

# On the Impact of Storage in Residential Power Distribution Systems

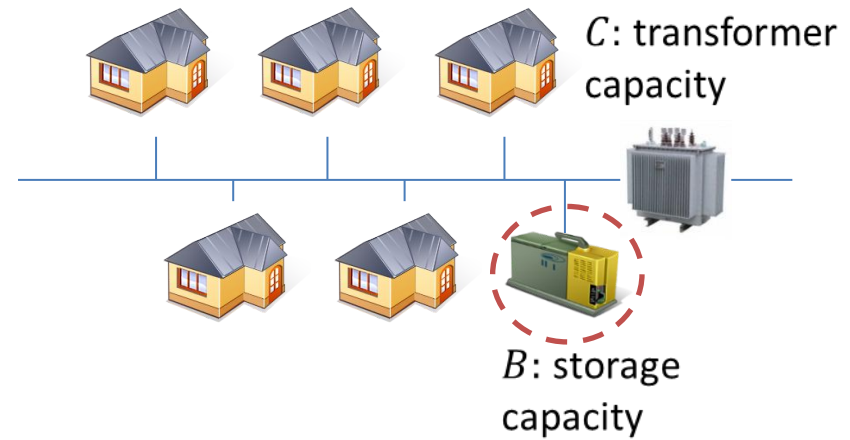
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GreenMetrics'12, June 11

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# A power distribution network consists of:

- Homes with stochastic electricity demand
- Shared lines, transformers, and storage (installed at different levels)



A certain level of “reliability” is guaranteed

## Reliability of the grid

- Loss of load probability is one measure of reliability
- Loss of load *may* happen when a transformer is *overloaded*

# Outline

- Introduction
- Background
- Upper bounds on the content of storage
  - Deterministic
  - Stochastic
- Load modelling
- Experimental Results
- Related work
- Conclusion

# Motivation

- Electric utilities will install storage in power distribution and transmission networks to
  - Enhance DG integration
  - Reduce the peak demand
  - **Defer system upgrade**
  - ...
- Its impact on the distribution network is not well-studied
  - **How it affects the loading of neighbourhood pole-top transformers**
  - How much it can reduce the aggregate peak demand

# Transformer upgrade

- A transformer is *overloaded* when the load supplied by it is higher than its nameplate rating
  - An overloaded transformer may overheat and fail
  - This decreases reliability of the grid
- Even low levels of EV adoption may increase the number of overloaded transformers depending on their charging strategies
- Distribution system upgrade is unaffordably expensive

# Transformer and storage

- Storage can partially supply the demand when the transformer is overloaded
  - Smoothing the demand curve
- Instead of sizing a pole-top transformer for the peak, it can be sized closer to the long-term average demand of the neighbourhood
  - The same reliability level is maintained
  - Transformer upgrade can be deferred
- A trade-off between the transformer size and the storage capacity

