The 3B1b3W Endgame

Robert Lake
Jonathan Schaeffer
Norman Treloar

Department of Computing Science,
University of Alberta,
Edmonton, Alberta
CANADA T6G 2H1

The 3B1b3W endgame, 3 Black kings and one Black checker against 3 White kings, can be a difficult endgame. Many interesting problems have been composed for this endgame, often involving long and intricate solutions. Since this endgame does occur in tournament play, and is not just a realm for the problem composers, understanding some of the subtleties required to win the man-up or draw the man-down side has practical benefits.

The checkers program *Chinook* has enumerated and solved all possible checker positions with 6 or fewer pieces on the board - 2.7 billion positions. The program is currently working on solving all endgames with 7 pieces on the board, a further 34.8 billion positions. The 3B1b3W endgames are now complete. From this, we can extract some useful information for the checkers community. We hope that this article might attract the interest of some endgame specialists who would like to do a more comprehensive analysis of this endgame.

Tables 1 and 2 show the breakdown of the positions, based on the rank of the checker. Table 1 enumerates all positions with Black (the strong side) to move, while Table 2 covers all the White to move positions (weak side). The 3B1b(4)3W entry in Table 1, for example, refers to this endgame with the Black checker on the fourth rank. For each of the endgames, the total number of possible legal positions is given in column 2. Column 3 subtracts out all the positions where the side to move has an immediate capture - the remaining positions are the ones of interest. Of these positions, columns 4, 5 and 6 give the number of winning, losing and drawing positions, respectively, both as an absolute number and as a percentage of the total non-capture positions (column 3). For example, if you randomly set up this endgame, put the Black checker on the fourth rank, there is no immediate capture and it is Black to move, then there is a 91.74% chance the position is a win for Black, 0.07% chance that it is lost for Black and 8.19% chance that it is drawn.

Endgame	Positions (millions)	No Capture (millions)	Wins (millions,%)	Losses (millions,%)	Draws (millions,%)
3B1b(7)3W	58,902,480	33,151,340 100%	30,193,121 91.08%	37,869 0.11%	2,920,350 8.81%
3B1b(6)3W	58902480	30,202,582 100%	27,803,491 92.06%	11,480 0.03%	2,387,611 7.91%
3B1b(5)3W	58902480	30,054,286 100%	27,570,210 91.73%	17,291 0.06%	2,466,785 8.21%
3B1b(4)3W	58902480	29,975,252 100%	27,500,244 91.74%	21,196 0.07%	2,453,812 8.19%
3B1b(3)3W	58902480	29,985,968 100%	27,933,663 93.16%	14,709 0.05%	2,037,596 6.79%
3B1b(2)3W	58902480	29,199,398 100%	27,009,062 92.50%	20,839 0.07%	2,169,497 7.43%
3B1b(1)3W	58902480	28,388,086 100%	27,757,098 97.77%	1562 0.01%	629,426 2.22%

Table 1. 3B1b3W statistics, Black to move.

The program that computes these endgames does it in a systematic manner. It finds all the positions that it can prove lead to a win or loss in 1 move, then all positions that lead to wins or losses in 2, then 3, etc. When there are no more wins or losses to be found, the remaining positions are drawn. Of interest are the positions that appear to be the most difficult to win. Unfortunately, to speed up the computation of this endgame, our program takes a shortcut which prevents us from knowing with certainty how many moves it takes to win a given position. We do have, though, one measure of a position's difficulty: the number of moves to *conversion*. A position is considered converted when an irreversible change occurs. The two obvious irreversible changes are any capture or checker advance. Once either of these events occur, it is impossible to ever repeat a previously seen position. Our program can provide us with a lower bound on the number of moves to reach conversion; the actual number may be slightly greater. If the conversion number for a position is, for example, 68 moves (a move is a play by both White and Black), then the *minimum* number of moves required to safely advance a checker or force an exchange of men is 68 moves, but in fact may be a few more. This, of course, assumes that the opponent does not make a mistake and always plays the move to offer the most resistance.

