

# TWIST AND TURN

## THE STORY OF HEX

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computing.science   university of alberta

2018 april

## THANKS

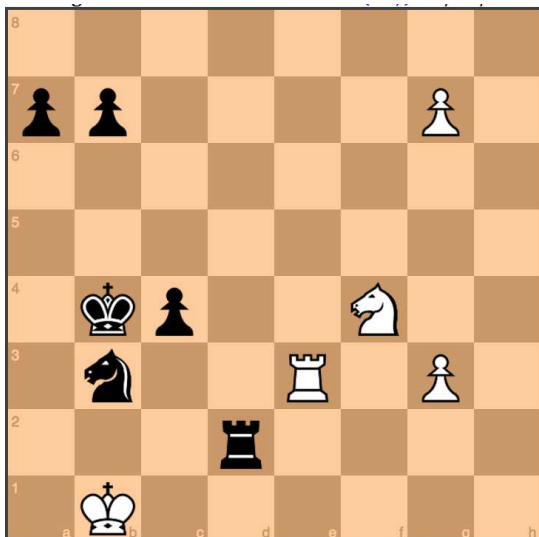
- **Computer Hex Group** Michael Johanson, Yngvi Björnsson, Morgan Kan, Nathan Po, Jack van Rijswijk, Broderick Arneson, Philip Henderson, Jakub Pawlewicz, Aja Huang (AlphaGo), Kenny Young, Noah Weninger, Chao Gao, Martin Müller (Fuego) Bjarne Toft
- NSERC

## BOOK

- twist and turn: the story of Hex (2018)
- Martin Gardner archive, Stanford University
- Piet Hein archive, collection of Hugo Hein
- Jens Lindhard archive, Aarhus University

WHITE'S MOVE ?

(CHESSGAMES.COM)



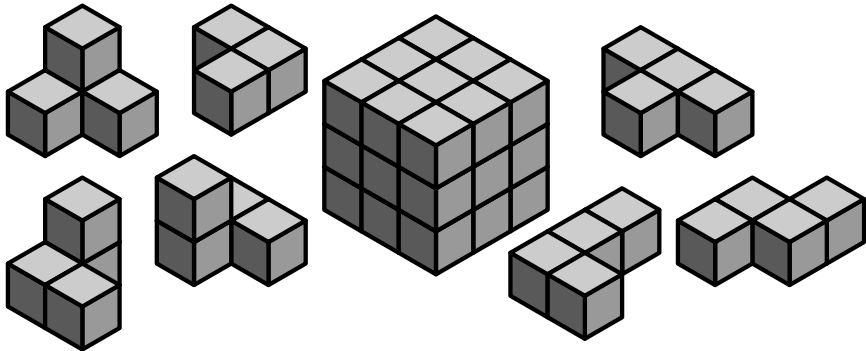
# JENS LINDHARD



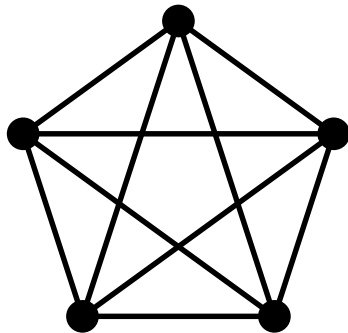
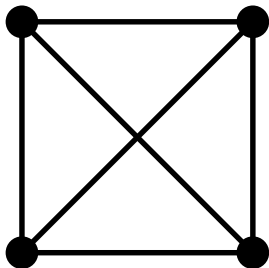
# PIET HEIN



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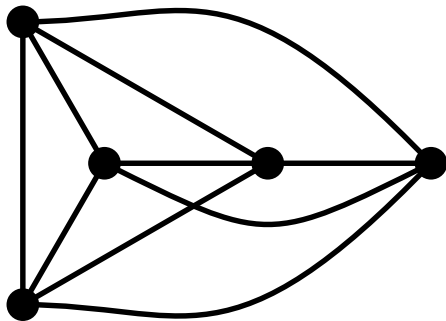
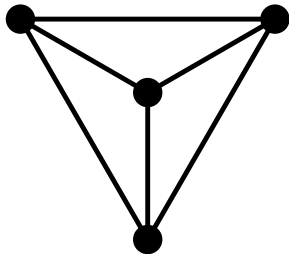


# PLANAR ?





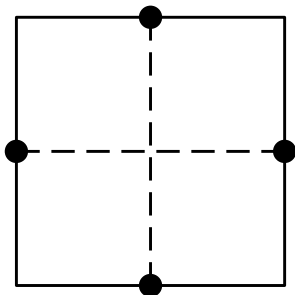
# PLANAR ?



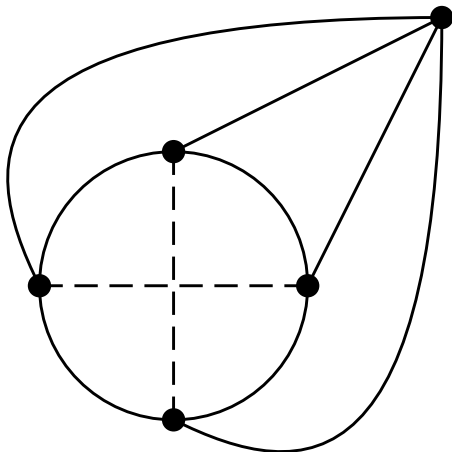
## K5 NOT PLANAR

- graph  $n$  points  $L$  lines
- if planar then  $L \leq 3 \times n - 6$  Euler
- say  $n = 5$
- if planar then  $L \leq 3 \times 5 - 6 = 9$
- graph with 5 points, 10 lines ? not planar

