Active surface model-based adaptive thresholding algorithm by repulsive external force

Fei Liu
Yupin Luo
Xiaodan Song
Dongcheng Hu
Tsinghua University
Department of Automation
Beijing, 100084, China
E-mail: liufei98@mails.tsinghua.edu.cn

Abstract. Inspired by the idea that threshold surface always intersects the image surface at high gradient points, an active surface-based adaptive thresholding algorithm is proposed to get the binarized result. In this model, the external force is designed to be repulsive from the image surface, thus at the equilibrium state the active surface tends to cover the supporting points of high gradient with smooth property, as well as be away from the image surface locally, which makes the obtained threshold surface properly separate the foreground and background. The description of the algorithm is in a simple and reasonable energy functional form, and only two parameters need to be tuned, which gives more convenience to the operation. Analysis and comparison for the experimental results reveal that it cannot only give the proper thresholding result but also restrain the occurrence of the ghost phenomenon. © 2003 SPIE and IS&T. [DOI: 10.1117/1.1556313]

1 Introduction

There are intensive interests for image analysis, which generally needs the result of image segmentation. In many applications of image processing, the gray levels of pixels belonging to the object are quite different from the gray levels of the pixels belonging to the background, and so can easily be distinguished. Image thresholding is a special progress to make the input gray scale image bilevel with dark objects against bright backgrounds or vice versa. It is one of the most popular image segmentation techniques, and a rich body of literature is available on this topic. Proper thresholding is important to the analysis result for the applications, such as document image analysis in which the binarization quality importantly affects the overall performance of layout analysis, character segmentation and character recognition,1 and medical image analysis. It is usually performed either globally or locally adaptively. The global thresholding method uses a fixed global threshold for all pixels, while adaptive thresholding changes the threshold dynamically over the image pixel by pixel on the basis of information contained in the neighborhood of the pixel.2

Inspired by the property of the active contour models that they exploit a priori knowledge of object shape and inherent smoothness, usually formulated as internal deformation energies to compensate for noise, gaps, and other irregularities in object boundaries,3 we present an active surface to estimate the background surface in Ref. 4. Under the assumption that the uneven illumination in the image occupies only lower space frequency,5 it uses an active surface model with an attractive force to estimate the background surface. Thus, the influence of nonuniform illumination and uneven background of the image can be removed by subtracting this estimated background surface from the original image surface. It is also pointed out that different external force models can be designed to meet different requirements more effectively.

Inspired by the idea that threshold surface always intersects the image surface at high gradient points, just as Yanowitz and Bruckstein use the gray-level values at high gradient places as known data to interpolate the threshold surface of the image,6 we propose another active surface-based adaptive thresholding algorithm following the active surface model in adaptive thresholding. In this model, a repulsive external force is devised to obtain an active threshold surface directly, thus at the equilibrium state the active surface tends to cover the supporting points of high gradient with smooth property, as well as be away from the image surface locally, which makes the obtained threshold surface properly separate the foreground and background.

The remainder of the work is arranged as follows. In Sec. 2, we give a simple review of thresholding techniques first, and then adaptive thresholding algorithms are reviewed in the order of the development. In Sec. 3, our active surface-based algorithm is described in detail. Then, after numerical implementation and discussions on parameters, experimental results are presented and compared in Sec. 4. Section 5 concludes the work.
2 Review Thresholding Algorithms and Active Contour Model

