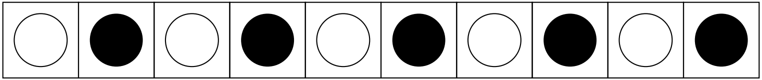
# Alternating Linear Clobber is a 1st-Player Win*

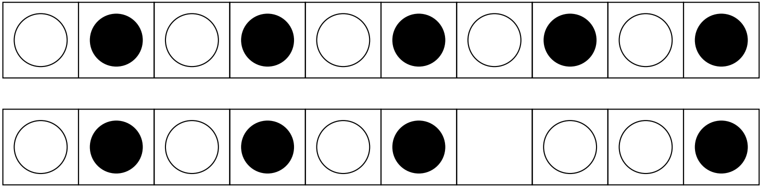
## Research by Chen et al. (2025)

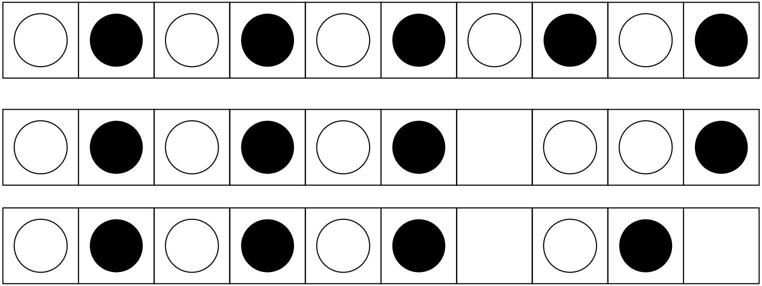

Abel Romer

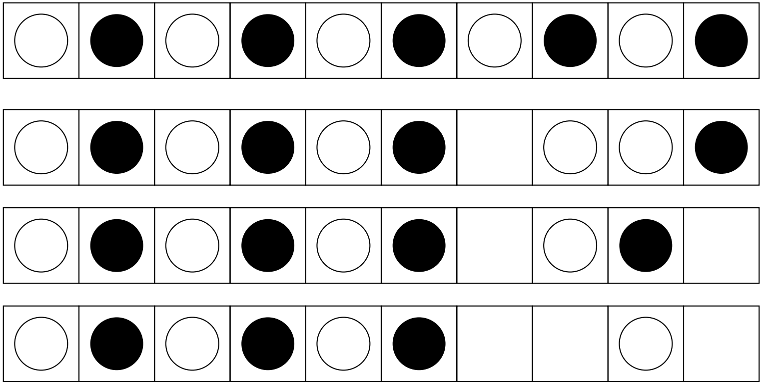
Department of Computing Science
University of Alberta