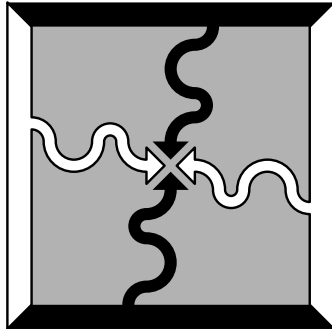
# K5 NOT PLANAR



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# K5 NOT PLANAR



# PIET HEIN

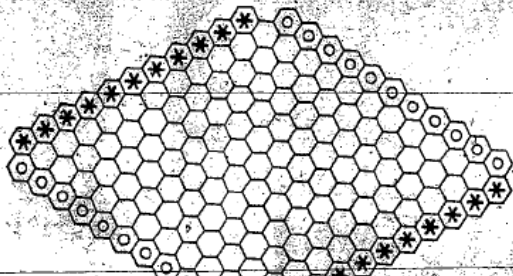
LIXEN

26. December 1942

## Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken“ udskriver i Dag en Præmieopgave, der vil volde Hovedbrud for Begyndere



ten kan afbryde Forbindelsen ved at besætte det mellemste bode Felt dens Anvendelighed beror paa Blykorpens Placering i den videre Omegn. I det hele taget viser det sig snart nødvendigt at læse en større Del af Spilletrøstet med Betragtning.

En anden Erfaring, som kommer senere, men som man kan lette Spillets Begyndelse ved at røbe, er, at det betaler sig at begynde i hvert Fald nogenlunde paa Midten. En rimelig, men paa ingen Maade nødvendig Aabning af Spillet er denne:

Paa Spillebrættet i Midten er Hvid begyndt i Midterfeltet. Sort har Sort sat i Kontaktfeltet til det ned imod Midten af Hvids Front og derved gjort to nyttige Felter, som staa i Vinkelstilling til Midterfeltet, usikre. Hvid har saa vaegt et Felt i Kontakt med sin første. Og nu svarer Sort med at besætte et Vinkelfelt, som vilde være meget nyttigt for Hvid. Hvor skal nu Hvid sætte? Der er forskellige gode Muligheder.

Saaen er dette Spil nu begyndt. Nu kan enhver fortsætte. Det er altsaa Hvids Tur! Man skal ikke være udspekuleret fra Begyndelsen. Der er ingen bedre



## RULES

- two players
- alternate turn
- turn    put a stone in any empty cell
- win    if you join your two sides

# CHESS

- complicated rules
- positions can recur  
stalemate, 3-repetition, 50-move
- draws (boring)
- challenging



# HEX

- simple rules
- positions never recur
- no draws
- challenging

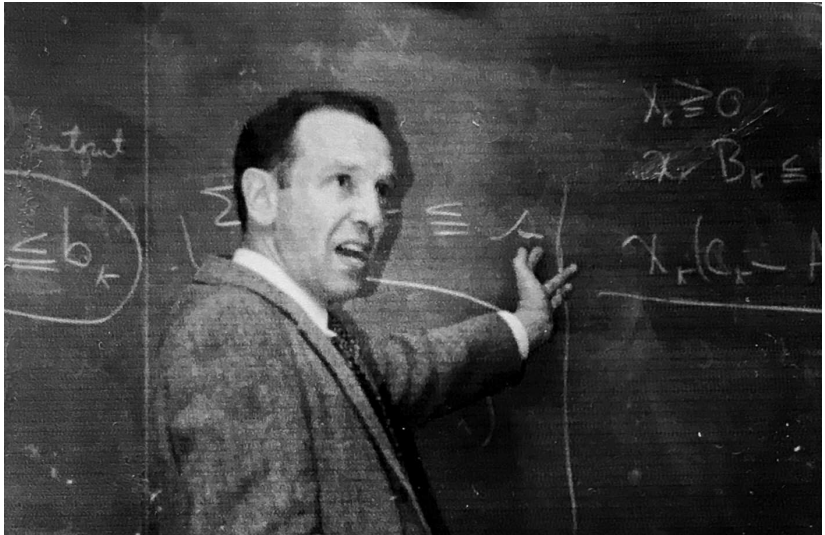
# CHESS

- can you prove 1st player wins?
- can you prove 1st player can (at least) draw?