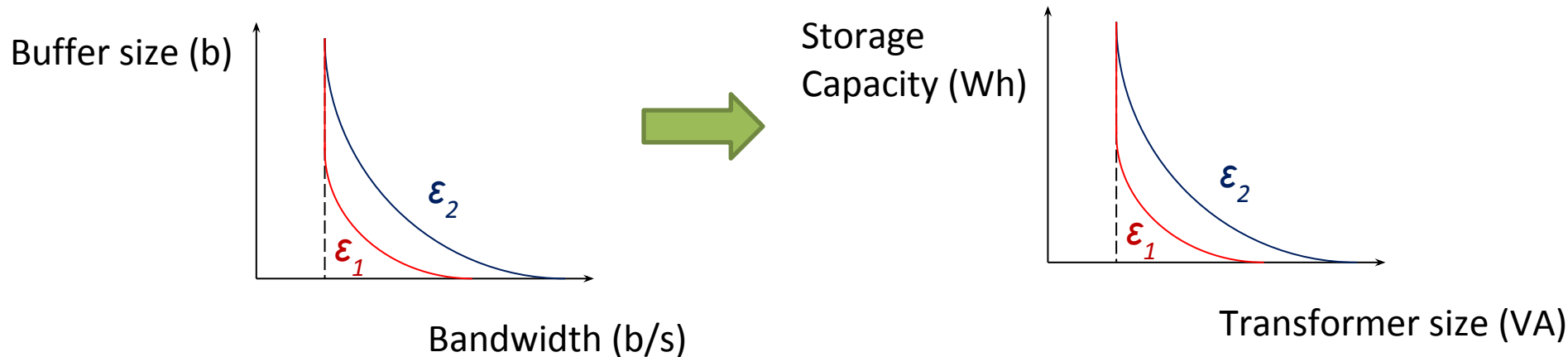
# Contributions

- We present a theoretical foundation for joint sizing of storage and transformers
- We demonstrate how to map electricity demand to the standard dual leaky-bucket parameters
- We present a joint sizing guideline in a residential neighbourhood and show the impact of the aggregate load parameters on this guideline



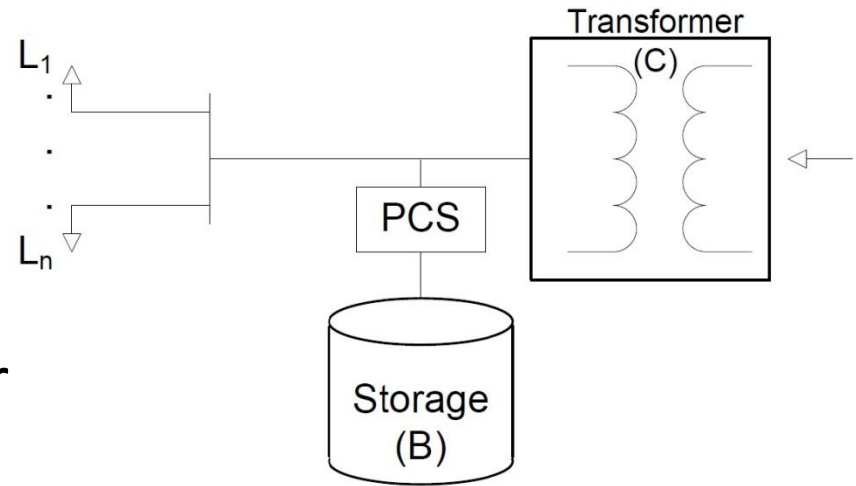
# Background

- **Equivalence Theorem**
- **The buffer/bandwidth trade-off**



# Observations:

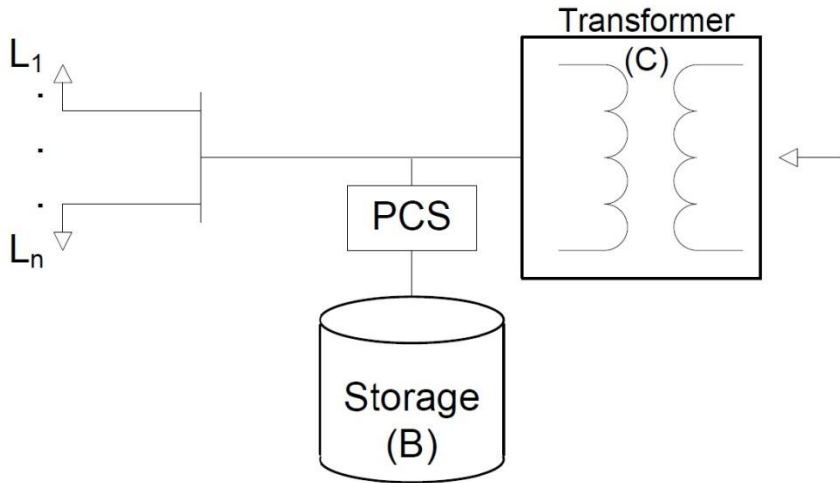
- Storage is a buffer
- A transformer charges storage as traffic sources fill the buffer
- Loads discharge storage as a router empties the buffer
- The loss of load probability is similar to the packet loss probability



