Week 1: Introduction

Agenda:

- Welcome to **CMPUT 272**
- Official course information
- Logical form and logical equivalence
- Conditional Statements

Reading:

- Course homepage (http://ugweb.cs.ualberta.ca/~c272/W07/)
- textbook: pages 1-28

- Discrete Mathematics: study of mathematics of discrete objects

- Why do we study Disc. Math.? Why a CS student needs to take this course? main reasons:

  - To acquire new tools and techniques for solving and analyzing problems in different applications.

  - to learn to approach problems using math to

    * think abstractly

    * logically infer conclusions using present assumptions

    * make solutions rigorous and concise

    * being able to analyze the quality of different solutions

    * etc

- Course description:

  An introduction to the tools of logic, induction, set theory, and relations, and their use in the practice of reasoning about algorithms and programs:

  - Deductive logic (science of reasoning) and its proof systems

  - Elementary number theory, inductive definitions, and proofs by induction

  - Basic set theory

  - Counting and probability

  - Functions and relations

- Course homepage: http://ugweb.cs.ualberta.ca/~c272/W07/

- Some of the applications:
  - AI: knowledge based expert systems, logic based reasoning.
  - Hardware design: digital circuits
  - Database Theory
  - Correctness of iterative and recursive programs
- Assignment #1 is ready, and due at the **beginning** of the class on Jan 26, 2007
- All the information you need to know about this course are available on the webpage of the course. Please do check this page.
- All the announcements as well as the assignments and solutions to assignments will be posted on the web. Do check this page regularly.

Propositional Logic (Overview):

- "A collection of rules for deductive reasoning that are intended to serve as a basis for the study of every branch of knowledge."
- The concept of argument form (versus its content)
- Logical analysis is to analyze an argument's form
- Logic is also defined as "the science of reasoning"
- A few terms remain undefined: sentence, true, false

Key Notions:

- **Statement**: a declarative sentence that is either true or false, but not both
  - E.g., "two plus two is six" is a statement, although it is false but "x plus x is greater than zero" is **NOT** a statement because based on what the value of x is it can be either true or false.

- **Argument**: sequence of statements aimed at demonstrating the truth of an assertion. The final assersion is called *Conclusion*.

- In logic, the importance is the form of an argument instead of its content. Logical analysis is to verify whether the conclusion follows logically from the argument form.

  e.g. If I don't study hard or I go to too many parties then I will not get a good mark.
  I got a good mark.
  Therefore, I studied hard and did not go to too many parties.

- We can use variables to denote statements.

- Primitive statements: no way to break down into simpler statements

- Compound statements: obtained using other statements and operations:
  - Negation ($\sim$, $\neg$): for a statement p, the negation of p, denote by $\sim p$ has the opposite value of p.
  - Conjunction (AND, $\wedge$, $\cdot$): the conjunction of p and q, denoted by $p \wedge q$ is true when and only when both p and q are true.
  - Disjunction (OR, $\vee$, $+$; exclusive or, XOR, $\oplus$): the disjunction of p and q, denote by $p \vee q$ is true when p, or q, or both are true. $p \oplus q$ is true when exactly one of p or q is true and the other one is false.

- One way to demonstrate the values of statements is to use truth table: has one row for each possible combination of value assignment to different variables (corresponding to statements).

p	q	$\sim p$	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	F	T	T	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

- The order of operations
  - $\sim$ first, $\wedge$ and $\vee$ co-equal
  - Use of parentheses e.g. $\sim p \vee q = (\sim p) \vee q$

    $p \vee q \wedge r$ is ambiguous: we should either write $(p \vee q) \wedge r$ or $p \vee (q \wedge r)$.

- Note: "but" means "and": e.g. "it is not hot but it's sunny" means "it is not hot and it is sunny".

  "neither p nor q" means $\sim p \wedge \sim q$.

- **Statement/Propositional Form**: An expression made up of statement variables and logical connectives

- we can use truth table to display the truth values for all possible combinations of truth values of its components.

  e.g. truth table for $(p \vee \sim q) \wedge r$:

p	q	r	$p \vee \sim q$	$(p \vee \sim q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	F
F	T	F	F	F
F	F	T	T	T
F	F	F	T	F

- Logical equivalence: two statement forms P and Q having identical truth values

  for each substitution of statements for their statement variables

- Denoted as: $P \equiv Q$ (or $P \Longleftrightarrow Q$)

- Two statements are logical equivalent **iff** they have logical equivalent forms

- How do we prove it? — one way is by "truth table"

  e.g. show that $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

p	q	$\sim (p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Some of the simplifying laws of logic

- DeMorgan's Laws: $\sim (p \vee q) \equiv \sim p \wedge \sim q$ and $\sim (p \wedge q) \equiv \sim p \vee \sim q$

- Distribution Laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ and $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- Also $\sim (\sim p) \equiv p$; double negation law.

- Commutative: $p \wedge q \equiv q \wedge p$ and $p \vee q \equiv q \vee p$

  Associative: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ and $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

- Def: t — **tautology**, a statement that is always true. e.g. $p \vee \sim p$
  c — **contradiction**, a statement that is always false, e.g. $p \wedge \sim p$.

- If t is tautology then $p \vee t \equiv t$ and $p \wedge t \equiv p$.
  Also if c is contradiction then $p \vee c \equiv p$ and $p \wedge c \equiv c$.

- See all laws on page 14 of the text.

Conditional statements

- if p then q: $p \to q$; truth of p implies truth of q.

  e.g. "if you call me tonight then I will bring your book tomorrow"

  the only case that I am wrong is when you do call me but I don't bring your book.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

- Note: $p \to q$ is true when p is false, regardless of value of q.

- Order: $\sim$, then $\vee$ and $\wedge$, then $\to$.

- Note: $p \to q \equiv \sim p \vee q$. Verify using truth table.

  e.g. "If you don't go to work on time you will be fired" is the same as "either you go to work on time or you will be fired".

- $$\begin{aligned} \sim (p \to q) &\equiv \sim (\sim p \vee q) \\ &\equiv \sim (\sim p) \wedge (\sim q) \quad \text{by De Morgan's} \\ &\equiv p \wedge \sim q \end{aligned}$$

- $p \to q \equiv \sim q \to \sim p$; verify by truth table

- For $p \to q$:
  - Contrapositive: $\sim q \to \sim p$ is called the contrapostive of $p \to q$ and as said above is equivalent to $p \to q$.
  - Converse: $q \to p$; Check using truth table that $q \to p \not\equiv p \to q$.

    e.g. "If today is Christmas then the university is closed" is NOT equivalent to "if the university is closed then it is Christmas".
  - Inverse: $\sim p \to \sim q$; check using truth table that $\sim p \to \sim q \not\equiv p \to q$.
  - p is sufficient condition for q and q is neccessary condition for p.
  - p only if q; which means: if not q then not p.

- e.g. Show $p \wedge q \to r \equiv p \to (q \to r)$.

$$
\begin{aligned}
p \wedge q \to r &\equiv \sim (p \wedge q) \vee r \\
&\equiv (\sim p \vee \sim q) \vee r \text{ by De Morgan's} \\
&\equiv \sim p \vee (\sim q \vee r) \text{ by associative} \\
&\equiv p \to (\sim q \vee r) \\
&\equiv p \to (q \to r)
\end{aligned}
$$

Biconditional condition (neccessary and sufficient)

- p if and only if (iff) q; $p \leftrightarrow q$

- p is a necessary and sufficient condition for q.

- Definition of biconditional $\leftrightarrow$ via truth table?

  Textbook page 24

- $\leftrightarrow$ and $\to$ are co-equal, and follows the other three connectives