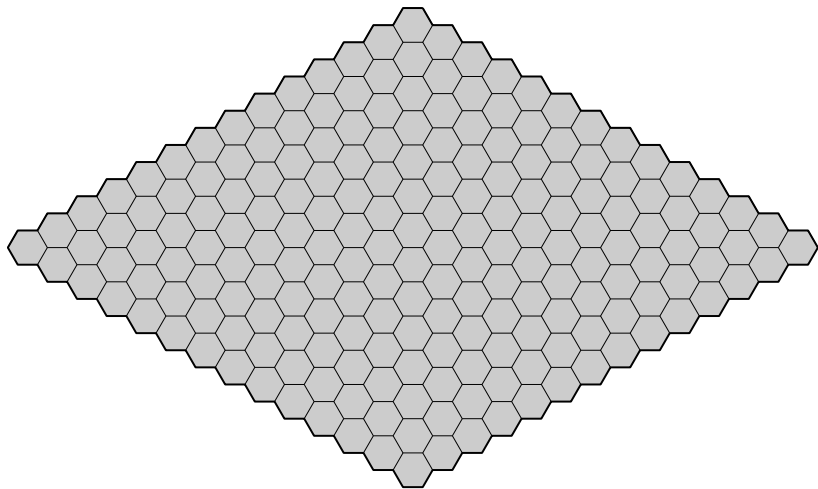
# 1949 NASH-GALE



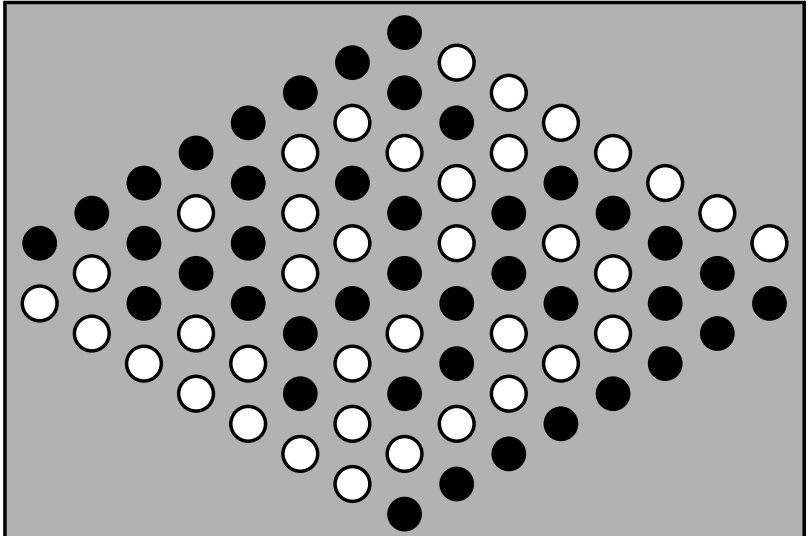
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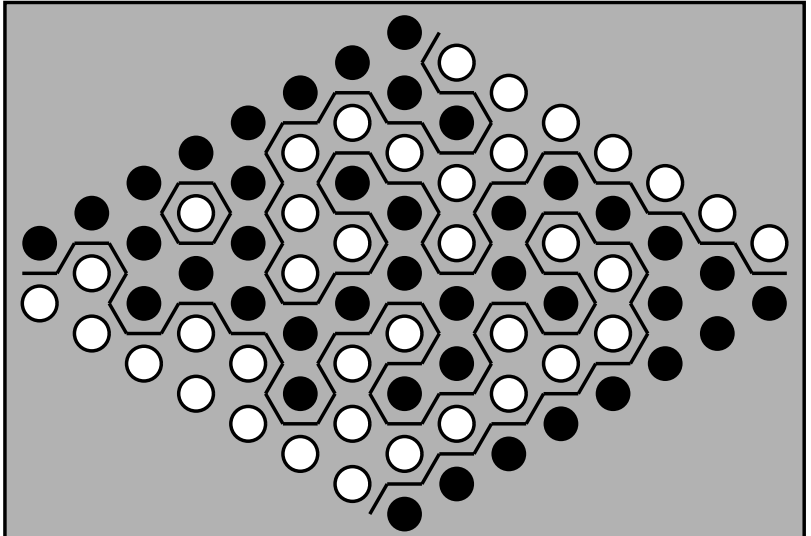
# 1949 NASH-GALE



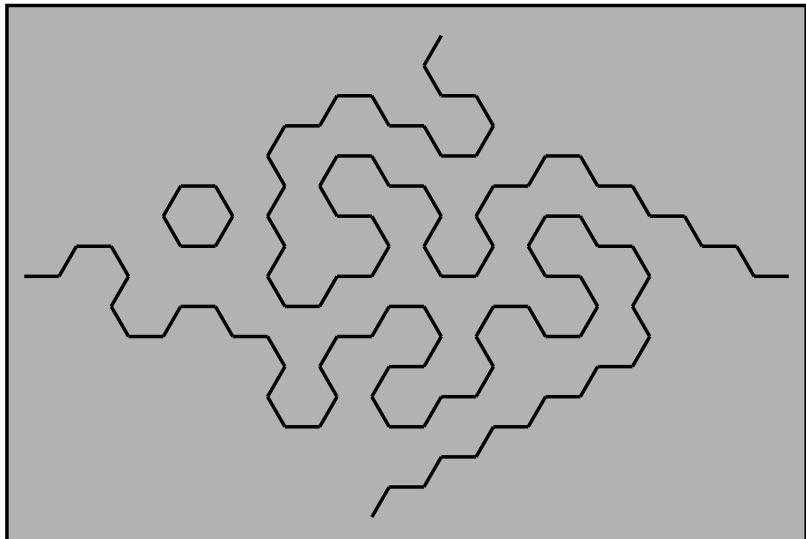
# NO DRAWS



# NO DRAWS



# NO DRAWS





## FIRST PLAYER WINS

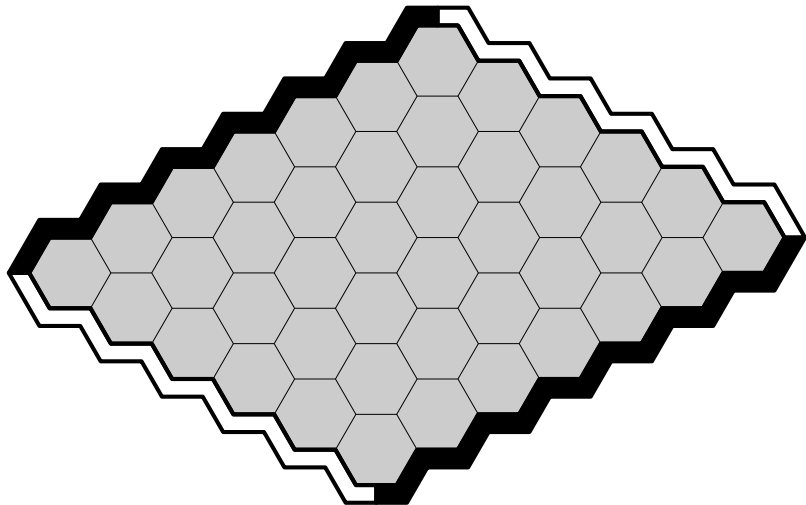
- ① ~~field~~ When the board is ~~field~~ filled one or the other of the players will have connected but not both.
- ② ~~One~~ Either the first player or the second will have a winning strategy.
- ③ Suppose the second player could force a win.
- ④ Consider a defensible strategy