Endgame	**Positions** (millions)	**No Capture** (millions)	**Wins** (millions, %)	**Losses** (millions,%)	**Draws** (millions,%)
3B1b(7)3W	58,902,480	25,358,739 100%	116,622 0.46%	22,879,214 90.22%	23,362,903 9.32%
3B1b(6)3W	58902480	25,995,862 100%	81,706 0.31%	23,324,832 89.73%	2,589,324 9.96%
3B1b(5)3W	58902480	26,418,055 100%	145,450 0.55%	23,085,352 88.09%	3,001,734 11.36%
3B1b(4)3W	58902480	26,418,055 100%	154,960 0.59%	23,085,352 87.38%	3,177,743 12.03%
3B1b(3)3W	58902480	25,995,862 100%	122,898 0.47%	23,365,766 89.88%	2,507,198 9.65%
3B1b(2)3W	58902480	25,358,739 100%	120,559 0.47%	23,209,873 91.53%	2,028,307 8.00%
3B1b(1)3W	58902480	32,201,320 100%	138,278 0.43%	30,965,608 96.16%	1,097,434 3.41%

Table 2. 3B1b3W statistics, White to move.

Diagram 1 shows one of the 4 positions that are the most difficult to win when Black has a checker on the seventh rank. Here Black requires at least 68 moves for conversion. In other words, assuming White defends properly, Black cannot exchange down into a winning 5-piece endgame, or force the crowning of the checker on 28 in less than 68 moves! This position forms a nice problem. Can you find the winning line?

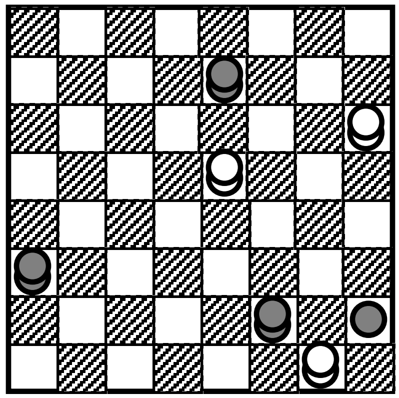

Diagram 1: 3B1b(7)3W conversion in 68 or more moves.

For brevity, we only include the first few moves of the solution: 1. 27-24* (* is used to indicate that this is the only winning move. 27-31 is a draw) 12-16 (all other White moves lose quickly) 2. 24-20* 16-19 (16-12 does not appear to offer much resistance. In response, any of 21-25, 21-17, 7-2 and 7-3 preserve the win) 3. 21-25* 19-23 (or 15-18 4. 25-30*) 4. 25-22*. Now White has 3 moves: for 23-27 and 23-19, the response is 7-2*; for 15-19, Black can play either 7-10 or 7-2 to win. After this, there are still over 60 moves to go! I doubt there are many players who can successfully find the correct shortest winning line. Conversely, if faced with playing the weak side of this position in a tournament game, few players could find the correct defensive moves to prolong the game, giving the opponent the opportunity of crossing the fine line between a win and a draw. In fact, we doubt many people would recognize that in Diagram 1 the move 27-31 is a draw.

If a rule is enforced that says a player has 40 moves to demonstrate progress towards a win, otherwise the game is a draw, then Diagram 1 poses a dilemma. If Black plays perfectly and White defends accurately, then after 40 moves Black is still 28 moves away from crowning the checker or forcing a favorable exchange of men, a demonstration of "progress". Without knowing this endgame thoroughly, it is difficult to determine if progress has been made. *Chinook*, for example, might reach this endgame in a tournament and play it perfectly. After 40 moves without conversion occurring, does the opponent have the right to declare a draw since it may not be obvious that progress is being made? These problems have arisen in the chess world, with FIDE (the governing body of international chess) resolving the situation by extending their draw rule from 50 to 100 to 75 and back to 50 moves. Obviously these cases occur infrequently in practice, but it only takes one unfortunate incident in an important game (such as a World Championship) to call this rule into question.

Diagrams 2 through 7 show the hardest positions for Black to win with checkers on the sixth through first ranks. These problems have been chosen based on the length of the solution, not based on the aesthetics of problem composition. For each Diagram, a sample line of play is given (defensive moves given may not offer the most resistance). Note that the number of moves given is for conversion, the minimum number of moves to *safely advance the checker one square only*. After that, there may be many moves yet required to crown the checker. For example, in Diagram 2, a minimum of 36 moves are required to safely advance the checker to the seventh rank. After that, depending on the position reached after the advance, it may take an additional 68 moves to crown the checker! Having done that, the worst case endgame of 4 Black kings against 3 White kings requires an additional 26 moves to force a winning exchange.

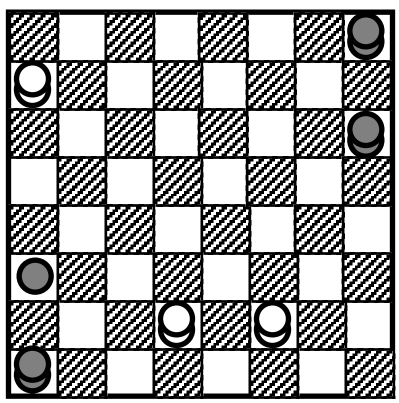

Diagram 2: 3B1b(6)3W conversion in 36 or more moves.