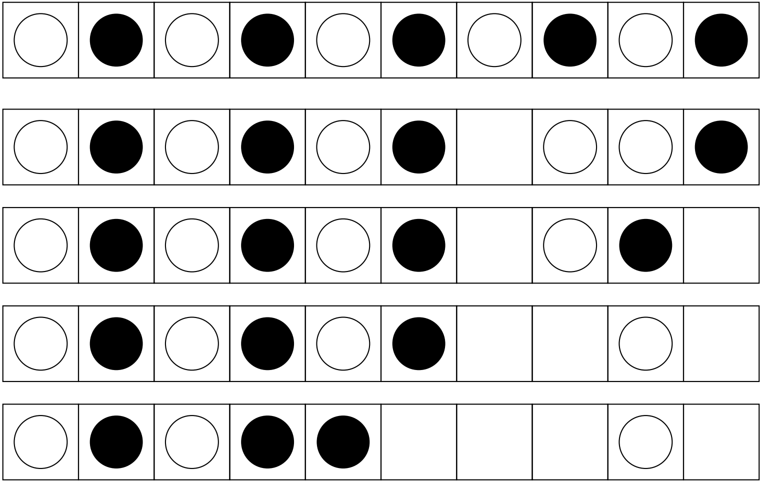

November 4, 2025

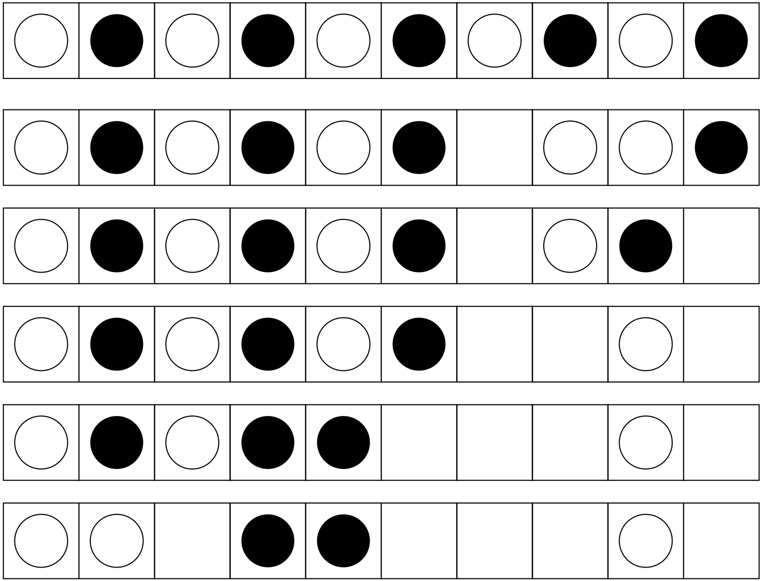




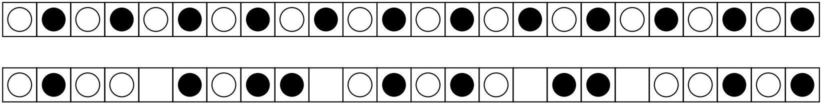




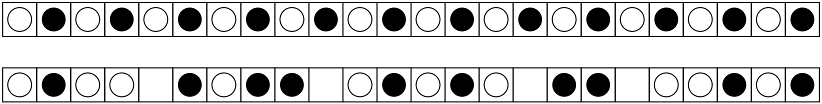




# Classes of components



# Classes of components



$$\mathcal{A} = (\bigcirc\bullet)^n$$
$$\mathcal{O} = (\bigcirc\bullet)^n\bigcirc$$
$$o\mathcal{A} = \bigcirc(\bigcirc\bullet)^n$$
$$o\mathcal{O} = \bigcirc(\bigcirc\bullet)^n\bigcirc$$
$$o\mathcal{O}o = \bigcirc(\bigcirc\bullet)^n\bigcirc\bigcirc$$
$$o\mathcal{A}x = \bigcirc(\bigcirc\bullet)^n\bullet$$

$$-\mathcal{O} = \bullet(\bigcirc\bullet)^n$$
$$-o\mathcal{A} = \bullet\bullet(\bigcirc\bullet)^n\bigcirc$$
$$-o\mathcal{O} = \bullet\bullet(\bigcirc\bullet)^n$$
$$-o\mathcal{O}o = \bullet\bullet(\bigcirc\bullet)^n\bullet$$

# Proof outline

1. Assume Left (●) plays first

# Proof outline

1. Assume Left (●) plays first $\rightarrow$ winning strategy for 1st player

# Proof outline

1. Assume Left (●) plays first $\rightarrow$ winning strategy for 1st player
2. We show that Left (●) can force Right (○) into a set of ultimately losing positions

# Proof outline

1. Assume Left (●) plays first → winning strategy for 1st player
2. We show that Left (●) can force Right (○) into a set of ultimately losing positions

Losing positions for Right (*not actually this simple...):

$$(\bigcirc\,\bullet)^n\,\bigcirc$$

$$\bigcirc\,(\bigcirc\,\bullet)^n\,\bigcirc$$

# Proof outline

1. Assume Left (●) plays first → winning strategy for 1st player
2. We show that Left (●) can force Right (○) into a set of ultimately losing positions

Losing positions for Right (*not actually this simple...):

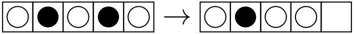
$$(\bigcirc\bullet)^n\bigcirc$$

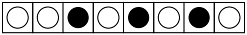
$$\bigcirc(\bigcirc\bullet)^n\bigcirc$$

*Why are these losing?*

# Why losing: heuristic argument

If they are small, Right cannot move to a winning position (some irregular exceptions).

