A Comparison of Knowledge-Based
GBFS Enhancements and Knowledge-Free Exploration

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Abstract

GBFS-based satisficing planners often augment their search with knowledge-based enhancements such as preferred operators and multiple heuristics. These techniques seek to improve planner performance by making the search more informed. In our work, we will focus on how these enhancements impact coverage and will use a simple technique called $\epsilon$-greedy node selection to demonstrate that planner coverage can also be improved by introducing knowledge-free random exploration into the search. We then revisit the existing knowledge-based enhancements so as to determine if the knowledge these enhancements employ is offering necessary guidance, or if the impact of this knowledge is to add exploration which can be achieved more simply using randomness. This investigation provides further evidence of the importance of preferred operators and shows that the knowledge added when using an additional heuristic is crucial in certain domains, while not being as effective as random exploration in others. Finally, we demonstrate that random exploration can also improve the coverage of LAMA, a planner which already employs multiple enhancements. This suggests that knowledge-based enhancements need to be compared to appropriate knowledge-free random baselines so as to ensure the importance of the knowledge being used.

1 Introduction

Greedy Best-First Search (GBFS) is a popular algorithm that is used in many heuristic search-based satisficing planners including LAMA (Richter and Westphal 2010) and Fast Downward (Helmert 2006). In its basic form, GBFS iteratively selects the most promising node for expansion, as suggested by a single heuristic which, along with a tie-breaking policy, will completely determine how the search progresses through the state-space. If GBFS does not work well on a given problem, it means that the heuristic and the tie-breaking policy are not effectively guiding the search.

To improve the performance of GBFS-based planners on such problems, the standard algorithm is often enhanced with techniques such as preferred operators or multi-heuristic best-first search. These enhancements use automatically generated knowledge as an alternative source of guidance. The goal of using such enhancements is to improve the way that the state-space is examined by making the search more informed. As the variation introduced into the search by these enhancements is based on knowledge, we refer to such enhancements as being knowledge-based.

Variation can also be introduced into GBFS by using random exploration. For example, we can change the way that nodes are iteratively selected for expansion by making the algorithm occasionally select a random node from the open list instead of the node suggested by the heuristic. This technique, which we refer to as $\epsilon$-greedy node selection, is easy to implement, has a low execution overhead, and does not require additional knowledge about the problem.

In this paper we compare the impact on coverage of knowledge-based enhancements and knowledge-free exploration. We begin by using $\epsilon$-greedy node selection to demonstrate that there is substantial value in adding random exploration to GBFS. This result suggests a need to revisit the existing knowledge-based enhancements so as to determine if the knowledge they use is offering important guidance, or if the main impact of this knowledge is to add variation that can be replicated using simpler knowledge-free approaches. We therefore evaluate the impact of the knowledge used in the knowledge-based enhancements by comparing these enhancements to equivalent systems in which the knowledge has been replaced by randomness. This investigation confirms that preferred operators are offering much more to the search than simply adding random variation, and that the knowledge employed when using secondary heuristics is crucial in some domains while not offering enough variation in others. Finally, we will show that random exploration can even benefit a fully enhanced planner like LAMA, and argue that knowledge-based planning enhancements need to be compared against stochastic alternatives to better understand the importance of the added knowledge.

2 The Value of Knowledge-Free Exploration

In this section we will demonstrate that there is value to adding knowledge-free random exploration to GBFS. We do so using $\epsilon$-greedy node selection, which is a simple modification to GBFS that allows for random exploration to be explicitly added to the algorithm. This technique is inspired by the $\epsilon$-greedy policies often used for multi-armed bandit problems (Sutton and Barto 1998). It requires the user to set a parameter $\epsilon$ with some value in the range $[0, 1]$. With probability $(1 - \epsilon)$, $\epsilon$-greedy node selection uses the same
rule as GBFS to select a node for expansion: it selects the node with the lowest heuristic value. However, with probability $\epsilon$ this technique selects a node uniformly at random from amongst all nodes in the open list. This means that the value of $\epsilon$ determines how often this node selection policy chooses greedily according to the heuristic, and how often the algorithm explores randomly.

