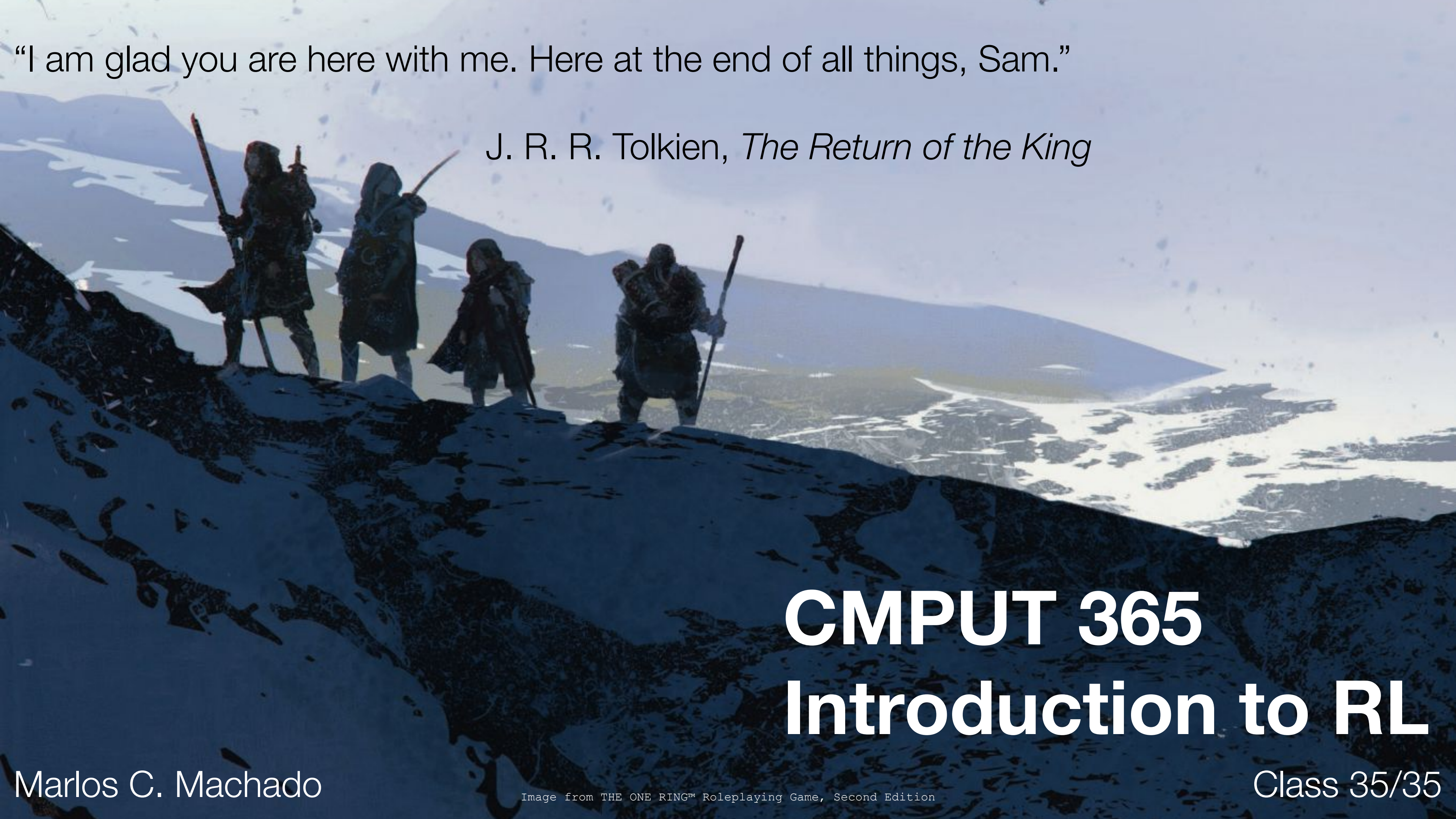


"I am glad you are here with me. Here at the end of all things, Sam."

J. R. R. Tolkien, *The Return of the King*

CMPUT 365
Introduction to RL

Marlos C. Machado

Image from THE ONE RING™ Roleplaying Game, Second Edition

Class 35/35

Reminder I

You **should be enrolled in the private session** we created in Coursera for CMPUT 365.

