Metropolis–Hastings Algorithm for Extended object tracking

\[ N = \# \text{ of snaxels} \]

\[ k = \text{frame number (time point)} \]

\[ x_k = [x_k^1, \ldots, x_k^N] \] is the state vector

Posterior density for \( k \)th frame

\[ P(x_k | z_{1:k}) \propto P(z_k | x_k) \sum_{i=1}^{M} P(x_k | x_{k-1} = s_k^i) \]

Measurements

\[ s_{k-1}^i \sim P(x_{k-1} | z_{1:k-1}), \quad i = 1, \ldots, M. \]

are samples from previous posterior density.

Actually, \( s_{k-1}^i \) are samples from the form (matrix)

\[ \{ s_{k-1}^i \}_{j=1, i=1}^{N, M} \]

We want to generate samples \( \{ s_k^j, i \}_{j=1, i=1}^{N, M} \) from the posterior \( P(x_k | z_{1:k}) \) (essentially a mixture density)
Component-wise MHT Algorithm

Because \( P(x_k | z_{1:n}) \) is a mixture density, we first need to choose an index \( l \in \{1, \ldots, M\} \) uniformly distributed.

Then sample from \( P(z_k | x_k) P(x_k | x_{k-1} = s_k^l) \).

The MHT ratio for \( i \)th sample:

\[
\gamma = \frac{P(z_k | \underbrace{x_k^i = s_{\text{new}}, x_k^{-i}}_{\text{new}}) P(x_k^i = s_k^l, x_k^{-i} | x_{k-1} = s_{k-1}^{l'})}{P(z_k | \underbrace{x_k^i = s_{\text{cur}}, x_k^{-i}}_{\text{cur}}) P(x_k^i = s_{\text{cur}}, x_k^{-i} | x_{k-1} = s_{k-1}^{l'}) + \frac{Q(s_{\text{cur}})}{Q(s_{\text{new}})}}
\]

\( Q(\cdot) \) : proposal density
\( s_{\text{new}} \) : new sample from \( Q(\cdot) \)
\( s_{\text{cur}} \) : current sample for \( x_k^i \)

If \( Q(x_k^i) = P(x_k^i | x_k^{-i}, x_{k-1}) \) then

\[
\gamma = \frac{P(z_k | \underbrace{x_k^i = s_{\text{new}}, x_k^{-i}}_{\text{new}})}{P(z_k | \underbrace{x_k^i = s_{\text{cur}}, x_k^{-i}}_{\text{cur}})}
\]
MHP: Algorithm for Extended Object

\[
\{ S_k^{j,i} \}_{j=1}^{N} = \text{MCMC} \left[ \{ S_k^{j,i} \}_{j=1}^{N}, \{ S_k \}_{j=1}^{N} \right]
\]

- Choose initial samples \( \left\{ S_k^{j,0} \right\}_{j=1}^{N} \)

- for \( i = 1 \) to \( M \)
  - Choose an index \( k \in \{ 1, \ldots, M \} \) uniformly distributed

  - for \( j = 1 \) to \( N \)
    - \( S \sim P(X_k^{j} | X_k^{-j}, \{ X_k^{j,i} = S_k^{j,i} \}_{j=1}^{N}) \)
    - \( Y = \frac{P(Z_k | X_k^{j} = S, X_k^{-j})}{P(Z_k | X_k^{j,i} = S_k^{j,i}, X_k^{-j})} \)
    - \( u \sim U[0,1] \)
    - if \( u \geq Y \),
      - \( S_k^{j,i} \leftarrow S \)
    - else
      - \( S_k^{j,i} \leftarrow s_k^{j,i-1} \)
  - end
- end
- end