## FIRST PLAYER WINS

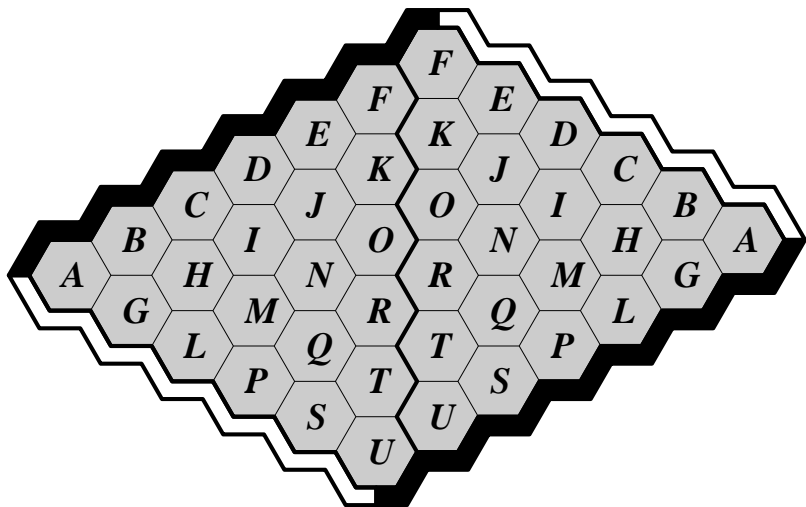
the board is always an asset,  
never a handicap in connecting,  
at the end of the game  
first player will be <sup>(assumed)</sup> better off  
using the adapted second player  
strategy than he would have  
been if simply playing as  
second player. So he will  
win.

⑥ Since this contradicts the  
hypothesis ③ that second  
player can win.

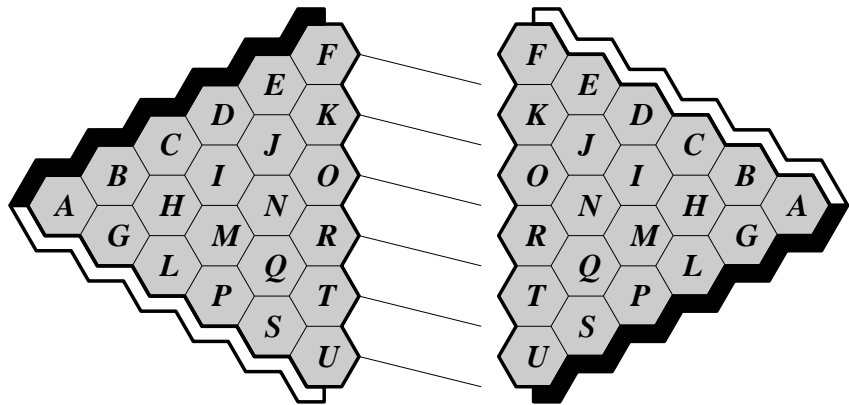
## SHORTER DISTANCE WINS



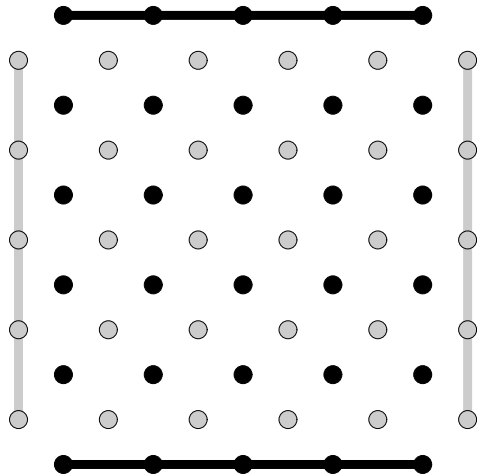
## SHORTER DISTANCE WINS



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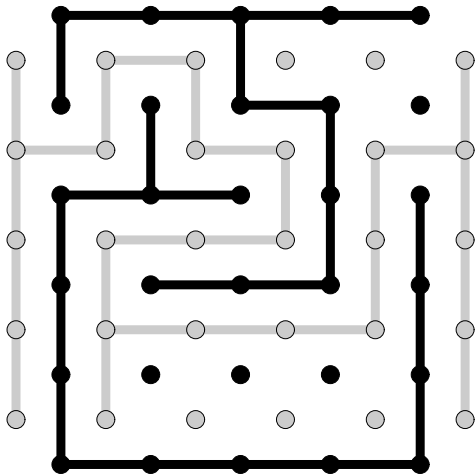
# GALE'S GAME BRIDG-IT



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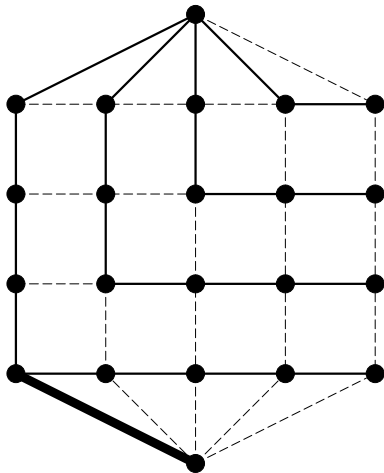
- on your turn  
put a stick  
between any two  
of your adjacent dots

