“The rotten tree-trunk, until the very moment when the storm-blast breaks it in two, has all the appearance of might it ever had.”

Isaac Asimov, *Foundation*

CMPUT 365
Introduction to RL

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Class 11/ 35

Plan

- Value Functions and Bellman Equations
  - Non–comprehensive overview
  - We are still not talking about solution methods, we are only formalizing things

Reminder

You **should be enrolled in the private session** we created in Coursera for CMPUT 365.

I **cannot** use marks from the public repository for your course marks.

You **need** to **check**, **every time**, if you are in the private session and if you are submitting quizzes and assignments to the private section.

The deadlines in the public session **do not align** with the deadlines in Coursera.

If you have any questions or concerns, **talk with the TAs** or email us `cmput365@ualberta.ca`.

Please, interrupt me at any time!

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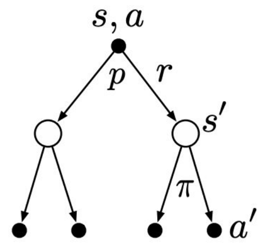
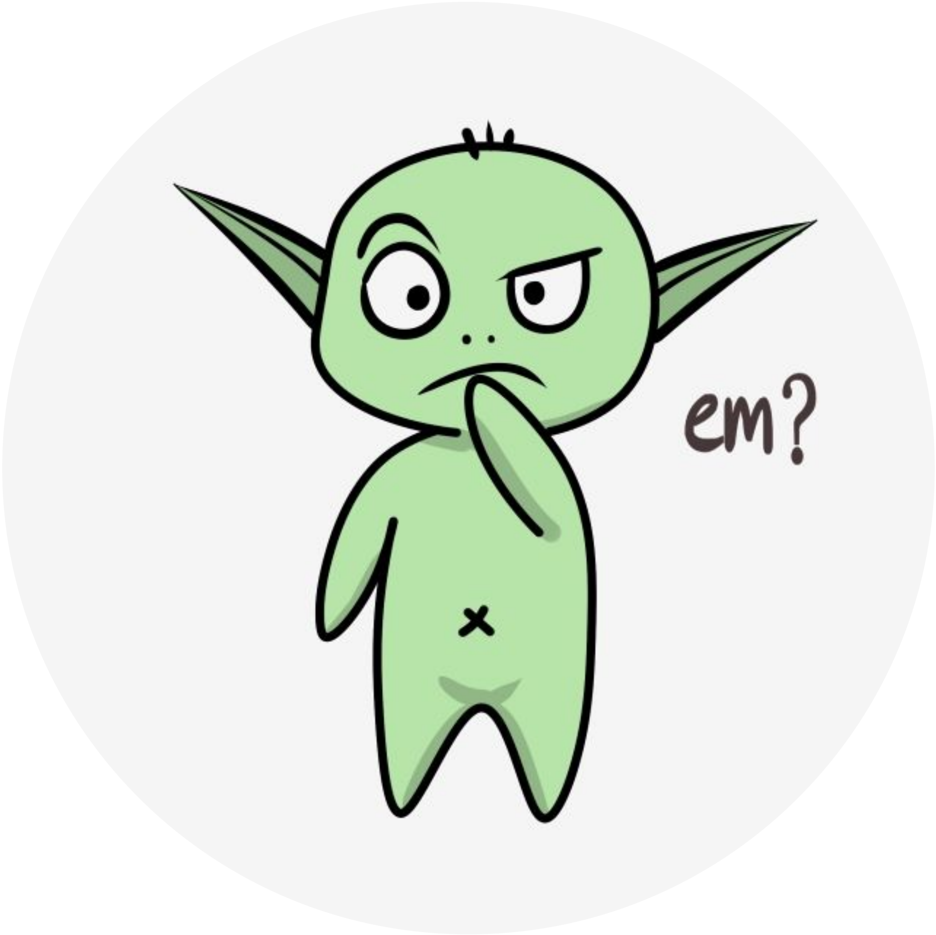
State-Action Value Functions Satisfy Recursive Relationships

Exercise 3.17 What is the Bellman equation for action values, that is, for q_π? It must give the action value $q_\pi(s, a)$ in terms of the action values, $q_\pi(s', a')$, of possible successors to the state–action pair (s, a). Hint: The backup diagram to the right corresponds to this equation. Show the sequence of equations analogous to (3.14), but for action values. □



q_π backup diagram



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Optimal Policies and Optimal Value Functions

- Value functions define a partial ordering over policies.
  - $\pi \geq \pi'$ iff $v_\pi(s) \geq v_{\pi'}(s)$ for all $s \in \mathcal{S}$.
  - There is always at least one policy that is better than or equal to all other policies. The *optimal policy*.

$$v_*(s) \doteq \max_\pi v_\pi(s)$$

$$q_*(s,a) \doteq \max_\pi q_\pi(s,a)$$

$$q_*(s,a) = \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a]$$

Optimal Policies and Optimal Value Functions

- Because v_* is the value function for a policy, it must satisfy the self-consistency condition given by the Bellman equation for state values.