I **cannot** use marks from the public repository for your course marks.

You **need** to **check**, **every time**, if you are in the private session and if you are submitting quizzes and assignments to the private section.

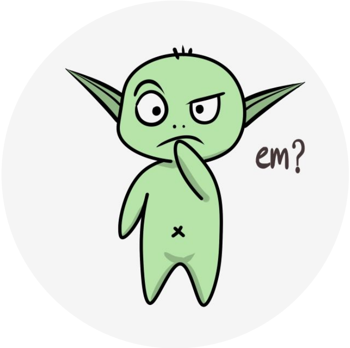
The deadlines in the public session **do not align** with the deadlines in Coursera.

If you have any questions or concerns, **talk with the TAs** or email us `cmput365@ualberta.ca`.

Reminder II

- Final Exam Schedule released
  - 12/14/2023 at 14:00 in CCIS L1-160. It will be 90 minutes long.
- The activity for extra marks is due today.
  - Policy gradient methods
- The Student Perspectives of Teaching (SPOT) Survey is available.

Please, interrupt me at any time!



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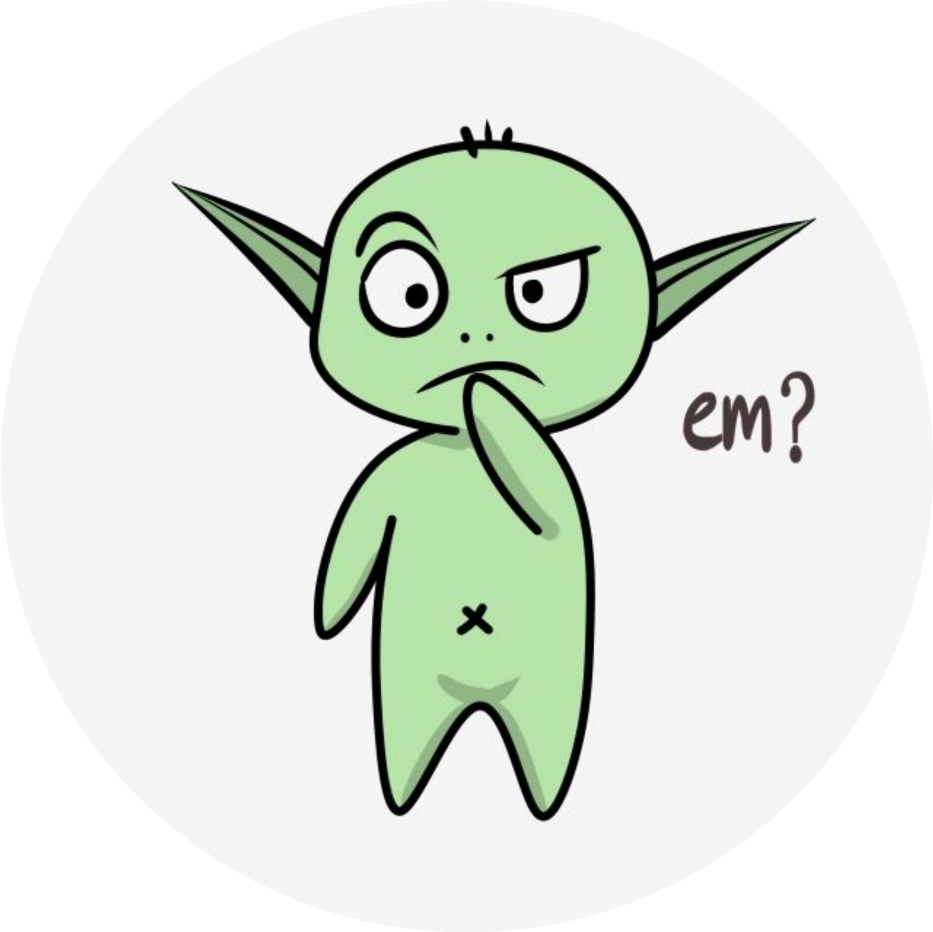
Last Class: Episodic Semi-gradient Sarsa

Episodic Semi-gradient Sarsa for Estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \to \mathbb{R}$
Algorithm parameters: step size $\alpha > 0$, small $\varepsilon > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)