2.1 Thresholding Algorithms

Literature\(^7\) presents a good survey of thresholding techniques and evaluates the performance of several automatic global thresholding methods. Most global thresholding methods try to find a threshold value depending on the gray-scale level histogram of the image. Among them, Ot- su’s method appears to be the most efficient,\(^8\) which is based on the analysis of a histogram and selects an optimal threshold according to the discriminant theory. O’Gorman proposed another global thresholding method, which is based on connectivity rather than intensity.\(^9\) He suggested that if a range of threshold values is found that leads to a stable number of regions, a value from this range will provide a suitable threshold. Since connectivity is a local measure and it is measured throughout the entire image, this is a global thresholding method based on a local measure. However, when the background is uneven or in nonuniform illumination conditions, a fixed (or global) gray-level threshold cannot segment the image correctly. In such cases, although locally the objects will still be lighter or darker than the background, the histogram may not be bimodal at all, or it may be bimodal by chance only.\(^5\) Furthermore, no single thresholding scheme gives satisfactory segmentation results on a variety of images. Thresholding methods are notoriously sensitive to parameters like ambient illumination, object shape and size, noise level, variance of gray levels within the object and background, and contrast. It is desirable to have an adaptive thresholding method that can adjust its parameters as conditions change in the image.\(^4,10\) Thus, the locally adaptive thresholding method, which tends to construct a threshold surface, is more suitable to give proper results.

Nakagawa and Rosenfeld\(^11\) proposed a threshold surface based on local histogram information. It suggested dividing the image into nonoverlapping cells of equal area forming a regular grid, and determine the histograms of the gray levels of pixels in each cell. Then the subhistograms, which are judged to be bimodal, are used to determine local threshold values for the corresponding cell centers, and the local thresholds are interpolated over the entire image to yield a threshold surface. This idea is an improvement to global thresholding and obtains a better thresholding result in some samples. However, it fails in some other samples. The major problem is due to the misdivision of the image.

Yanowitz and Bruckstein\(^6\) presented a threshold surface that is determined by interpolating the image gray levels at points where the gradient is high, which serves as supporting points to indicate the probable object edge. It contains the major steps of image smoothing, large gradient point locating, and threshold surface interpolating. Based on it, Chan, Lam, and Zhu\(^12\) derived a variational method to combine the steps of large gradient point location and threshold surface interpolation, which can be viewed as a variational translation of Yanowitz and Bruckstein’s algorithm. The ideas of these two methods are both aimed at locating the threshold surface directly via largest gradient points and interpolation. Subsequent performance evaluation of several binarization methods showed that this method was one of the best binarization methods. However, a ghost phenomenon may occur because of this type of interpolation-based threshold surface\(^9\) (Fig. 1).

2.2 Active Contour Model

Deformable models, which include popular active contour models or snakes,\(^13\) are a powerful segmentation technique designed to locate the object boundary. They tackle the segmentation problem by considering an object boundary as a single, connected structure. The classical snakes approach is based on deforming an initial contour or surface toward the boundary of the object to be detected. For the general case of image segmentation in 2-D, a snake is a curve \(X(s)=[x(s),y(s)]\), which moves through an image to minimize a global energy designed such that its (local) minima are obtained at the boundary of the object. The energy is basically composed of an internal term \(E_{\text{int}}\), which controls the smoothness of the deforming curve, and an external term \(E_{\text{ext}}\), which attracts it to the boundary.

\[
E = \int_{0}^{1} E_{\text{int}}(X(s)) + E_{\text{ext}}(X(s)) ds. \tag{1}
\]

The most frequently used internal energy is:

\[
E_{\text{int}}(X(s)) = w_1 |X'(s)|^2 + w_2 |X''(s)|^2, \tag{2}
\]

where \(w_1\) and \(w_2\) are weighting parameters that control the snake’s tension and rigidity, respectively, and \(X'(s)\) and \(X''(s)\) denote the first and second derivatives of \(X\) with respect to \(s\). The external energy function \(E_{\text{ext}}\) is derived from the image to meet the desired requirements so that it takes on its smaller values at the features of interest, such as boundaries. If the snake is expected to represent the contour of the object, \(E_{\text{ext}}\) is generally related to the gradient of the image. When a local minimum is obtained, \(X(s)\) will satisfy the Euler-Lagrange equation

\[
\begin{align*}
-(w_1X'(s))' + (w_2X''(s))'' + \nabla E(X) &= 0 \\
X(0), X'(0), X(1), X'(1) &\text{ given}
\end{align*}
\tag{3}
\]

where each term represents a force applied to the curve. The first two terms compose the internal force, which discourages stretching and bending, while the third term is the external potential force that pulls the deformable contour toward the desired object boundaries. Thus, the solution can be viewed either as realizing the equilibrium of the forces in the equation or reaching the minimum of the energy.
3 Proposed Adaptive Thresholding Algorithm