A fluid queueing model can be associated to a radial power distribution network

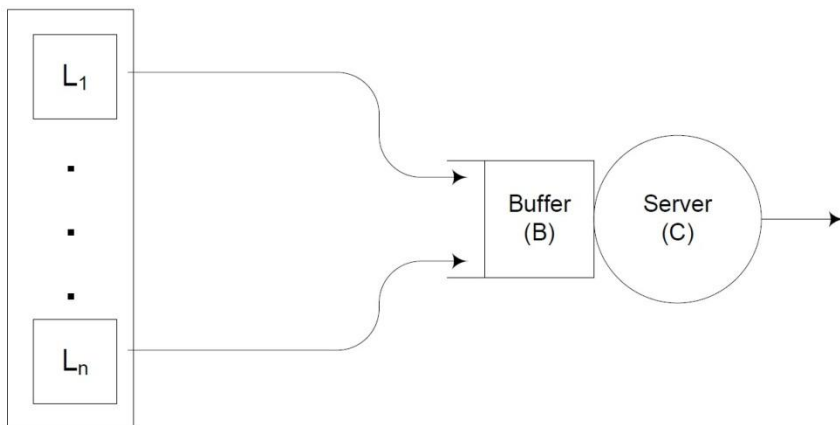
- A fluid queue with constant arrival rate and arbitrary service rate
- We want to quantify the buffer (storage) underflow probability in this model
- Unfortunately teletraffic analysis does not deal with this question

# A dual queueing model



$\varepsilon$ : storage underflow probability

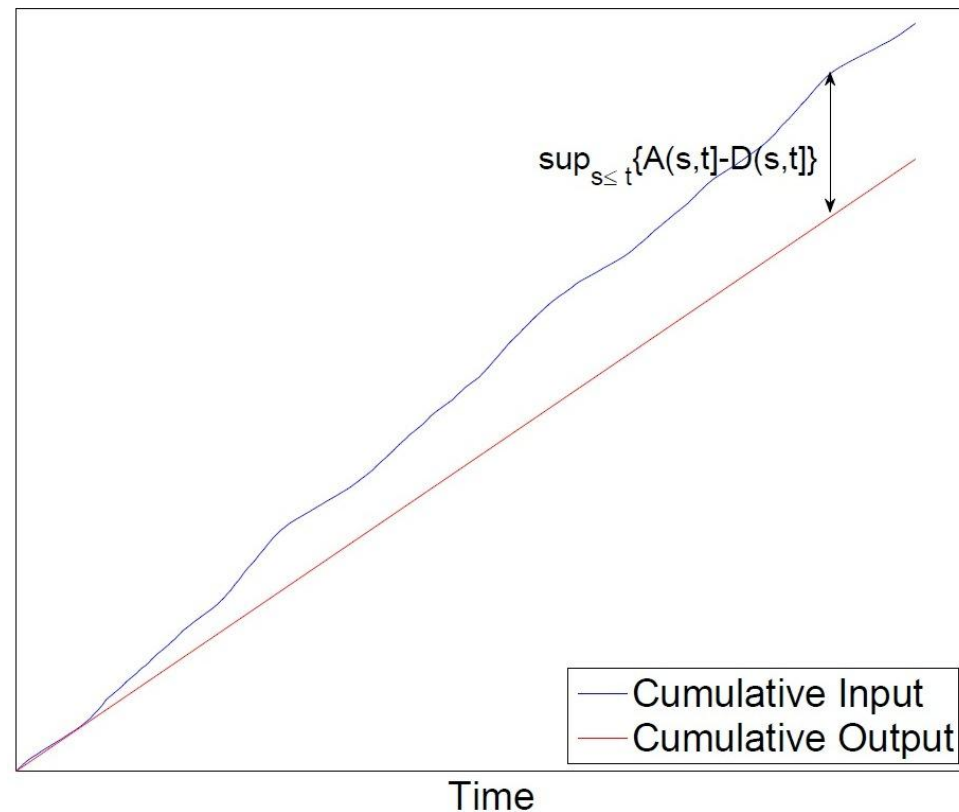
## The Equivalence Theorem



buffer overflow probability  $\cong \varepsilon$

# A deterministic upper bound on the content of the store

- Arrival curve  $A$ 
  - e.g.,  $A(s, t] = \max\{\pi(t - s), \rho(t - s) + \sigma\}$
- Service curve  $D$ 
  - e.g.,  $D(s, t] = C(t - s)$   
Where  $\rho < C < \pi$
- To guarantee no overloading  
 $B \geq \sup_{s \leq t} \{A(s, t] - D(s, t]\}$