Loop for each episode:
    $S, A \leftarrow$ initial state and action of episode (e.g., ε-greedy)
    Loop for each step of episode:
        Take action A, observe R, S'
        If S' is terminal:
            $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$
            Go to next episode
        Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha\big[R + \gamma\hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})\big]\nabla\hat{q}(S, A, \mathbf{w})$
        $S \leftarrow S'$
        $A \leftarrow A'$



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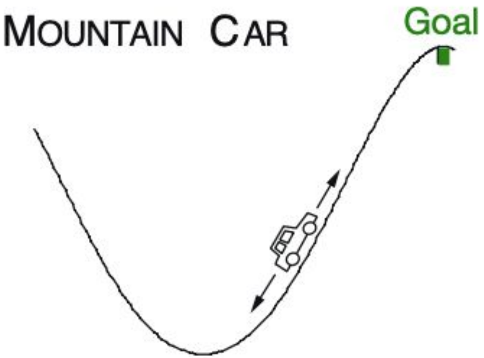
Example: Mountain Car Task

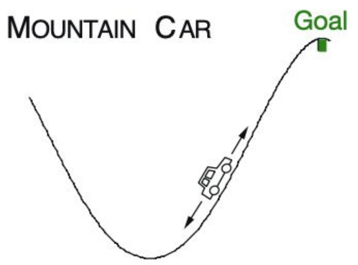


- Observations: $(x, \dot{x})$
- Actions:
  - Full throttle forward: +1
  - Full throttle reverse: -1
  - Zero throttle: 0
- Rewards: -1 at every time step, until end of episode.
- Dynamics:

$$x_{t+1} \doteq bound\left[x_t + \dot{x}_{t+1}\right]$$

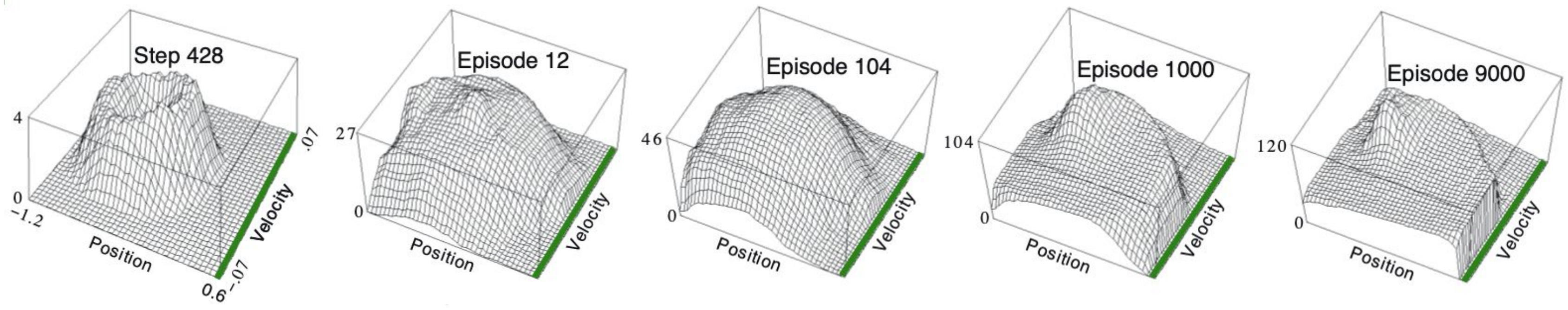
$$\dot{x}_{t+1} \doteq bound\left[\dot{x}_t + 0.001 A_t - 0.0025 \cos(3x_t)\right]$$