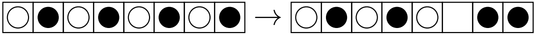




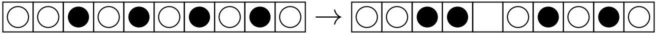
If they are big, any move by Right can be countered by a move back to these positions.

# Left strategy: Examples

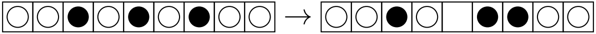
$$(\bigcirc\,\bullet)^n \to (\bigcirc\,\bullet)^n\,\bigcirc$$



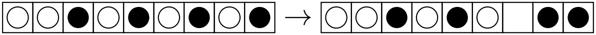
$$\bigcirc\,(\bigcirc\,\bullet)^n\,\bigcirc \to (\bigcirc\,\bullet)^n\,\bigcirc$$



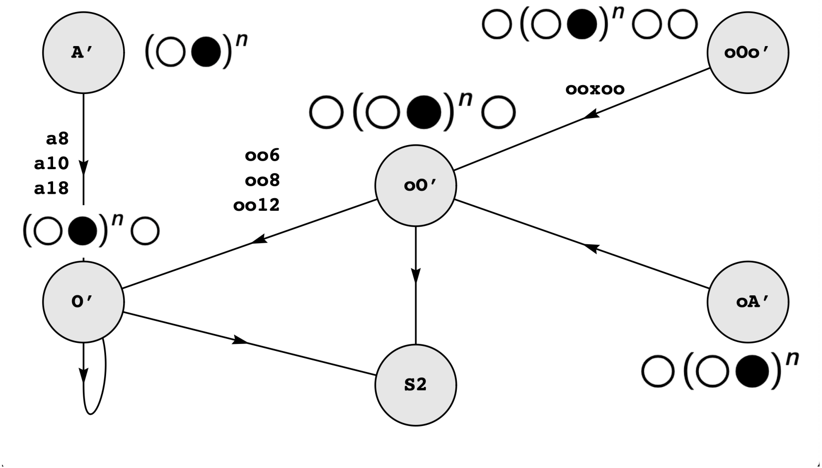
$$\bigcirc\,(\bigcirc\,\bullet)^n\,\bigcirc\,\bigcirc \to \bigcirc\,(\bigcirc\,\bullet)^n\,\bigcirc$$



$$\bigcirc\,(\bigcirc\,\bullet)^n \to \bigcirc\,(\bigcirc\,\bullet)^n\,\bigcirc$$



# Left strategy: Examples

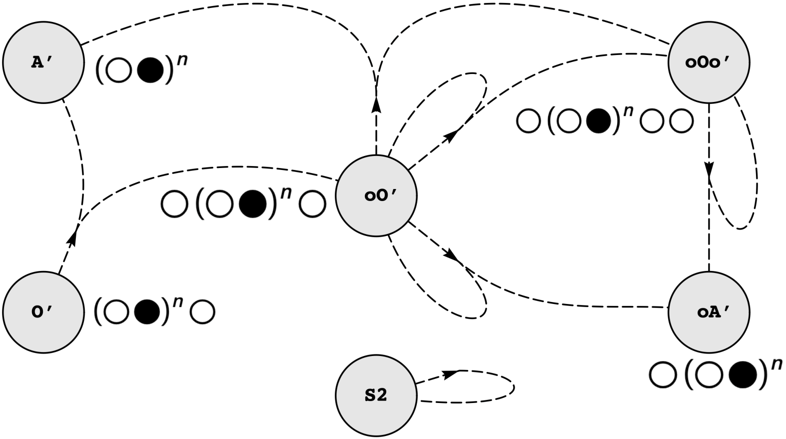


# Right options: Examples

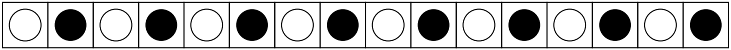
$$
\begin{aligned}
(\bigcirc\bullet)^n\bigcirc &\to (\bigcirc\bullet)^n \\
&\to \bigcirc(\bigcirc\bullet)^n\bigcirc
\end{aligned}
$$

