual queueing model

\( \varepsilon: \text{storage underflow probability} \)

The Equivalence Theorem

buffer overflow probability \( \equiv \varepsilon \)
Transformer Sizing

A Case Study
Resource allocation and effective bandwidth

• It is shown that the overflow probability is defined in terms of the aggregate *effective bandwidth* of homes supplied by a transformer

• Effective bandwidth of a stochastic source represents the amount of resource that should be reserved for it

• Computing effective bandwidth requires modelling the electricity demand of each home
Load Modelling

- A neighbourhood of 20 homes
- Classified into 4 classes
- Busy hour electricity demands of homes in each class are used to construct the Markov model of this class
- The aggregate effective bandwidth of the neighbourhood is the sum of effective bandwidths of all homes
Teletraffic-based Sizing of Power Distribution Networks

Measurement
Modelling
Analysis
Results of the Teletraffic-based Sizing Approach

• The transformer capacity computed for a neighbourhood given the industry standard loss of load probability is 107 kVA

• The utility guideline based on decades of field experience recommends a 100 kVA transformer for the same neighbourhood
Validation

Measurement

Modelling

Analysis

Validation
Comparison of the teletraffic-based sizing with the sizing guideline of a utility
Conclusions

- A distribution network can be modelled as a fluid queueing system

- Teletraffic theory can be applied to size
  - Transformers
  - Transformers and storage jointly