Since $\epsilon$-greedy node selection only changes GBFS by introducing random node selection, we can use this technique to evaluate the impact of adding knowledge-free random exploration to a GBFS-based planner. For the evaluation of this technique and the others considered below, we will only consider planner coverage due to space constraints. However, these techniques could be used as only the first iteration of a restarting weighted A* search (Richter, Thayer, and Ruml 2010), or with a post-processor in Diverse Any-time Search (Xie, Valenzano, and Müller 2013). We expect that when doing so, the quality of solutions found would be similar.

The experimental setup for both this section and the rest of this paper was then as follows. All experiments are performed using the Fast Downward planning system. The test set is composed of the 790 problems from IPCs 2006, 2008, and 2011, and these tasks are treated as being unit-cost since our focus is on coverage. The experiments were performed on a cluster of 8-core machines, each with two 4-core 2.8 GHz Intel Xeon E546s processors and 6 MB of L2 cache. Planners were run with a 4 GB per-problem memory limit, and a 30 minute per-problem time limit which did not include the time for translation from PDDL to SAS+. All tested planners were set to break ties in a first-in first-out manner, do not re-open closed nodes, and use deferred heuristic evaluation (Helmert 2006) unless otherwise stated. For stochastic planners, the coverage shown is the average coverage seen over 10 runs on each problem.

Table 1 shows the coverage of a baseline GBFS planner and instances of that baseline planner that are enhanced with $\epsilon$-greedy node selection. The baseline runs standard GBFS with the FF heuristic (Hoffmann and Nebel 2001). The third column shows on how many of the 30 domains the enhanced planner could solve at least one more problem than the baseline, on average. The fourth column shows on how many domains the enhanced planner solved at least one fewer problem on average. The table shows that by adding random exploration to a GBFS search we can substantially improve the coverage on the test set. This is true for a wide range of values of $\epsilon$ and in many domains. In some of these domains the magnitude of this increase is also quite high. For example, for all values of $\epsilon$ tested, $\epsilon$-greedy was able to solve no less than an average of 29.4 of 30 2008 cybersecurity problems, 22.8 of 30 2008 woodworking problems, and 16.8 of the 20 2011 barman problems, while the baseline solved 20, 15, and 12, respectively. In the few domains in which $\epsilon$-greedy node selection decreased the coverage, the effect was minimal unless $\epsilon$ was high. For example, for all values of $0.05 \leq \epsilon \leq 0.3$, the coverage never decreased in any domain by more than an average of 2.5 problems.

To ensure this behaviour is not specific to the use of the FF heuristic and deferred heuristic evaluation, we also tested $\epsilon$-greedy node selection with standard heuristic evaluation and with a different heuristic. In these cases the results were similar. For example, the performance of GBFS using the FF heuristic and standard heuristic evaluation improved from a total of 533 problems when not using $\epsilon$-greedy node selection to an average of 596.3 when $\epsilon = 0.3$. Similarly, a GBFS that uses deferred heuristic evaluation and is guided by the context-enhanced additive (CEA) heuristic (Helmert and Geffner 2008) solves 491 problems when not using $\epsilon$-greedy node selection and an average of 536.0 when $\epsilon = 0.3$.

These results indicate that while the popular heuristics offer effective guidance for standard GBFS in many cases, there is significant value in adding variation through knowledge-free random exploration. When GBFS fails it is because it is too sensitive to the errors in the heuristic. The results suggest $\epsilon$-greedy can help decrease this sensitivity.

## 3 Knowledge-Based GBFS Enhancements

In this section, we describe how preferred operators and multi-heuristic best-first search use additional knowledge to add variation to GBFS.

### 3.1 Preferred Operators

The preferred operators of a node $n$ are operators that are applicable to $n$ and which are identified—typically as a byproduct of heuristic calculation—as being more likely to be part of a solution path. Preferred operators were first introduced under the name of helpful actions by Hoffmann and Nebel (2001) who used them for pruning actions in an enhanced hill-climbing search. They were then adapted for use in GBFS by Helmert (2006) in the Fast Downward planning system. In this system and other GBFS-based planners like LAMA, preferred operators are most often used in a dual-queue search (Richter and Helmert 2009). The first queue contains the entire open list, while the second queue only contains those nodes reached with a preferred operator. The simplest approach to using these two queues is to alternate between them when selecting a node to be expanded, where the heuristic determines the most promising node in each queue. Some GBFS-based planners also use boosting, which increases the proportion of time in which the preferred operator queue is used to select the next node to be expanded (Richter and Westphal 2010).