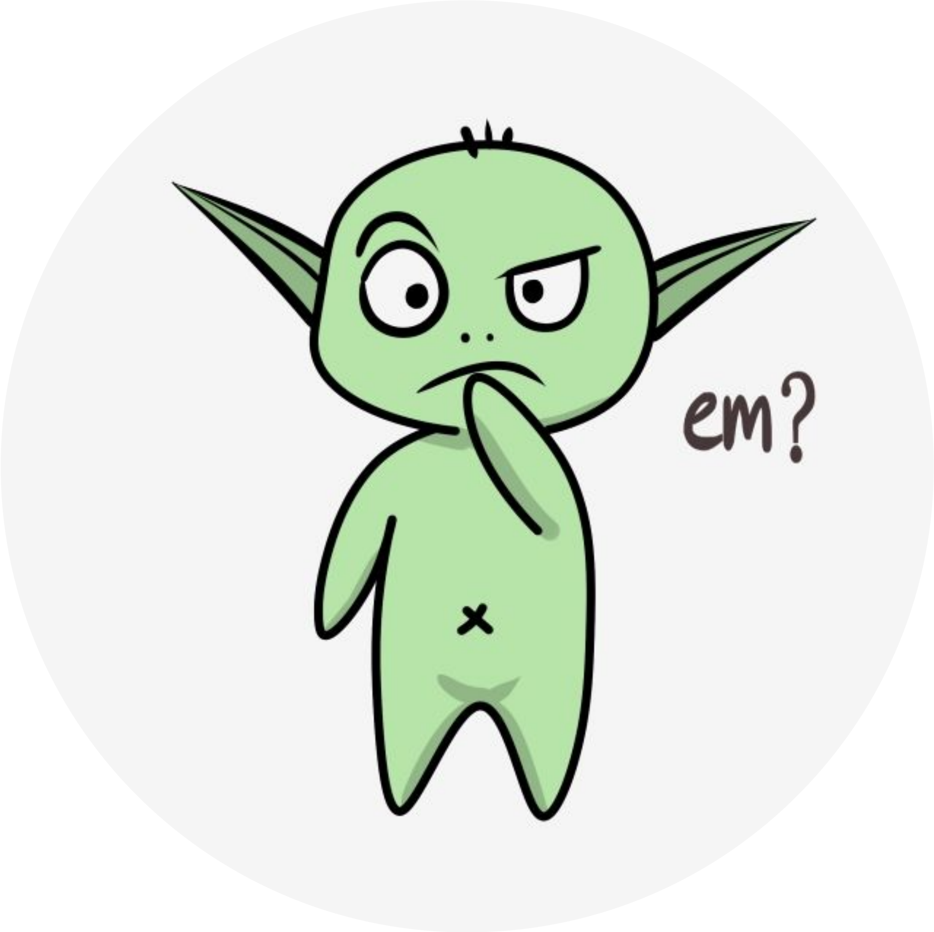
$$v_*(s) = \max_{a \in \mathcal{A}(s)} q_{\pi_*}(s, a)$$

Optimal Policies and Optimal Value Functions

- Because v_* is the value function for a policy, it must satisfy the self-consistency condition given by the Bellman equation for state values.

$$\begin{aligned} v_*(s) &= \max_{a \in \mathcal{A}(s)} q_{\pi_*}(s,a) \\ &= \max_a \mathbb{E}_{\pi_*}[G_t \mid S_t = s, A_t = a] \\ &= \max_a \mathbb{E}_{\pi_*}[R_{t+1} + \gamma G_{t+1} \mid S_t = s, A_t = a] \\ &= \max_a \mathbb{E}[R_{t+1} + \gamma v_*(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \max_a \sum_{s',r} p(s',r|s,a)\big[r + \gamma v_*(s')\big]. \end{aligned}$$

$$\begin{aligned} q_*(s,a) &= \mathbb{E}\Big[R_{t+1} + \gamma \max_{a'} q_*(S_{t+1},a') \Bigm| S_t = s, A_t = a\Big] \\ &= \sum_{s',r} p(s',r|s,a)\Big[r + \gamma \max_{a'} q_*(s',a')\Big]. \end{aligned}$$



