

Computing Science (CMPUT) 657

Algorithms for Combinatorial Games

Martin Müller

Department of Computing Science
University of Alberta
`mmueller@ualberta.ca`

Fall 2025

Notes on When to Play One-Sided Sente

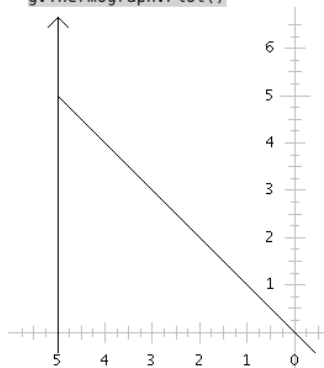
CMPUT 657

- Cryptic remark in (Berlekamp 1996), page 368:
Although the “size of a move” in any region is not necessarily unique, it may always safely be taken as the temperature of that region.
- What does he mean?
- I think he is talking about one-sided sente situations

Example

CMPUT 657

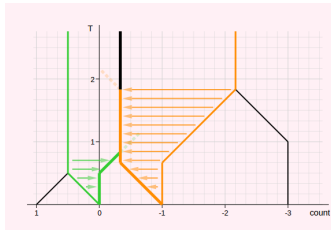
```
> g := {{20|5}|0}  
{20|5||0}  
> g.Thermograph.Plot()
```



- Example from recent sum game:
 $G^3 = \{\{20|5\}|0\}$
- Mean 5, temperature 5, mast at 5, right scaffold diagonally from 0
- Move to G^L raises temperature to 7.5
- Orthodox play: from G to G^L to $G^{LR} = 5$, left sente

One-Sided Sente Thermograph Construction

CMPUT 657



In picture, the taxed left scaffold of the right option (fat orange line) is vertical at t , and for a while above t

- It *coincides* with the mast even above t
- Eventually, it branches off from mast diagonally (faint fat orange dashed line)
- The taxed right scaffold of the left option (fat green) is diagonal
- It coincides with the mast in only one point (the base point)
- It immediately branches off diagonally from the mast (faint fat green dashed line)

One-Sided Sente vs Tax

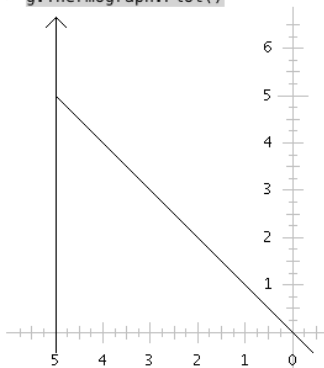
CMPUT 657

- Tax below 5: both are eager to play
 $G^3_t = \{\{20 - 2t|5\}|t\}$
- Taxed scaffolds: $LS(t) > RS(t)$
- At tax = 5, the game is inf. close to the number 5
- At tax above 5, Right would rather pass
- But: between 5 and 7.5, Left can still play without a loss
- Left plays tax t with $5 \leq t \leq 7.5$, leaving
- $G^L - t = \{20|5\} - t$
- Right is eager to answer, so Left gets $+t$ back
- $G^{LR} - t + t = 5$
- If Right did not answer, left would be glad to move to
 $G^{LL} - 2t = 20 - 2t \geq 20 - 15 = 5$
- Above 7.5 both prefer to pass, $20 - 2t < 5$

Summary

CMPUT 657

```
> g := {{20|5}|0}  
{20|5||0}  
> g.Thermograph.Plot()
```



- In one-sided sente games G , the sente player can play even at a temperature higher than $t(G)$
- There is an interval from $t(G)$ upwards to the point where the taxed scaffold in the construction diverges from the mast
- There is no standard notation for this upper limit of the range
- In our TDS paper, we call it $\hat{t}(G, p)$ for player p