Computing Science (CMPUT) 657

Algorithms for Combinatorial Games

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Example - The Book for Economic Rules

Goal: Play the sum game G perfectly under Economic Rules

- $G = G1 + G2 + G3 + G4 + G5$
- $G1 = \{2|0\}$
- $G2 = \{\{10|1\}|\{-1|-10\}\}$
- $G3 = \{\{20|5\}|0\}$
- $G4 = \{-3|5\}$
- $G5 = \{8|7||6|5|||0|-1||-2|-3\}$
- How to play perfectly? We need "the book" with all means and temperatures of subgames

The Book for Game G: Games G1 and G4

- G1 is simple switch:
- $G1 = \{2|0\}$, $\mu(G1) = 1$, $t(G1) = 1$
- $G1^L = 2$, $\mu(G1^L) = 2$, $t(G1^L) = -1$
- $G1^R = 0$, $\mu(G1^R) = 0$, $t(G1^R) = -1$

- G4 is zugzwang:
- $G4 = \{-3|5\} = 0$, $\mu(G4) = 0$, $t(G4) = -1$

The Book for Game G: Game $G3$

- G3 is one-sided sente for Left
- $G3 = \{\{20|5\}|0\}$, $\mu(G3) = 5$, $t(G3) = 5$
- $G3^L = \{20|5\}$, $\mu(G3^L) = 12.5$, $t(G3^L) = 7.5$
- $G3^R = 0$, $\mu(G3^R) = 0$, $t(G3^R) = -1$
- $G3^{LL} = 20$, $\mu(G3^{LL}) = 20$, $t(G3^{LL}) = -1$
- $G3^{LR} = 5$, $\mu(G3^{LR}) = 5$, $t(G3^{LR}) = -1$

The Book for Game G: Game $G2$

- G2 is "double sente", but not too hot:
- $G2 = \{\{10|1\}|\{-1|-10\}\}$, $\mu(G2) = 0$, $t(G2) = 5.5$
- $G2^L = \{10|1\}$, $\mu(G2^L) = 5.5$, $t(G2^L) = 4.5$
- $G2^R = \{-1|-10\}$, $\mu(G2^R) = -5.5$, $t(G2^R) = 4.5$
- $G2^{LL} = 10$, $G2^{LR} = 1$
- $G2^{RL} = -1$, $G2^{RR} = -10$

The Book for Game G: Game $G5$ and sum

- G5 can be written as 2.5 + a sum of switches:
- $G5 = 2.5 \pm 0.5 \pm 1 \pm 4,\ \mu(G5) = 2.5,\ t(G5) = 4$

- $G = G1 + G2 + G3 + G4 + G5$
- $\mu(G) = \mu(G1) + \mu(G2) + \mu(G3) + \mu(G4) + \mu(G5) =$ $= 1 + 0 + 5 + 0 + 2.5 = 8.5$
- $t(G) \leq \max(t(G1), t(G2), t(G3), t(G4), t(G5)) =$ $= \max(1, 5.5, 5, -1, 4) = 5.5$
- Fair bid for playing Black: 8.5
- Fair initial tax rate: 5.5