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Reinforcement learning is very related to search algorithms

“Heuristic search methods can be viewed as expanding the right-hand side of the equation below several times, up to some depth, forming a “tree” of possibilities, and then using a heuristic evaluation function to approximate v_*, at the “leaf” nodes.”

$$v_*(s) = \max_a \sum_{s',r} p(s', r | s, a) \big[r + \gamma v_*(s') \big].$$

Yay! We solved sequential decision-making problems

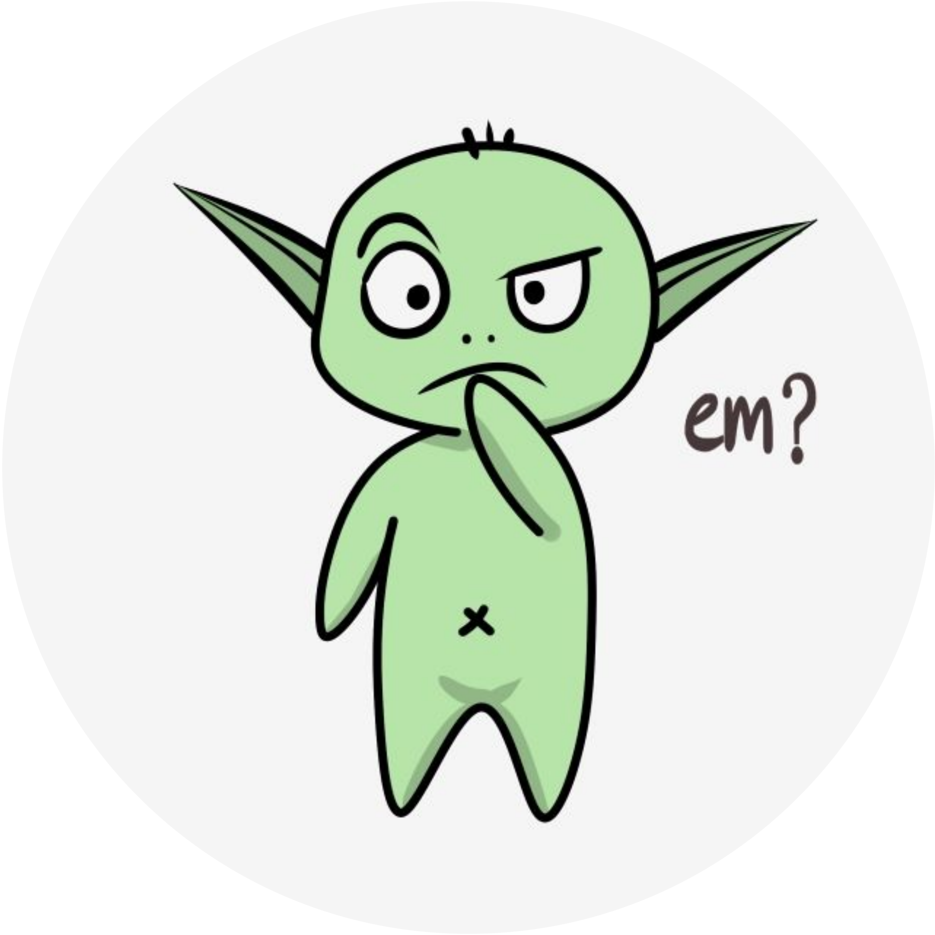
Except…

1.
2.
3.

Yay! We solved sequential decision-making problems

Except…

1. we need to know the dynamics of the environment
2. we have enough computational resources to solve the system of linear eq.
3. the Markov property



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Exercises from the Textbook

Exercise 3.11 If the current state is S_t, and actions are selected according to a stochastic policy π, then what is the expectation of R_{t+1} in terms of π and the four-argument function p (3.2)? □

Exercise 3.12 Give an equation for v_π in terms of q_π and π. □

Exercise 3.13 Give an equation for q_π in terms of v_π and the four-argument p. □

Next class

- What I plan to do:
  - Non-comprehensive overview of Dynamic Programming (Chapter 4 of the textbook)

- What I recommend YOU to do for next class:
  - Read (most of) Chapter 4 of the textbook.
  - Submit Practice Quiz for Fundamental of RL: Dynamic Programming (Week 4).

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