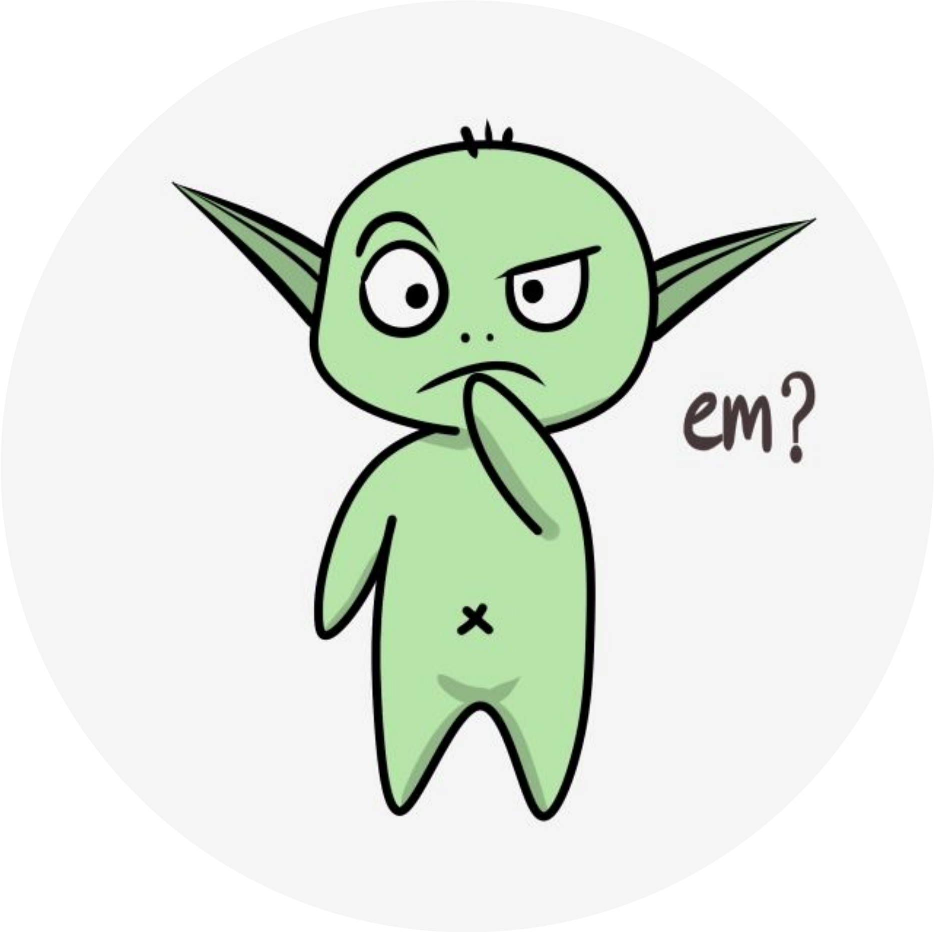
$$-1.2 \leq x_{t+1} \leq 0.5 \text{ and } -0.07 \leq \dot{x}_{t+1} \leq 0.07$$



"Solution": Mountain Car Task

- Feature representation:
  - Grid-tilings with 8 tilings and asymmetrical offsets.
  - $\hat{q}(s, a, \mathbf{w}) \doteq \mathbf{w}^\top \mathbf{x}(s, a) = \sum_{i=1}^{d} w_i \cdot x_i(s, a)$
- Sarsa
  - Weights initialized at zero. Effectively optimistic initialization.





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Avg. Reward: A Problem Setting for Continuing Tasks

- Continuing problems without discounting.
    - The agent cares about all rewards equally.

Avg. Reward: A Problem Setting for Continuing Tasks

- Continuing problems without discounting.
  - The agent cares about all rewards equally.
- Quality of a policy is defined by the average rate of reward, r(π):

$$\begin{aligned} r(\pi) &\doteq \lim_{h\to\infty} \frac{1}{h}\sum_{t=1}^{h} \mathbb{E}[R_t \mid S_0, A_{0:t-1} \sim \pi] \\ &= \lim_{t\to\infty} \mathbb{E}[R_t \mid S_0, A_{0:t-1} \sim \pi], \\ &= \sum_s \mu_\pi(s) \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) r \end{aligned}$$

If the MDP is *ergodic*: the starting state and any early decision made by the agent can have only a temporary effect; in the long run the expectation of being in a state depends only on the policy and the MDP transition probabilities.

Avg. Reward: A Problem Setting for Continuing Tasks

- (Differential) Return:

$$G_t \doteq R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + R_{t+3} - r(\pi) + \cdots .$$

Avg. Reward: A Problem Setting for Continuing Tasks

- (Differential) Return:

$$G_t \doteq R_{t+1} - r(\pi) + R_{t+2} - r(\pi) + R_{t+3} - r(\pi) + \cdots.$$

- Differential value functions:

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{r,s'} p(s', r|s, a)\Big[r - r(\pi) + v_\pi(s')\Big],$$

$$q_\pi(s,a) = \sum_{r,s'} p(s', r|s, a)\Big[r - r(\pi) + \sum_{a'} \pi(a'|s') q_\pi(s', a')\Big],$$

$$v_*(s) = \max_a \sum_{r,s'} p(s', r|s, a)\Big[r - \max_\pi r(\pi) + v_*(s')\Big], \text{ and}$$

$$q_*(s,a) = \sum_{r,s'} p(s', r|s, a)\Big[r - \max_\pi r(\pi) + \max_{a'} q_*(s', a')\Big]$$

Avg. Reward: A Problem Setting for Continuing Tasks

- Differential value functions:

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{r,s'} p(s', r | s, a) \Big[r - r(\pi) + v_\pi(s') \Big],$$

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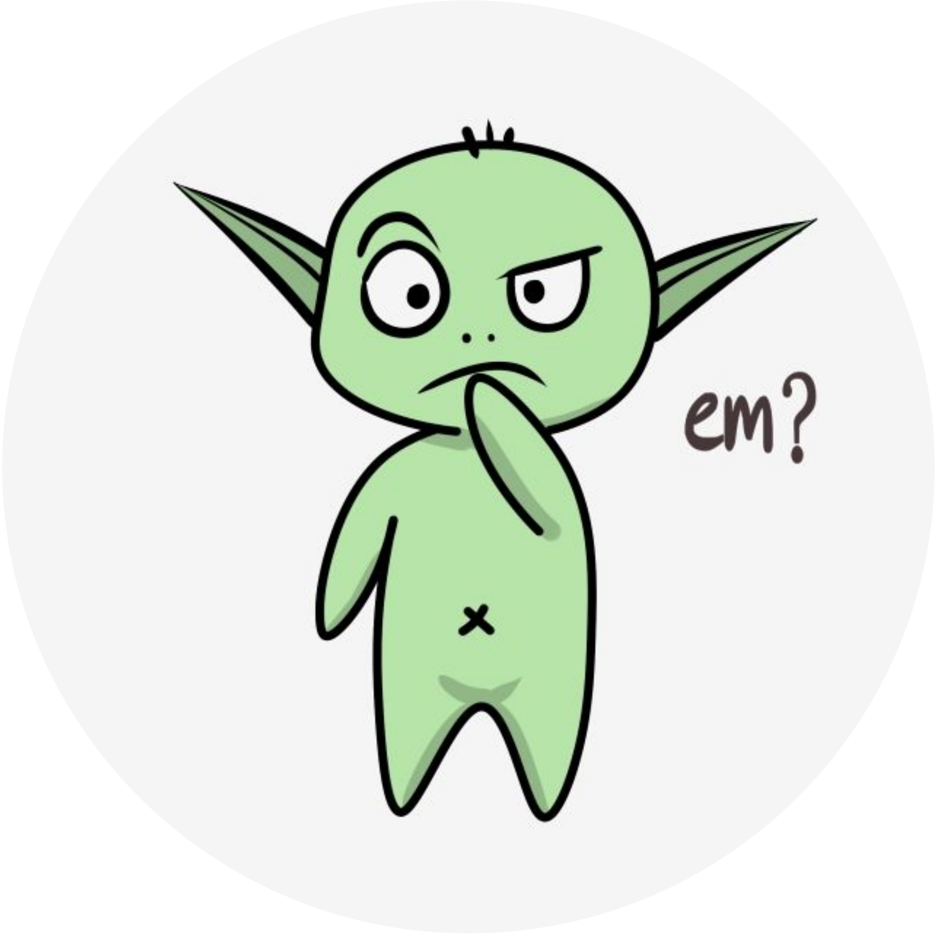
$$v_*(s) = \max_a \sum_{r,s'} p(s', r | s, a) \Big[r - \max_\pi r(\pi) + v_*(s') \Big], \text{ and}$$

$$q_*(s,a) = \sum_{r,s'} p(s', r | s, a) \Big[r - \max_\pi r(\pi) + \max_{a'} q_*(s', a') \Big]$$

- Differential TD error:

$$\delta_t \doteq R_{t+1} - \bar{R}_t + \hat{v}(S_{t+1}, \mathbf{w}_t) - \hat{v}(S_t, \mathbf{w}_t),$$

$$\delta_t \doteq R_{t+1} - \bar{R}_t + \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}_t) - \hat{q}(S_t, A_t, \mathbf{w}_t)$$