3.1 Active Surface Model

Assume the gray-level image \( g \) (ranged from 0 to 255) is defined in a rectangle region \( \Omega \), and the estimated active surface is \( u \). To make the calculation simple, we ignore the high order smoothness component, which is related to the rigidity, and keep only the first-order smoothness component to design the energy functional. Thus the proposed model is designed as:

\[
E = \int_{\Omega} w |\nabla u|^2 + E_{\text{ext}}(u) \, dx \, dy. \tag{4}
\]

The first term in Eq. (4) is an inner energy to maintain the smoothness of the active surface, which is used to fulfill the assumption mentioned in Sec. 1 that the illumination causing variation of the background employs only lower space frequency. The second term is an external energy related to the image surface and makes the active surface relative with the given image in a particular way. Parameter \( w \) determines the trade off between surface smoothness and image surface proximity.

The local minimum will satisfy the corresponding Euler-Lagrange equation with the Neumann boundary condition as

\[
\begin{cases}
 w\Delta u + F_{\text{ext}}(u) = 0 & u \in \Omega \\
 \frac{\partial u}{\partial n} = 0 & u \in \partial \Omega
\end{cases} \tag{5}
\]

where the force function \( F_{\text{ext}}(u) = -\nabla E_{\text{ext}}(u) \) is an external force to drive the active surface to meet particular requirements. Due to the properties of the adaptive thresholding problem, the force is defined only in a vertical direction other than the case for snakes, where the force can be in any direction. There can be various choices for this external potential energy to meet the requirements in different conditions.

3.2 Active Surface Model with Repulsive External Force

Derived from the idea of Yanowitz and Bruckstein's algorithm that the image gray levels at points where the gradient is high are good choices for local thresholds, we choose the external energy as:

\[
E_{\text{ext}}(u) = \exp(- (u - g)^2 / \sigma^2). \tag{6}
\]

The energy will get the maximum when \( u = g \). The corresponding external force is a repulsive force in the form:

\[
F_{\text{ext}}(u) = (u - g) \exp(- (u - g)^2 / \sigma^2). \tag{7}
\]

In the evolution progress, the initial state is set at the original image surface to obtain the satisfactory local minimum of the functional. At the points with high gradient, there are two neighboring points belonging to the foreground and background, respectively (Fig. 2). It is obvious that the energy reaches local minimum at the mean of these two points. Thus the active surface will reach stable equilibrium at this mean point. Furthermore, since the energy is decreasing with the distance increasing, a larger difference of two neighboring pixels will result in less energy. Thus, the minimum energy-seeking progress is also a high gradient-seeking progress. Then with the obtained supporting points with high gradient and cooperation of the external and inner force simultaneously, a smooth active threshold surface is gained to separate the image into upper and lower parts, representing foreground and background, respectively.

Unlike the interpolation in Yanowitz and Bruckstein's algorithm, which does not consider the relationship between the threshold surface and the image surface, the repulsive force in our proposed model makes the active surface repel away from the image surface, thus avoiding the conditions in which the active surface intersects the image surface at the points other than the supporting points. Therefore, the occurrence of the ghost phenomenon is restrained in the proposed model.

Figure 3 gives an illustration of this evolution progress. The initial state at the image surface is an unstable state with external energy being at maximum. With the cooperation of repulsive force and tension force, the surface will keep moving until it steps at the equilibrium position, which is the adaptive threshold surface of the image.

4 Numerical Implementation and Discussions

4.1 Numerical Implementation

To solve Eq. (5), the active contour surface is obtained dynamically by treating \( u \) as a function of time \( t \) as well as \( x \) and \( y \), thus it has the form of \( u(t, x, y) \). The partial derivative of \( u \) with respect to \( t \) is set equal to the left-hand side of the equation as follows

\[
\frac{\partial u}{\partial t} = w \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + (u - g) \exp \left( - \frac{(u - g)^2}{\sigma^2} \right). \tag{8}
\]

It is a heat equation. When the solution \( u \) stabilizes, the left side vanishes, and a solution for Eq. (5) is achieved. This approach is equivalent to applying a gradient descent algorithm to find the local minimum of Eq. (4).