1. 12-16* 27-23 2. 29-25* (4-8 is a mistake: 23-18* draws) 5-9

3. 4-8* 9-14 4. 8-12*, etc.

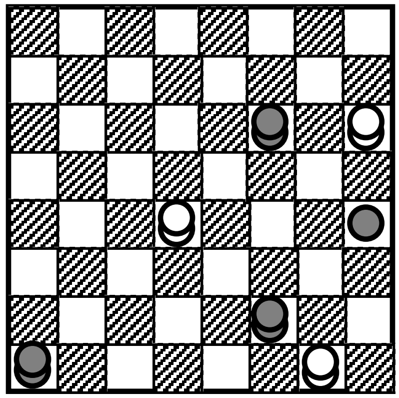

Diagram 3: 3B1b(5)3W conversion in 42 or more moves.

1. 27-24* 18-22 2. 11-15 or 24-19, etc.

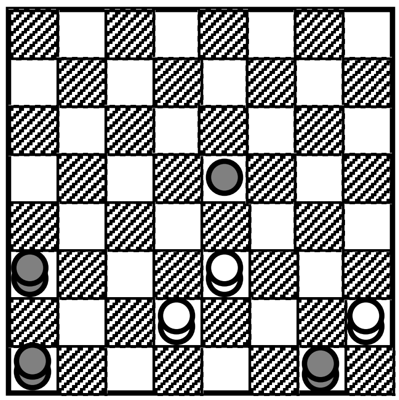

Diagram 4: 3B1b(4)3W conversion in 41 or more moves.

1. 21-17* 26-30 17-14*, etc.

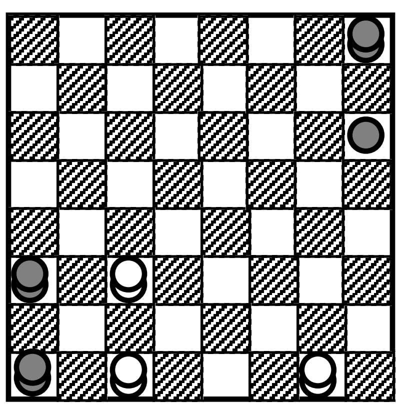

Diagram 5: 3B1b(3)3W conversion in 41 or more moves.

1. 4-8* 32-27 2. 8-11* 27-24 3. 11-15 (or 11-07) 24-20 4. 15-10*, etc.

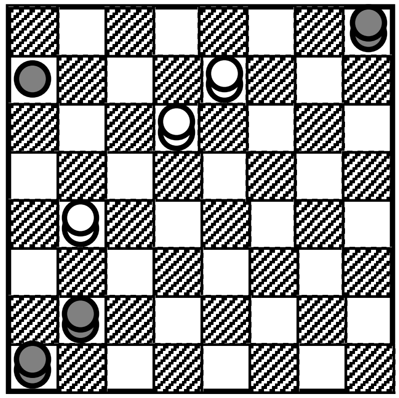

Diagram 6: 3B1b(2)3W conversion in 26 or more moves.

1. 25-21* 17-14 2. 4-8* 10-15 3. 29-25*, etc.

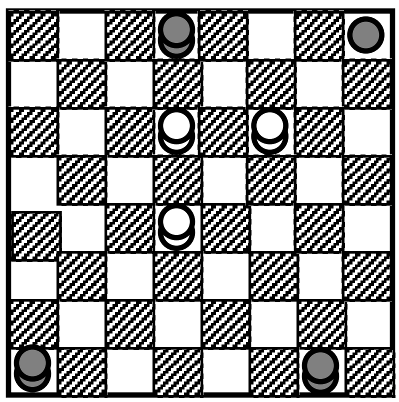

Diagram 7: 3B1b(1)3W conversion in 26 or more moves.

1. 29-25* 18-23 2. 25-22 (or 25-21), etc.

We have extracted the 3B1b3W positions from Boland's *Masterpieces in the Game of Checkers* and *Famous Positions in the Game of Checkers* and fed them to *Chinook*. In all cases, Boland had correctly classified each position as a win, loss or draw. We did not verify that the line of play given was correct.

Obviously these endgames offer more surprises than this short article can convey. In the future, we hope to publish additional articles on other 6 and 7 piece endgames. We invite readers to submit to us interesting positions with 7 or fewer men on the board.

Acknowledgements

Thanks to Edward Scheidt, who provided the encouragement for us to start making *Chinook*'s results more available.