# A probabilistic upper bound

- Loss probability in a finite queueing system (with capacity  $B$ ) can be approximated with the probability of up-crossing the level  $B$  in an infinite queueing system (under some conditions) <sup>†</sup>
- Upper bound on the stationary probability of crossing level  $B$  in an infinite capacity queueing system <sup>\*</sup>

$$- \mathbb{P}(Q(0) \geq B) = \frac{\sigma - \frac{\pi - \rho}{\pi - C} B}{\frac{C}{\rho} \sigma - B}, \quad \rho < C < \pi, \quad \frac{\pi - C}{\pi - \rho} \sigma \geq B$$

<sup>†</sup>Y. Ying, F. Guillemin, R. Mazumdar, and C. Rosenberg. “Buffer overflow asymptotics for multiplexed regulated traffic”. *Performance Evaluation*, 65:555-572, July 2008.

<sup>\*</sup>G. Kesidis and T. Konstantopoulos. “Extremal shape-controlled traffic patterns in high-speed

# Admissible region

• Admissible region for a given reliability requirement  $\varepsilon$ :

$$\text{Admissible region} = \left\{ (B, S) \mid B \geq \frac{\sigma(1 - \varepsilon \frac{sf}{\rho})}{\frac{\pi - \rho}{\pi - sf} - \varepsilon} \right\}$$

**Definition:**  $S(B)$  is the minimum transformer size such that  
 $(B, S(B)) \in \text{Admissible region}$

# Mapping electricity demand to the standard dual leaky-bucket parameters

- Utilities have a guideline to size transformers when it is not supplemented with storage (*i.e.*,  $S(0)$ )

- We set:

$$\pi = S(0)f$$

$$\rho_1 = \bar{L}_{peak\ day}$$

$$\rho_2 = \bar{L}_{peak\ week}$$

$$\rho_3 = \bar{L}_{peak\ month}$$

} 3 estimators for  $\rho$

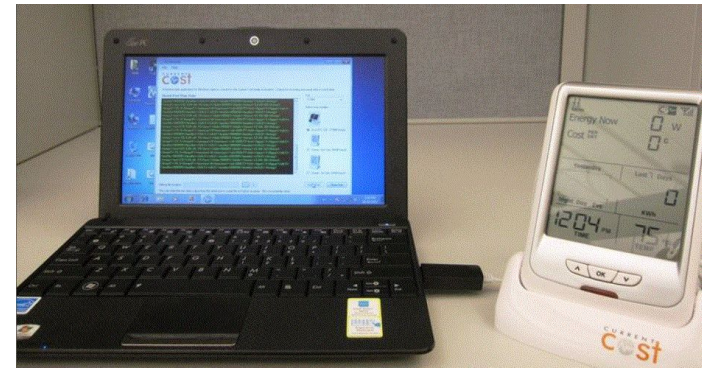
$$\sigma_1 = (\pi - \rho)T$$

$$\sigma_2 = \{\min B \mid (B, \rho/f) \in \text{Admissible region}\}$$

} 2 estimators for  $\sigma$

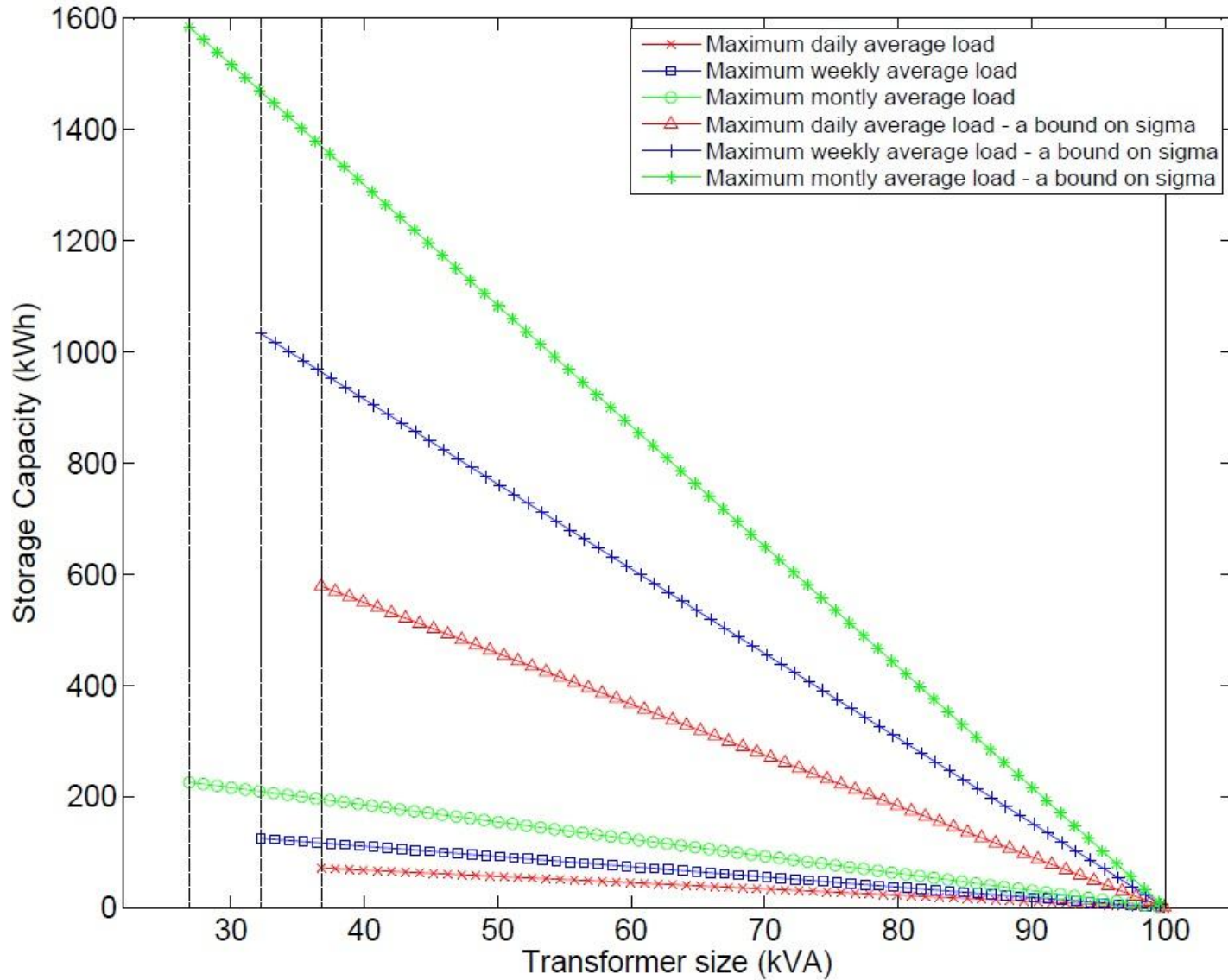


# Results



- Measurement nodes
  - Are installed at a residential neighbourhood consisting of 20 homes
  - Active power consumption of each home is measured every 6sec
  - The measurement period includes peak day, peak week, and peak month of year
- Homes are classified into 4 classes
  - Each class is associated with a unit value
  - Total unit value of the neighbourhood determines the transformer size ( $S(0)=100\text{kVA}$  for this neighbourhood)
- The power factor is set to 0.95

# Results



# Related work

- Proving the Equivalence Theorem and showing its application in sizing distribution transformers<sup>1</sup>
- Sizing storage for ‘demand response via quantity’ applications<sup>2</sup>
- Joint sizing of storage and renewable energy sources<sup>3</sup>

<sup>1</sup> O. Ardakanian, S. Keshav, and C. Rosenberg. “On the use of teletraffic theory in power distribution systems”. In Proc. e-Energy, May 2012.

<sup>2</sup> J.-Y. Le Boudec and D.-C. Tomozei. “A demand-response calculus with perfect batteries”. In WoNeCa, 2012.

<sup>3</sup> K. Wang, S. Low, and C. Lin. “How stochastic network calculus concepts help green the power grid”. In IEEE Smart Grid Communications 2011, pages 55-60, October 2011.

# Conclusions

- A theoretical foundation for joint sizing of transformers and storage is presented
- A mapping from electricity demand to standard dual leaky bucket parameters is proposed
- The impact of the aggregate load parameters on the sizing guideline is shown using 6 trade-off curves obtained for the same neighbourhood