In a GBFS that is enhanced by preferred operators, at least every second node expanded will have been reached with a preferred operator. The enhancement is thereby us-
The knowledge-based enhancements are added. Each planner used above and the coverage achieved when the knowledge-based enhancements are added. Each planner was also tested with different operator orderings. The secondary heuristic used is the landmark count (LM) heuristic (Richter and Westphal 2010). The tested planner configurations are GBFS using the FF heuristic (FF), GBFS using multi-heuristic best-first search with the FF and LM heuristics (FF & LM), GBFS using the FF heuristic and preferred operators (FF, Pref), GBFS using the FF heuristic and boosted preferred operators (FF, BP), and GBFS using multi-heuristic best-first search with the FF and LM heuristics and boosted preferred operators (FF & LM, BP). The last of these configurations, “FF & LM, BP”, corresponds to the first iteration of LAMA (Richter and Westphal 2010). As the subsequent iterations are only used to improve the quality of the solution found during the first iteration, “FF & LM, BP” and LAMA are equivalent in terms of coverage.

The operator orderings tested are the standard ordering (Stand), the reverse of the standard ordering (Rev), random operator ordering (RO), preferred operators first (P 1st), and random operator ordering with preferred operators first (P 1st RO). Random operator ordering means that the successors of a node are randomly shuffled before they are added to a queue, while preferred operators first means that the preferred operators are put at the front of the generated successor list. By default, Fast Downward and LAMA use “P 1st” when using preferred operators, and “Stand” otherwise.

Operator ordering has previously been shown to have a substantial effect on planner performance due to its impact on the way ties are broken between successors of the same node (Howe and Dahlman 2002). The table shows that this is also true of Fast Downward, though the best operator ordering changes depending on the planner configuration. For example, the standard ordering is better than the reverse ordering for “FF & LM”, while the opposite is true for other configurations like “FF”. The table also shows that the knowledge-based enhancements are able to improve the coverage regardless of the operator ordering. While the magnitude of this improvement changes depending on the operator ordering, the relative ordering of the planners does not.

### 4.2 Evaluating the Variation Added by Preferred Operators

The previous section shows that enhancing GBFS with preferred operators substantially improves planner coverage. Recall that the knowledge being exploited by this enhancement is given by the preferred operators suggested by the heuristic, and that the variation introduced by using these operators is the result of putting a higher priority on nodes that are reached using a preferred operator. As the search would most likely vary if any proper subset of the open list was prioritized using a second queue, we can evaluate the effectiveness of this knowledge by populating the second queue using randomly selected nodes instead of with those corresponding to preferred operators.

In this experiment we ensured that the number of random successors of a given node that are put in the second queue was equal to the actual number of preferred operators suggested by the heuristic. We use random operator ordering so as to avoid the inherent bias introduced through the use of a static operator ordering with first-in-first-out tie-breaking, and we did not use boosting for the sake of

<table>
<thead>
<tr>
<th>Planner</th>
<th>Stand</th>
<th>Rev</th>
<th>RO</th>
<th>P 1st</th>
<th>P 1st RO</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>528.0</td>
<td>543.0</td>
<td>526.9</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>FF &amp; LM</td>
<td>604.0</td>
<td>599.0</td>
<td>587.4</td>
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<td>NA</td>
</tr>
<tr>
<td>FF, Pref</td>
<td>616.0</td>
<td>615.0</td>
<td>606.8</td>
<td>616.0</td>
<td>613.8</td>
</tr>
<tr>
<td>FF, BP</td>
<td>654.0</td>
<td>665.0</td>
<td>656.3</td>
<td>675.0</td>
<td>657.5</td>
</tr>
<tr>
<td>FF &amp; LM, BP</td>
<td>680.0</td>
<td>692.0</td>
<td>676.0</td>
<td>713.0</td>
<td>677.7</td>
</tr>
</tbody>
</table>
simplicity. The results are shown in Table 3, in which the columns are labelled as follows. “No Prefs” refers to the use of a single-queue GBFS that does not use preferred operators. “Prefs” refers to the use of the actual preferred operators suggested by the heuristic for populating the second queue. “Rand. Prefs” refers to the use of randomly selected operators for populating the second queue. The final column, “Avoid Prefs” refers to the use of operators that are selected randomly such that those selected are restricted from including those identified as preferred operators by the heuristic. Intuitively, a search using the “Avoid Prefs” approach is prioritizing operations against the advice of the preferred operators.