# GALE'S GAME BRIDG-IT





# BRIDG-IT CRACKS



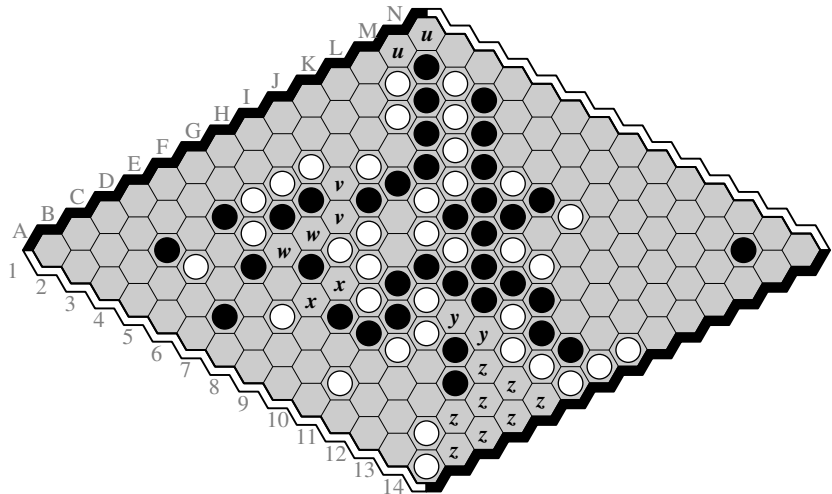
## ... BUT HEX IS HARD

Reisch    Hex is P-space complete

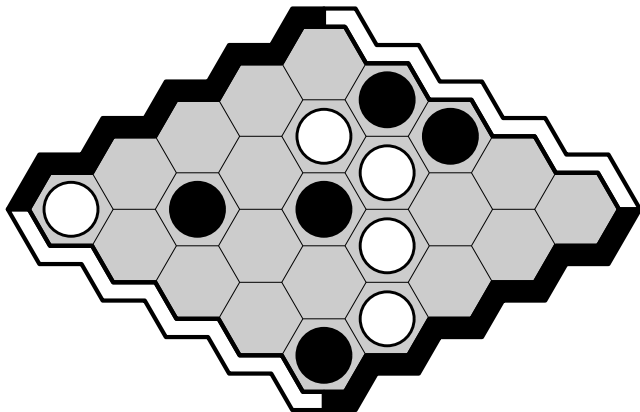
# CLAUDE BERGE



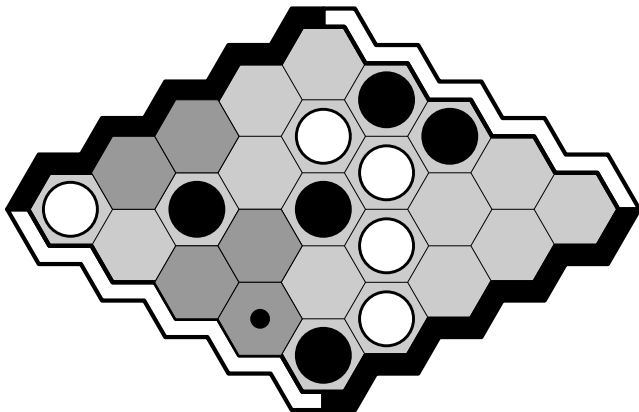
# VIRTUAL CONNECTION



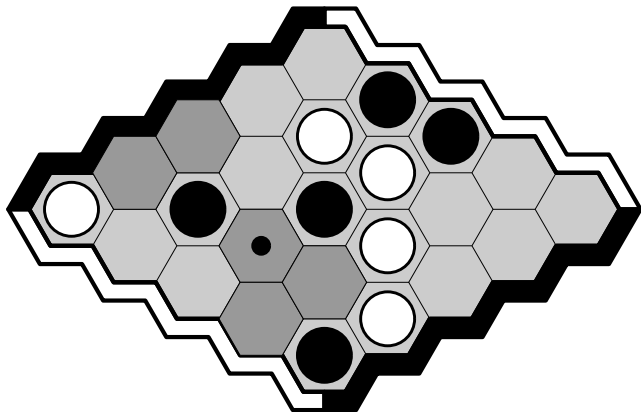
# MUSTPLAY



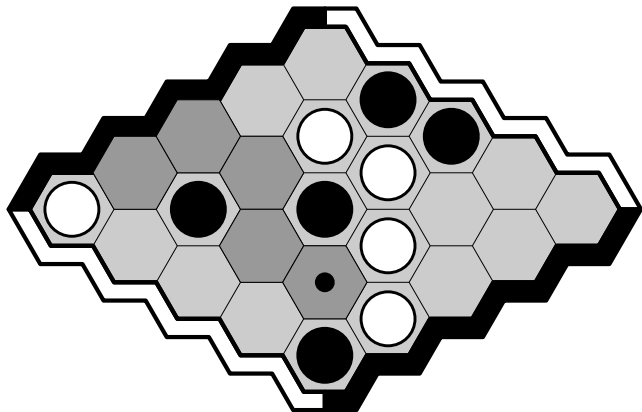
# MUSTPLAY



# MUSTPLAY

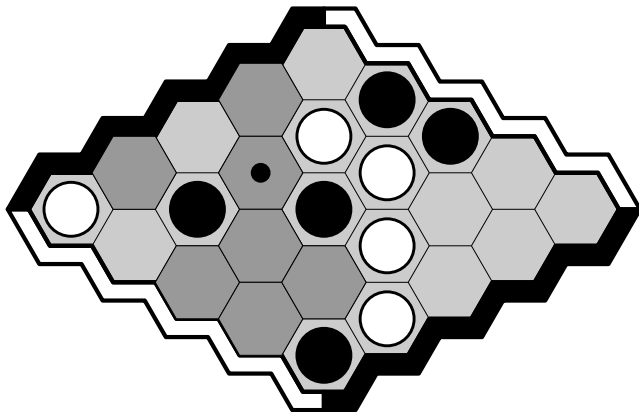


# MUSTPLAY

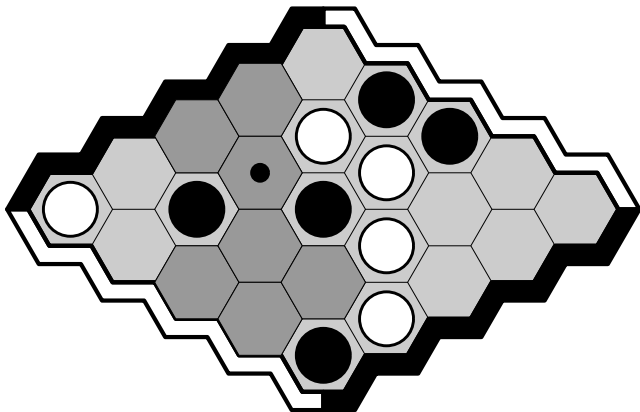




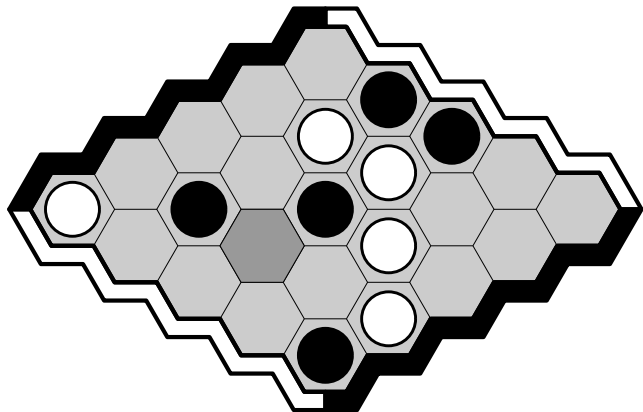
# MUSTPLAY



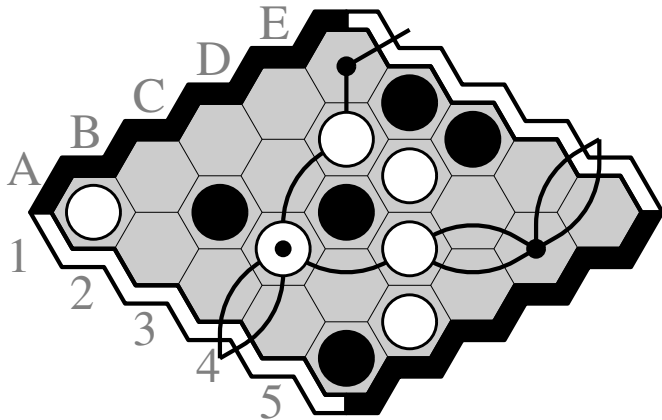
# MUSTPLAY



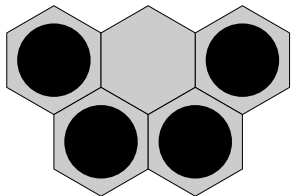
# MUSTPLAY



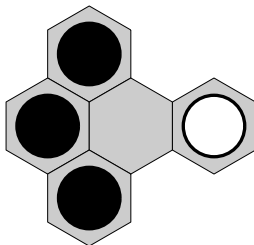
