SOME QUESTIONS ON HEX

U Montana talk

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computing UAlberta

2014 april 14

THANK YOU

- invitation Prof Mark Kayll UMontana
- solving 10×10 Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
- Natural Sciences and Engineering Research Council of Canada

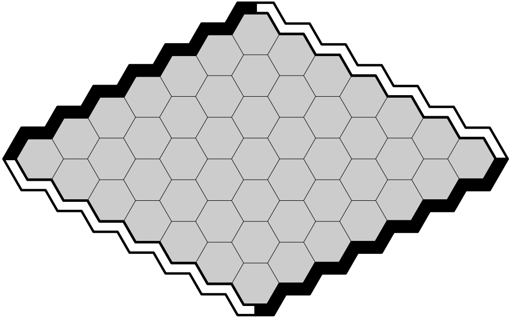
SOME QUESTIONS

- is hex fair ?
- when will computers solve 11×11 hex ?
- 11×11 hex, 10-1 odds, 1st 2 stones: wager ?
- write hex player in 8 hours: algorithms ?

1942 HEX

RULES

- 2 players, alternate moves
- win: connect your two sides



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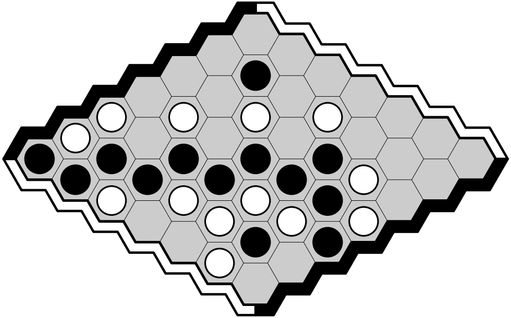
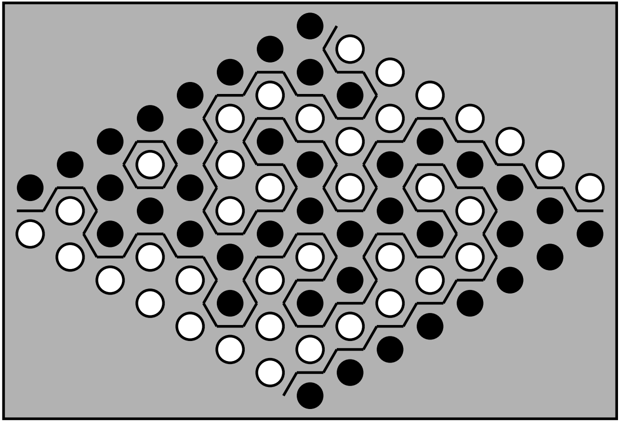


n x n Hex: 1st-player win

PROOF

- lemma: extra X-cell ok for player X
- lemma: no draws
- suppose P2 has win strategy S2
- then P1 can move anywhere, forget move, and follow S2
- thus P1 has win strategy, contradiction □

NO-DRAW



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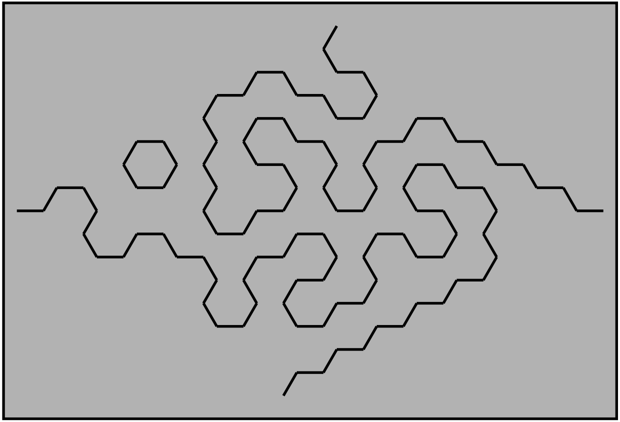
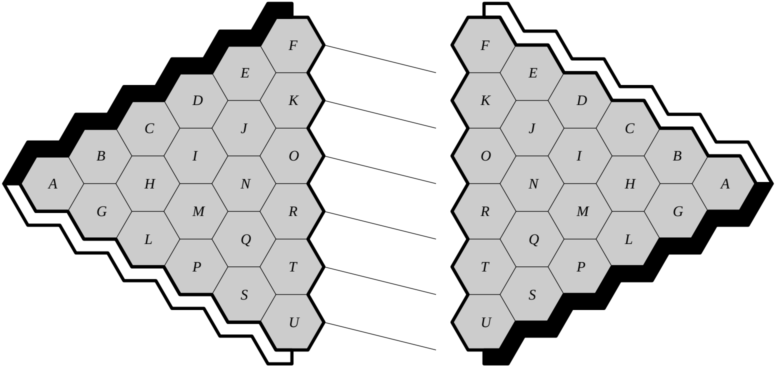
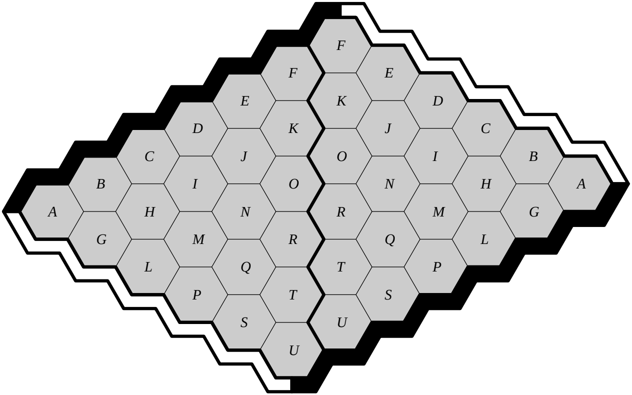