$$
\begin{aligned}
\bigcirc(\bigcirc\bullet)^n\bigcirc &\to (\bigcirc\bullet)^n \\
&\to \bigcirc(\bigcirc\bullet)^n\bigcirc \\
&\to \bigcirc(\bigcirc\bullet)^n\bigcirc\bigcirc \\
&\to \bigcirc(\bigcirc\bullet)^n
\end{aligned}
$$

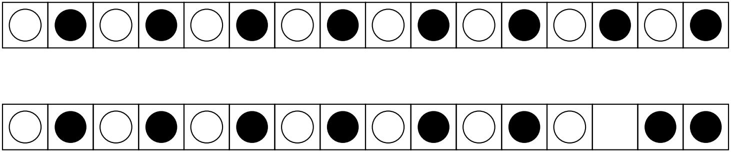
# Right options: Examples



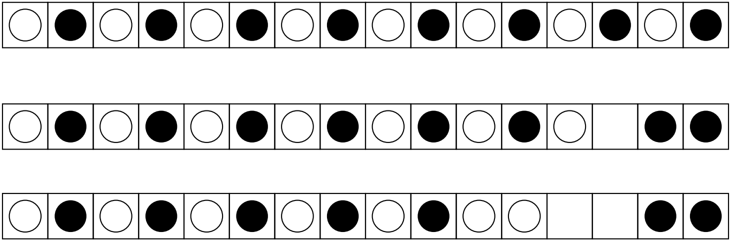
# An example game



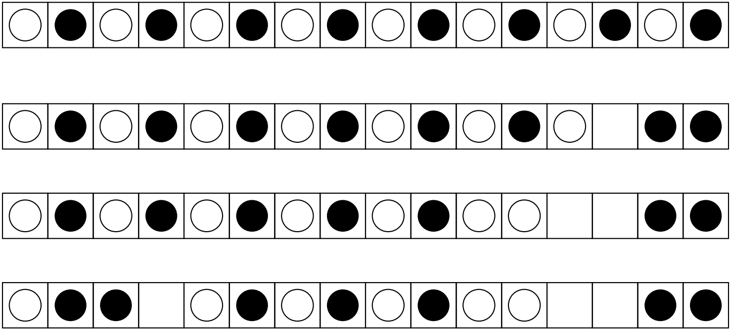
# An example game



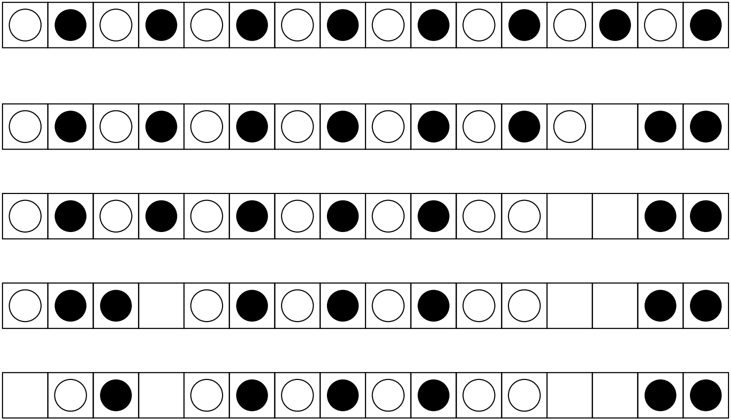
