Computing Science (CMPUT) 657
Algorithms for Combinatorial Games

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Fall 2025

Relation Between Thermographs and Stops

- We can determine how games relate to numbers by looking at their stops
- We can use that for rough but powerful pruning
- Thermographs are closely related to stops
- Remember Leftstop and Rightstop: minimax values of game *when stopping play at a number*

Thermograph vs Stops, Rough Version

- Siegel Proposition 3.17:
- Let x be a number and G a game
- If $RS(G) > x$ then $G > x$
- If $LS(G) < x$ then $G < x$
- So the value of a game G "almost" lies between the stops:
- $RS(G) - \epsilon < G < LS(G) + \epsilon$, for any $\epsilon > 0$.
- But how does G compare with LS, RS?

Thermograph vs Stops, Equality

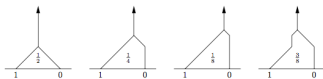**Figure 17.** Thermographs of loop-free infinitesimals.

Image source: (Berlekamp 1996)

- A game can be either $\geq$ its right stop, or be confused with it
- Example 1: $\{2|*\} > 0$ (win for left)
- Example 2: $\{2|0\} \not\geq 0$ (win for first player)
- A game can be either $\leq$ its left stop, or be confused with it
- Examples: e.g. inverses of previous examples, $\{*|-2\} < 0$, $\{0|-2\} \not\geq 0$
- Subzero thermographs can tell these two cases apart!

Comparing Two Games using Stops

- Easy observation: two games can be ordered if they can be separated by number
- Remember: if $LS(G) < x$ then $G < x$, if $RS(H) > x$ then $H > x$
- If $LS(G) < RS(H)$, then there exists a number x in between, so $G < x < H$ follows
- With subzero thermography we can sharpen that a bit
- If $G \leq x$ and $x < H$, then $G < H$
- If G and H are options in a game, we can prune the dominated one - G for Left options or H for Right options

- We can very often compute thermographs much faster than canonical forms
- Their complexity stays bounded by a small constant in practice
- In hot games, many options will be dominated from looking only at their stops
- Can we develop general CG algorithms that prune based only on TG?
- Tradeoff between how much pruning (less than with canonical forms) and speed (could be much faster)