To realize the numerical implementation, Eq. (8) is solved by a finite difference method. For the sake of simplicity in discussion, let us suppose \( u \) is a 1-D signal. Thus Eq. (8) becomes

\[
\frac{\partial u}{\partial t} - w \left( \frac{\partial^2 u}{\partial x^2} \right) = (u - g) \exp \left( - \frac{(u - g)^2}{\sigma^2} \right). \tag{9}
\]

Fig. 2 Illustration of two types of surfaces.
The numerical approximation uses the implicit form to ensure the stability of the solution as
\[
\frac{1}{\tau}(u_i^k - u_i^{k-1}) - \frac{w}{h^2}(u_{i-1}^k - 2u_i^k + u_{i+1}^k) = \left(u_i^{k-1} - g_i\right) \exp\left(-\frac{(u_i^{k-1} - g_i)^2}{\sigma^2}\right),
\]
where \(\tau\) and \(h\) are time and spatial intervals, respectively. It can be rewritten as
\[
u_i^k - \frac{\tau w}{h}(u_{i-1}^k - 2u_i^k + u_{i+1}^k) \\
= u_i^{k-1} + \tau (u_i^{k-1} - g_i) \exp\left(-\frac{(u_i^{k-1} - g_i)^2}{\sigma^2}\right),
\]
which composes a linear equation with the form \(AU = B\), where \(A\) is a symmetric tridiagonal positive defined matrix. The analysis for a 2-D signal is similar, and the corresponding \(A\) is a symmetric penta-diagonal positive defined matrix. For these large scale linear equations, iterative methods are frequently used to obtain the solution. There are many recent references available\(^{15,16}\) for the details of these algorithms. In this work, we selected a conjugate gradient (CG) algorithm.

4.2 Parameter Discussion

There are two tunable parameters in the model. Parameter \(\sigma\) is proportional to the standard deviation of the external energy function. Larger \(\sigma\) tends to remove the influence of noise and emphasize the importance of the higher gradient. Parameter \(w\) determines the trade off between surface smoothness and image surface proximity. When \(w\) is small, the active surface at the equilibrium will have a greater chance to possess supporting points with relatively high gradient. Correspondingly, more details will remain in the final thresholding result.

Figure 4(a) displays the force function with different parameter \(\sigma\) when \(g = 0\), and Figs. 4(c) and 4(d) show the corresponding energy surfaces for a 1-D signal [Fig. 4(b)], respectively. Figure 5 shows some solutions for threshold surface calculation for the given 1-D signal with different \(\sigma\) and \(w\). In the model, \(w\) is set to meet the requirement in which details are of interest or not, and \(\sigma\) controls the degree of removing the influence of the noise. Moreover, the external force covers a larger range with increasing \(\sigma\). This broad force will make the threshold surface a little farther away from the image surface [Fig. 5(a)], which is much different from the smooth threshold surface when \(\sigma\) is small [Fig. 5(b)]. However, the different shapes of the threshold surface do not influence the result in the thresholding problem, in which the threshold surface is just used to distinguish the foreground and background, independent of the shape of the surface.

4.3 Experimental Results

Here three examples are illustrated to show the performance of the proposed model.