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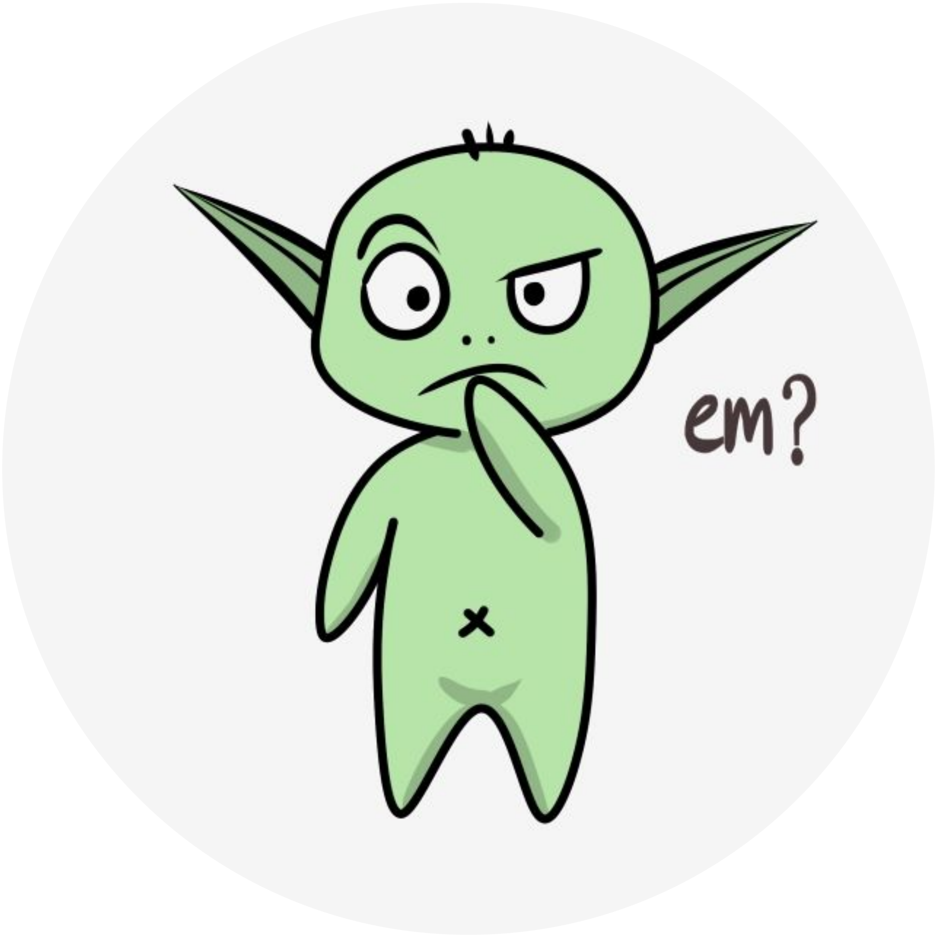
Differential semi-gradient Sarsa

Differential semi-gradient Sarsa for estimating $\hat{q} \approx q_*$

Input: a differentiable action-value function parameterization $\hat{q} : \mathcal{S} \times \mathcal{A} \times \mathbb{R}^d \rightarrow \mathbb{R}$
Algorithm parameters: step sizes $\alpha, \beta > 0$, small $\varepsilon > 0$
Initialize value-function weights $\mathbf{w} \in \mathbb{R}^d$ arbitrarily (e.g., $\mathbf{w} = \mathbf{0}$)
Initialize average reward estimate $\bar{R} \in \mathbb{R}$ arbitrarily (e.g., $\bar{R} = 0$)

Initialize state S, and action A
Loop for each step:
  Take action A, observe R, S'
  Choose A' as a function of $\hat{q}(S', \cdot, \mathbf{w})$ (e.g., ε-greedy)
  $\delta \leftarrow R - \bar{R} + \hat{q}(S', A', \mathbf{w}) - \hat{q}(S, A, \mathbf{w})$
  $\bar{R} \leftarrow \bar{R} + \beta\delta$
  $\mathbf{w} \leftarrow \mathbf{w} + \alpha\delta\nabla\hat{q}(S, A, \mathbf{w})$
  $S \leftarrow S'$
  $A \leftarrow A'$



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