As shown in the table, the use of preferred operators with either heuristic greatly outperforms both the single-queue search and the baselines which give preference to randomly selected operators. This suggests that the preferred operators are offering important knowledge. However, the results in the “Rand. Prefs” column indicate that useful variance is introduced into the search even if the second queue is populated using randomly selected operators. It is only when the second queue is used to bias the search against the advice of the heuristic — advice which is clearly informative — that the use of the second queue is not helpful.

### 4.3 Evaluating the Variation Added by Multi-Heuristic Best-First Search

Multi-heuristic best-first search was also shown to lead to coverage improvements in Section 4.1, with some of those improvements coming in domains in which preferred operators were not as effective. For example, when using random operator ordering, “FF” solved an average of 3.8 of the 20 problems in the 2011 visitall domain, while “FF, BP” solved an average of 4.2 and “FF & LM” solved an average of 18.4. As with preferred operators, we will evaluate the importance of the extra knowledge used in this technique by replacing that knowledge with randomness. This means that we will still use a second heuristic, but it will be a purely random heuristic. For the experiments below, this was done by defining the second heuristic so that the heuristic value of a node was given by a random integer in the range from 0 to 100.

The coverage of the planner using this random heuristic is shown in Table 4 in the row labelled “FF & Random” both when using boosted preferred operators and when not using preferred operators. The table shows that the variation added by the random heuristic leads to substantially better coverage than the single-heuristic baseline planner. We include the results over different operator orderings since this attribute did affect the relative ordering of the planners tested. When using random operator ordering, the use of a random heuristic led to better coverage than the knowledge-based heuristic, though the opposite is true with the other

<table>
<thead>
<tr>
<th>Heuristic</th>
<th>No Prefs</th>
<th>Prefs</th>
<th>Rand. Prefs</th>
<th>Avoid Prefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>526.9</td>
<td>606.8</td>
<td>554.1</td>
<td>531.6</td>
</tr>
<tr>
<td>CEA</td>
<td>491.9</td>
<td>583.6</td>
<td>534.6</td>
<td>486.8</td>
</tr>
</tbody>
</table>

Table 3: Comparing the use of preferred operators to the prioritization of randomly selected operators.

<table>
<thead>
<tr>
<th>Planner</th>
<th>No Prefs</th>
<th>Boosted Prefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>526.9</td>
<td>528.0</td>
</tr>
<tr>
<td>FF &amp; Random</td>
<td>586.0</td>
<td>584.6</td>
</tr>
<tr>
<td>FF &amp; LM</td>
<td>587.4</td>
<td>604.0</td>
</tr>
</tbody>
</table>

Table 4: Knowledge-based and knowledge-free multi-heuristic best-first search.

orderings. When using the random heuristic, the planner is less sensitive to the operator ordering because when selecting nodes for expansion according to this heuristic, the randomly assigned heuristic value will matter more for tie-breaking between children of the same node than will the operator ordering. However, this also means that if the operator ordering is introducing a beneficial bias, the planner using the random heuristic is less able to take advantage.

Despite the similarity in the total coverage results, domain-by-domain analysis showed that using the knowledge-based heuristic is resulting in variation that is quite different than random exploration. For example, consider the results when using standard operator ordering and no preferred operators. Just as with $\epsilon$-greedy node selection, the random exploration added when using the random heuristic increased coverage in the 2008 cybersecurity and 2008 woodworking domains, from 20 and 15 respectively when using a single heuristic, to averages of 30.0 and 25.4 when using the random heuristic. In contrast, the use of the knowledge-based LM heuristic as a secondary heuristic actually hurts coverage, as the resulting planner solves only 12 and 19 problems respectively. However, the LM heuristic does add important guidance in the 2008 transport, 2011 parking, and the 2011 visitall domains. In these domains, the single heuristic planner solved 34 of the 70 total problems, while adding the random heuristic improved coverage to 45.7 which was still not as much as the 67 solved when using the LM heuristic.