## WINNING STRATEGY



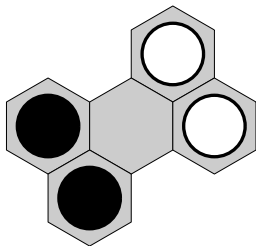
# INFERIOR CELLS: DEAD



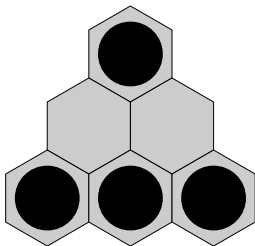
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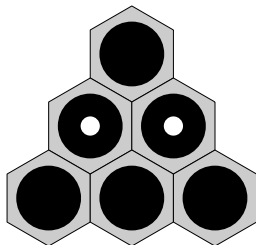


# INFERIOR CELLS: CAPTURED

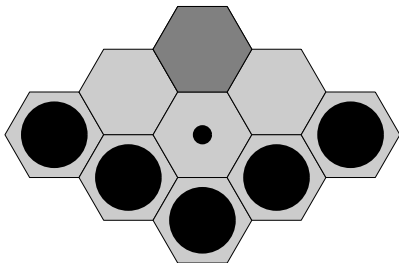




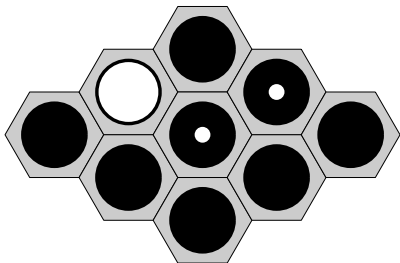
# INFERIOR CELLS: CAPTURED



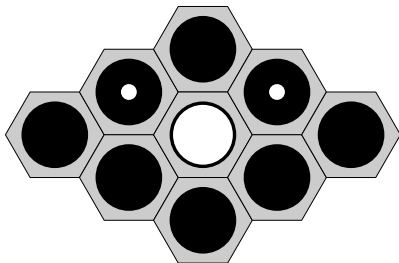
## INFERIOR CELLS: PERMANENT



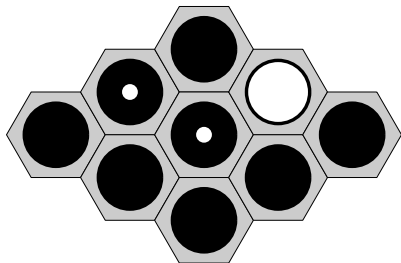
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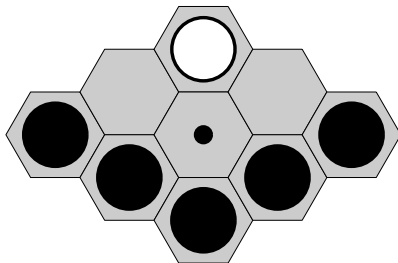
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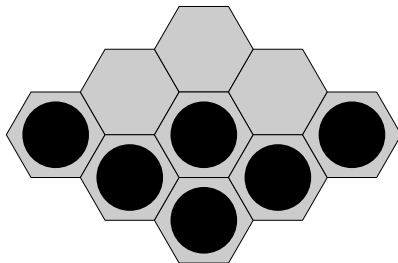
## INFERIOR CELLS: PERMANENT



