Point Operations

CMPUT 206
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Source: http://www.imagingbook.com/
What is a point operation?

• A point operation changes pixels values
• A new pixel value depends only on the previous pixel value at the same location
• Examples
  – Modifying image brightness or contrast
  – Quantizing
  – Global thresholding
  – Gamma correction
  – Color transformation
Point Operations: Formally

Each new pixel value \( a' = l'(u, v) \) depends exclusively on the previous value \( a = l(u, v) \).

The original pixel values are mapped to the new values by a function \( f(a) \):

\[
\begin{align*}
a' & \leftarrow f(a) \\
I'(u, v) & \leftarrow f(I(u, v))
\end{align*}
\]

If the function \( f(a) \) is independent of pixel location \((u, v)\), the point operation is called homogeneous point operation.

An example of homogeneous point operation is global thresholding.
Inverting Images

For intensity inversion the function $f$ is defined as:

$$ f_{\text{invert}}(a) = -a + a_{\text{max}} = a_{\text{max}} - a $$
Global thresholding

The function \( f \) for threshold point operation is defined as:

\[
f_{\text{threshold}}(a) = \begin{cases} 
  a_0 & \text{for } a < a_{\text{th}} \\ 
  a_1 & \text{for } a \geq a_{\text{th}} 
\end{cases}
\]
Effect of threshold operation on histogram
Automatic contrast adjustment

Suppose $a_{\text{low}}$ and $a_{\text{high}}$ are the lowest and the highest pixel values in the current image.

Further, suppose $[a_{\text{min}}, a_{\text{max}}]$ is the full range of intensity available. Then, automatic contrast adjustment maps $a_{\text{low}}$ to $a_{\text{min}}$ and $a_{\text{high}}$ to $a_{\text{max}}$. All in between values are mapped linearly as follows:

$$f_{ac}(a) = a_{\text{min}} + (a - a_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{a_{\text{high}} - a_{\text{low}}}$$

For an 8-bit image with $a_{\text{min}} = 0$, and $a_{\text{max}} = 255$, the above equation becomes:

$$f_{ac}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$
Automatic contrast adjustment....
Modified automatic contrast adjustment

• In practice, the linear mapping function can be affected by a few extreme pixel values (sometimes called outlier values)
• This can be avoided by saturating a fixed percentage \((s_{\text{low}}, s_{\text{high}})\) of pixels at the lower and upper ends of the target intensity range
• To do this, we determine two values \(\hat{a}_{\text{low}}\) and \(\hat{a}_{\text{high}}\) such that a predefined quantile slow of all pixel values in the image \(I\) is less than \(\hat{a}_{\text{low}}\) and a quantile shigh of the values are greater than \(\hat{a}_{\text{high}}\)
Modified automatic contrast adjustment...

This is how $\hat{a}_{\text{low}}$ and $\hat{a}_{\text{high}}$ are computed with the help of cumulative histogram $H(i)$:

$$\hat{a}_{\text{low}} = \min \{ i \mid H(i) \geq M \cdot N \cdot s_{\text{low}} \}$$

$$\hat{a}_{\text{high}} = \max \{ i \mid H(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \}$$

The mapping function $f$ looks like:

$$f_{\text{mac}}(a) = \begin{cases} a_{\text{min}} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\text{min}} + (a - \hat{a}_{\text{low}}) \cdot \frac{a_{\text{max}} - a_{\text{min}}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\text{max}} & \text{for } a \geq \hat{a}_{\text{high}} \end{cases}$$
MAC Adjustment: Show “boats” image in ImageJ
Gamma correction

- The term “gamma” originates from photography, where the relationship between the light energy and the resulting film density is logarithmic.
- The exposure function ($D$) is approximately linear over a range of the logarithm of the light intensity ($B$).
- The slope of the linear portion of this plot is called “gamma”.
- The same term was adopted in television broadcasting to describe the relationship between the voltage of video signal and the emitted light intensity.
- To compensate for the non-linearities of the receivers, a “gamma correction” is applied to the TV signal once before broadcasting.
Gamma correction is based on gamma function:

\[ b = f_\gamma(a) = a^\gamma \quad \text{for } a \in \mathbb{R}, \gamma > 0 \]

Inverse gamma function is given by: \( a = f^{-1}_\gamma(b) = b^{1/\gamma} \)

Let’s look at an example of how gamma correction is applied to a digital camera
Applications of gamma correction: A simple example