Figure 6 shows the thresholding progress for an IC figure with nonuniform illumination. The image IC [Fig. 6(a)] is a good example of ghost objects that appear as white areas between the conductor lines in the result binarized by Yanowitz and Bruckstein’s algorithm [Fig. 6(f)]. They are almost absent in the image binarized by the proposed model [Fig. 6(g)]. It is close to the analysis in Sec. 3.2. Figure 6(b) is the calculated threshold surface with \(\sigma = 16\) and \(w = 30\), and Figs. 6(c) and 6(d) are the 3-D illustration of the original image and threshold surface, respectively. For the convenience of comparison, the thresholding result with Otsu’s algorithm is shown in Fig. 6(e).
The second example [Fig. 7(a)] is a real document image with uneven illumination and background. In this example, we discuss the effect of the boundary condition to the thresholding result. From the global thresholding result [Fig. 7(b)], we can see that when the upper part of the image is smeared, the lower part is still indistinct because of these uneven factors. In the thresholding result by the proposed algorithm [Fig. 7(d)], the characters can be prop-
erly segmented. However, because of the conflict of the slanting property of the margins and the horizontal requirement of the Neumann boundary condition, there are some ghost regions in the margins. For example, in the top margin of this image, especially the right top corner, the background surface has a slope \( \partial c \). But the Neumann condition restricts the threshold surface in a horizontal direction at the boundary. This boundary condition combined with the smoothness requirement makes the threshold surface cross the image surface at the corner, which results in the ghost phenomenon. This kind of ghost can be restrained by modifying the boundary condition, such as adding a rectangle black margin to the original image \( \partial e \). Thus, the threshold surface tends to intersect the image surface at the mean of the gray value of the added black margin and the original boundary, which restrains the occurrence of the ghost phenomenon \( \partial f \).

Another example in Fig. 8 shows the thresholding results for “Barbara,” which includes a person and a complex background. In the global thresholding result, some of the boundary information, such as Barbara’s right shoulder, is missing. However, all the binarization results using the proposed model with different parameter combinations reveal rich boundary information of the objects. Compared with the binarization results in Figs. 8(e) and 8(f), which have larger \( w \) and give more compact expression of the scene, Figs. 8(c) and 8(d) display more detailed information such as the texture of tablecloth and the pleat of the cloth at the right chest. From another point of view, in comparison with the results in Figs. 8(c) and 8(e), which are with smaller \( \sigma \), the boundaries in Figs. 8(d) and 8(f) are smoother and have little noise. These results reveal the properties of the parameters to binarization, including segmentation fineness and region boundary smoothness as discussed in Sec. 4.2.

5 Conclusions

We propose an active surface model-based adaptive thresholding algorithm by a repulsive external force. Inspired by the idea of Yanowitz and Bruckstein’s algorithm that the gray-level values at high gradient are a good choice for local threshold,\(^6\) this active surface-based adaptive thresholding algorithm follows the active surface model in adaptive thresholding.\(^4\) In this model, the repulsive external force is devised to obtain an active threshold surface directly, thus at the equilibrium state the active surface tends to cover the supporting points with high gradient and smooth property as well as be away from the image surface locally, which makes the obtained threshold surface properly separate the foreground and background.

Comparing with the interpolation in Yanowitz and Bruckstein’s algorithm, which does not consider the relationship of the threshold surface and the background or foreground, the threshold surface is repelled away from the image surface by the repulsive external force. Thus, the occurrences of ghost phenomenon are restrained. Furthermore, the description of the algorithm is in a simple and reasonable energy functional form and only two parameters need to be tuned, which gives more convenience to the operations. These are much different from Yanowitz and
Fig. 7 Thresholding result for document image with uneven illumination and background: (a) original document image, (b) global thresholding result with Otsu's algorithm, (c) 3-D illustration of the image, (d) thresholding result by the proposed algorithm for the original image with $s=8$, $w=5$, (e) document image with modified boundary condition, and (f) thresholding result by the proposed algorithm for the modified image with $s=8$, $w=5$.

Fig. 8 Thresholding results of Barbara: (a) original image, (b) thresholding result by Otsu's algorithm, (c) through (f) thresholding results by the proposed model with different parameter combinations (c) $s=3$, $w=3$, (d) $s=16$, $w=1$, (e) $s=3$, $w=30$, and (f) $s=16$, $w=30$. 

Active surface model-based adaptive thresholding algorithm...
Brustein’s algorithm, which needs several separated steps and the supporting points must be pointed out explicitly by object boundary point selection, which involves many parameter decisions. Experimental results in several types of images show the high performance of this proposed algorithm.

Acknowledgments

We would like to thank the anonymous reviewers for their effort to improve the quality of this work.

References


Biographies and photographs of the authors not available.