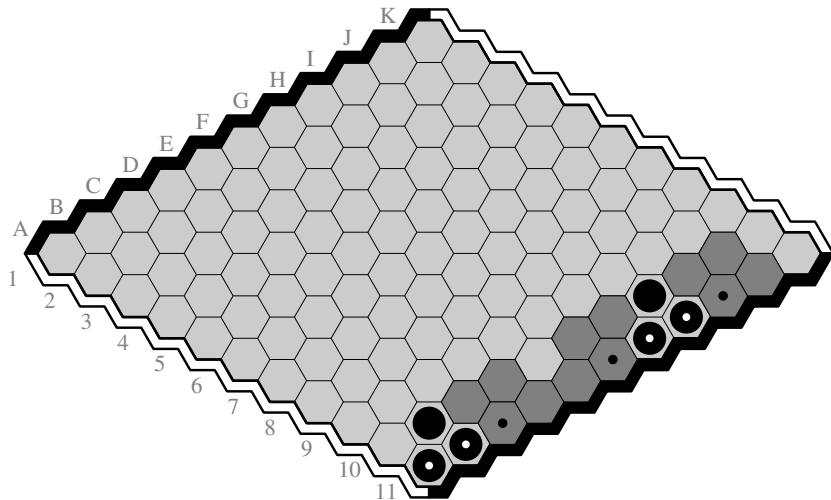
## INFERIOR CELLS: PERMANENT



# INFERIOR CELLS: PERMANENT



# INFERIOR CELLS HENDERSON HANDICAP





# H BJÖRNSSON ARNESON HENDERSON KAN 2006



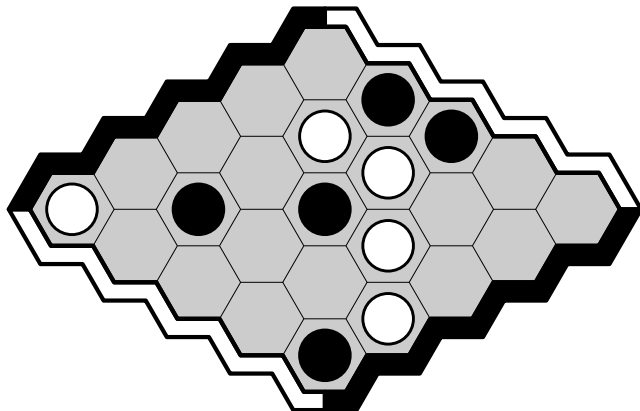
# H BJÖRNSSON ARNESON HENDERSON KAN 2006



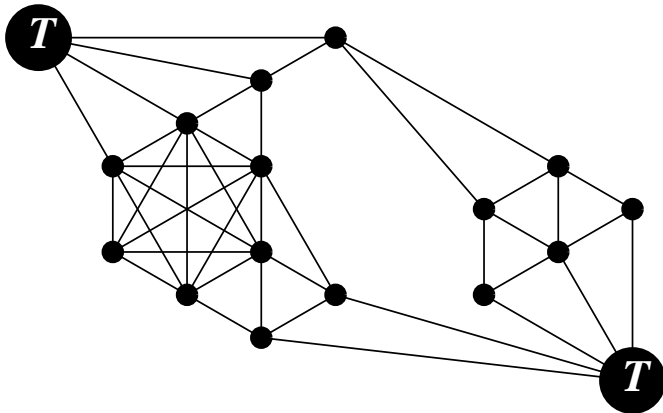
# H BJÖRNSSON ARNESON HENDERSON KAN 2006



# SHANNON RESISTANCE NETWORK



# SHANNON RESISTANCE NETWORK



# 2008 BEIJING SIX-MoHEX GAME 1

- Six
- Wolve
- MoHex
- Six-Wolve game 1 move 40

## THE QUEST FOR STRATEGIES

Gardner 1957 Scientific American

*Concerning the game of Hex, which may be played  
on the tiles of the bathroom floor*

- find 1pw ?      find win/loss for each 1st move ?
- 4x4 easy                      5x5 harder                      6x6 ?