# An example game



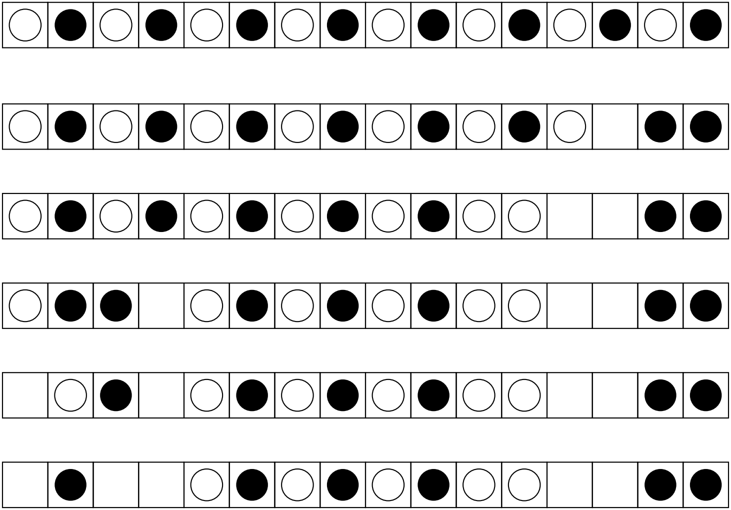
# An example game



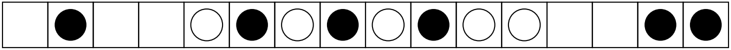
# An example game



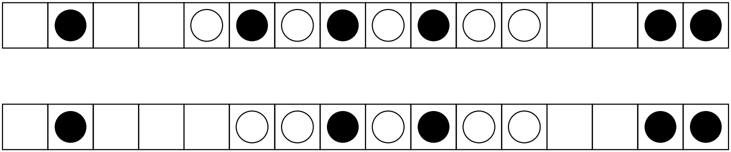
# An example game



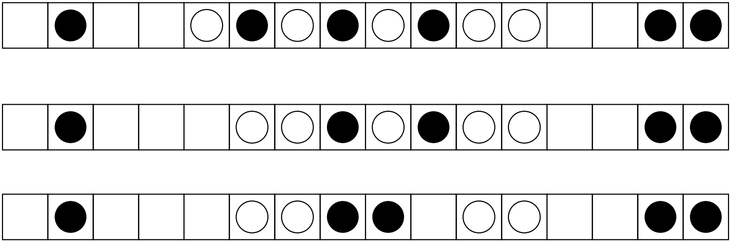
# An example game continued



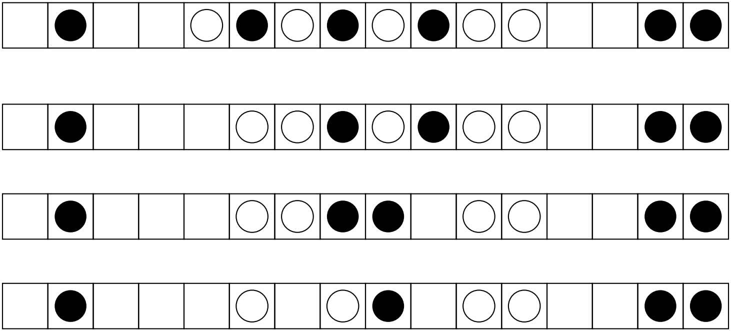
# An example game continued



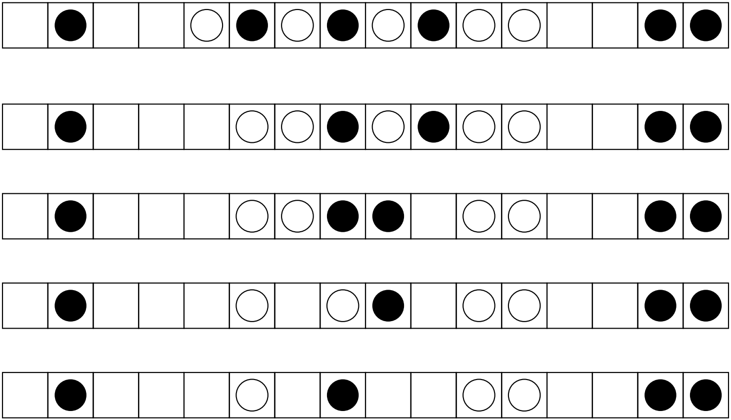
# An example game continued



# An example game continued



# An example game continued



THANKS