The use of a random heuristic was also an effective way to increase coverage when used alongside the CEA heuristic. For example, when using random operator ordering and boosted preferred operators, CEA solved an average of 617.3 problems, while the addition of a random heuristic increased coverage to an average of 650.9. The use of the random heuristic even compares well to a multi-heuristic best-first search that uses both the CEA and FF heuristics when using random operator ordering and boosted preferred operators, as such a system solves an average of 646.5 problems.

### 5 Related Work

Diverse best-first search (DBFS) is a search algorithm that also stochastically selects nodes from the open list (Imai and Kishimoto 2011). The execution of this algorithm consists of two phases. First, a node is randomly selected from the open list according to a distribution which favours nodes with a low $g$-cost and a low heuristic value. Secondly, an expansion-limited local GBFS search is initiated from the selected node. This process repeats until a solution is found.

Though $\epsilon$-greedy node selection may not increase coverage as much as DBFS when added to an otherwise unen-
that any newly developed knowledge-based enhancements through random exploration, it remains necessary to ensure proper random baselines to ensure that they are actually adding better guidance into the search instead of merely adding exploration which can be achieved in simpler ways. We performed such a comparison between appropriate randomized baselines and two existing enhancements. Our results indicate that the knowledge used by preferred operators is essential to the success of this technique, while the use of a secondary heuristic in a multi-heuristic best-first search is offering important guidance in certain domains while not varying the search effectively in others.

We have also tried adding ϵ-greedy node selection to the first iteration of LAMA, so as to test if random exploration is unnecessary in such a fully enhanced planner. For this experiment, each queue was set to individually use ϵ-greedy node selection. For example, if the next node expanded is to be selected from a preferred operator queue, the search will expand the most promising preferred operator with probability 1− ϵ, and a random preferred operator with probability ϵ. The results of this experiment are given in Table 5, which shows that the added exploration increases the coverage of LAMA over a wide range of ϵ values and over all the operator orderings considered. As there is still room for gains through random exploration, it remains necessary to ensure that any newly developed knowledge-based enhancements are also compared to appropriate random baselines.

6 Conclusion and Discussion

In this paper, we have demonstrated through a simple technique called ϵ-greedy node selection that there is substantial value in adding variation to GBFS through random exploration. This means that GBFS can be improved by both better guidance and by adding random exploration. It is therefore necessary to compare knowledge-based enhancements to proper random baselines to ensure that they are actually adding better guidance into the search instead of merely adding exploration which can be achieved in simpler ways. We performed such a comparison between appropriate randomized baselines and two existing enhancements. Our results indicate that the knowledge used by preferred operators is essential to the success of this technique, while the use of a secondary heuristic in a multi-heuristic best-first search is offering important guidance in certain domains while not varying the search effectively in others.

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### Table 5: Adding ϵ-greedy node selection to the first iteration of LAMA.

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<tbody>
<tr>
<td>Baseline</td>
<td>680.0</td>
<td>676.0</td>
<td>713.0</td>
<td>677.7</td>
</tr>
<tr>
<td>ϵ = 0.05</td>
<td>704.6</td>
<td>704.8</td>
<td>723.0</td>
<td>706.7</td>
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<tr>
<td>ϵ = 0.1</td>
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<td>ϵ = 0.2</td>
<td>703.8</td>
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<tr>
<td>ϵ = 0.3</td>
<td>702.9</td>
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In this paper, we have demonstrated through a simple technique called ϵ-greedy node selection to the first iteration of LAMA. The default LAMA configuration corresponds to the baseline with the P 1 st operator ordering.

enhanced planner, it still increases coverage substantially and it is considerably simpler. In any case, the main purpose of ϵ-greedy node selection is not to compete with DBFS, but to isolate the impact of knowledge-free random exploration on GBFS and to clearly demonstrate its positive impact. This is also the case when comparing ϵ-greedy node selection to other systems which search for solutions using stochastic techniques. These include Arvand (Nakhost and Müller 2009), which uses a random-walk based search, Roamer (Lu et al. 2011), which adds random-walks to GBFS, and Lamar and Randward (Olsen and Bryce 2011), which use a stochastic version of the FF heuristic that is constructed by adding randomness into the way in which the heuristic is computed.

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### Acknowledgments

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