• A digital camera with a gamma value $\gamma_c$, i.e., output signal $s$ is related to the incident light energy $B$ as:

$$s = B^{\gamma_c}$$

• To obtain a measurement $b$ that is proportional to the light intensity $B$, we apply a gamma correction:

$$b = f_{\gamma_c}(s) = s^{1/\gamma_c}$$

• Now the resulting signal is

$$b = s^{1/\gamma_c} = (B^{\gamma_c})^{1/\gamma_c} = B^{(\gamma_c \frac{1}{\gamma_c})} = B^1$$

The transfer characteristic of a device with gamma value $\gamma$ is compensated by a gamma correction with $\bar{\gamma} = 1/\gamma$
Gamma correction: the digital workflow
### Arithmetic point operations

<table>
<thead>
<tr>
<th>Operator</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ADD</strong></td>
<td>$ip1 \leftarrow ip1 + ip2$</td>
</tr>
<tr>
<td><strong>AVERAGE</strong></td>
<td>$ip1 \leftarrow (ip1 + ip2) / 2$</td>
</tr>
<tr>
<td><strong>DIFFERENCE</strong></td>
<td>$ip1 \leftarrow</td>
</tr>
<tr>
<td><strong>DIVIDE</strong></td>
<td>$ip1 \leftarrow ip1 / ip2$</td>
</tr>
<tr>
<td><strong>MAX</strong></td>
<td>$ip1 \leftarrow \max(ip1, ip2)$</td>
</tr>
<tr>
<td><strong>MIN</strong></td>
<td>$ip1 \leftarrow \min(ip1, ip2)$</td>
</tr>
<tr>
<td><strong>MULTIPLY</strong></td>
<td>$ip1 \leftarrow ip1 \cdot ip2$</td>
</tr>
<tr>
<td><strong>SUBTRACT</strong></td>
<td>$ip1 \leftarrow ip1 - ip2$</td>
</tr>
</tbody>
</table>
Point operations by lookup table

• What is a lookup table?
• How can a point operation be defined by a lookup table?
• Why lookup table for a point operation?
• JPEG compression uses such a lookup table point operation
Histogram equalization

• Also known as “histogram flattening”
Histogram equalization...

The mapping function $f$ for histogram equalization:

$$f_{\text{eq}}(a) = \left\lfloor H(a) \cdot \frac{K-1}{MN} \right\rfloor$$
Histogram equalization: example

Original image: (a) Original image
The image after histograms equalization: (b)

Histograms:
- (c) Original histogram
- (d) Equalized histogram

Cumulative histograms:
- (e) Original cumulative histogram
- (f) Equalized cumulative histogram
A histogram equalization example from Wikipedia

Original image

After histogram equalization

Histograms (red) and cumulative histograms (black)
Histogram specification

Also known as “histogram shaping” or “histogram matching”

Problem statement: find the point operation \( f_{hs}(a) \) that converts a histogram \( h_A(i) \) of an image \( I_A \) to a reference histogram \( h_R(i) \).

The solution principle: From \( I_A \) construct a new image \( I_R \) by a point operation \( f_{hs}(a) \), so that \( h_R(i) \) becomes the histogram of \( I_R \).

How?

\[
    f_{hs}(a) = a' = P_R^{-1}(P_A(a))
\]

Pictorially:

\( P_A \) and \( P_R \) are normalized cumulative histograms
Histogram shaping: derivation

We want to find out the mapping function $f_{hs}$:

$$i' = f_{hs}(i)$$

We can show this by the following basic concept from probability:

$$\text{Prob}[i' \leq a'] = \text{Prob}[f_{hs}(i) \leq a']$$
$$\Rightarrow \text{Prob}[i' \leq a'] = \text{Prob}[i \leq f_{hs}^{-1}(a')]$$
$$\Rightarrow \text{Prob}[i' \leq a'] = \text{Prob}[i \leq a]$$
$$\Rightarrow P_R(a') = P_A(a)$$
$$\Rightarrow a' = P_R^{-1}(P_A(a))$$

Can you now derive the formula for histogram equalization?
Histogram matching: example

(a) $I_A$
(b) $I_R$
(c) $I_{A'}$

(d) $h_A$
(e) $h_R$
(f) $h_{A'}$

(g) $H_A$
(h) $H_R$
(i) $H_{A'}$
Numerical Examples

• Look at the “midterm” or “final exam” type small numerical examples from the emailed pages on histogram shaping and flattening