# WINNING HEX OPENINGS

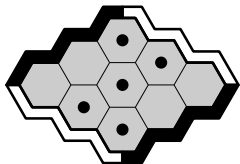




# WINNING HEX OPENINGS



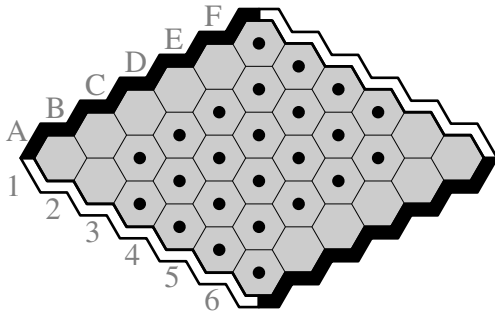
# WINNING HEX OPENINGS





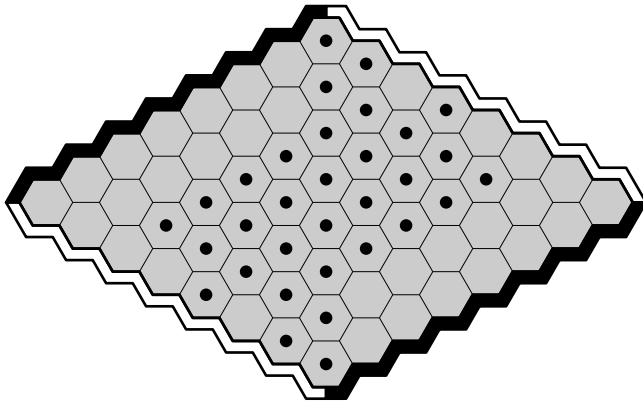


# WINNING HEX OPENINGS 1995 ENDERTON





# WINNING HEX OPENINGS 2009 HAH

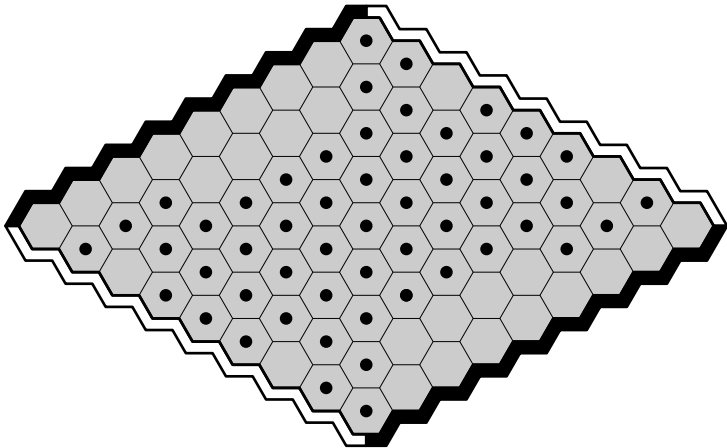


# 2013 PAWLEWICZ H HUANG

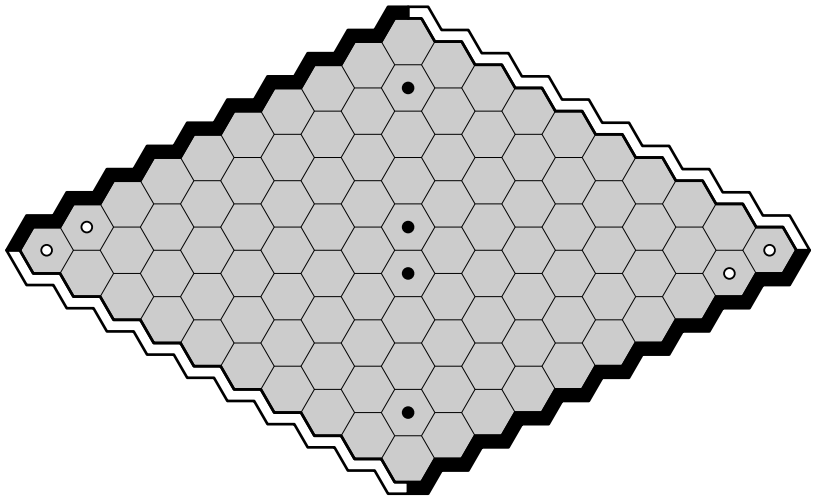




# WINNING HEX OPENINGS 2013 AHHP



# WINNING HEX OPENINGS 2014 PH



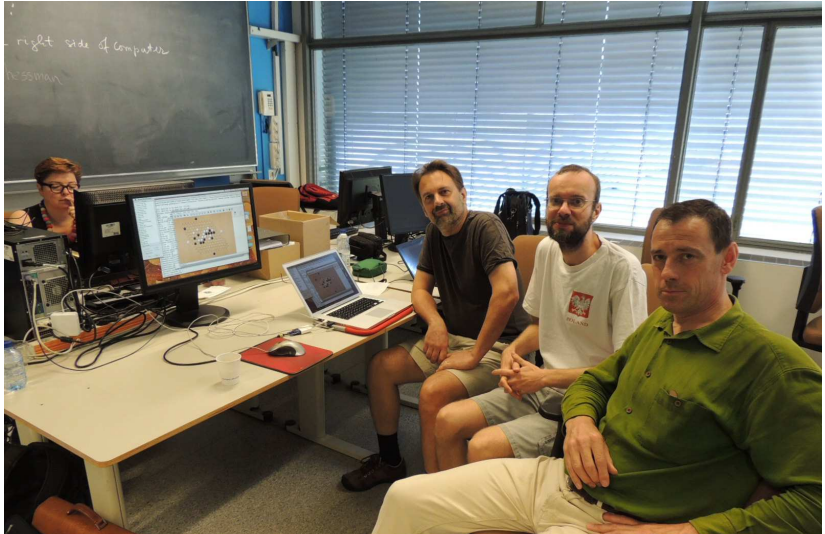
# ICGA LEIDEN 2015



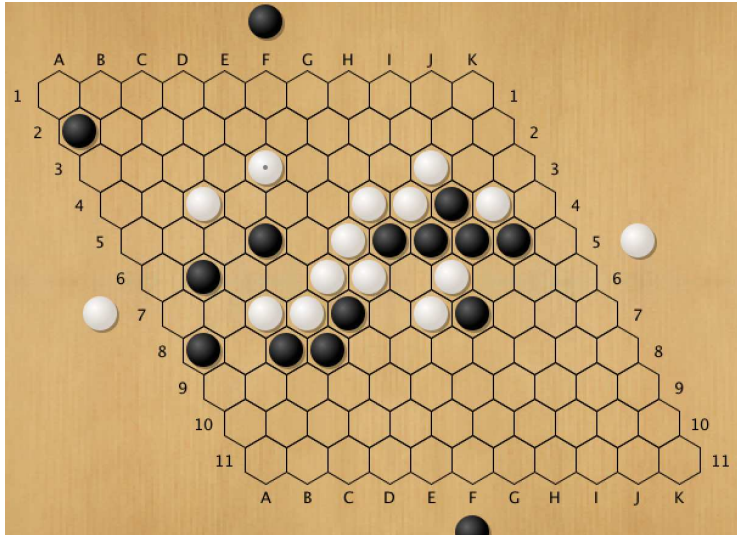
# ICGA LEIDEN 2015



# ICGA LEIDEN 2015



# ICGA LEIDEN 2015



# THANK YOU