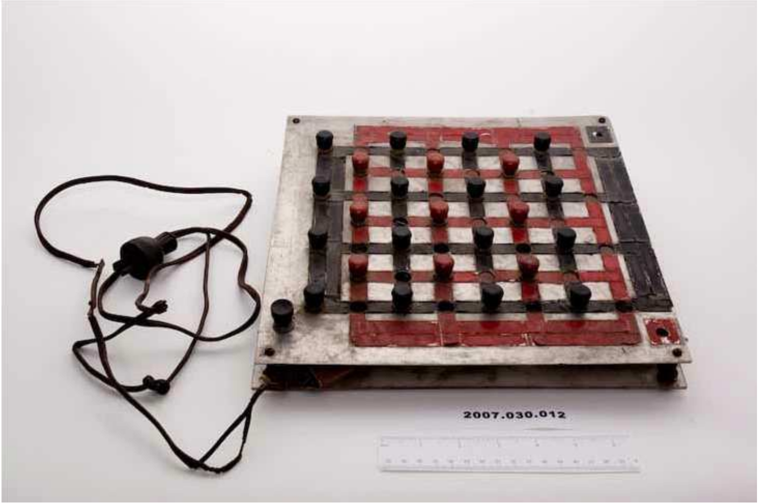
n x n+1 Hex: longer-side win



N X N+1 HEX: LONGER-SIDE WIN



1951 Shannon machine

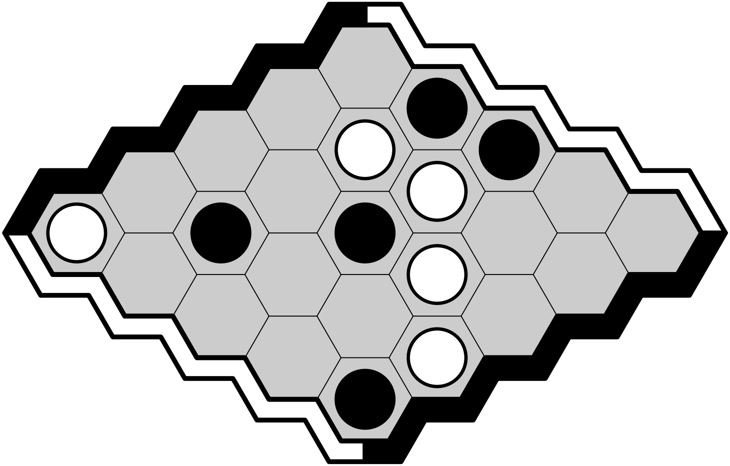


1951 Shannon machine

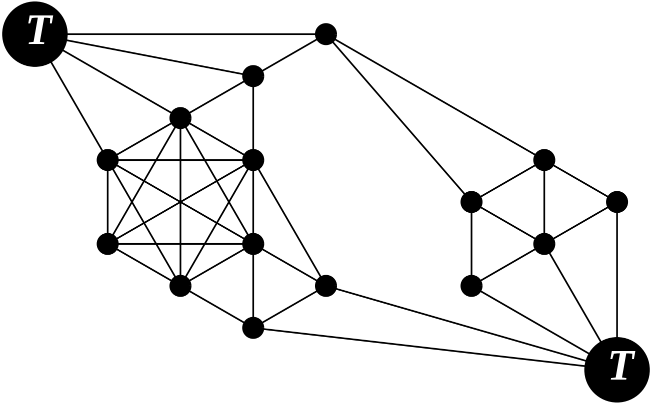
- play on any graph
- two marked vertices
- black move: ‘short’ any vertex (make nbrs clique)
- white move: ‘cut’ any vertex (delete)
- black wins iff two marked vertices are shorted (connected)

- generalizes Hex

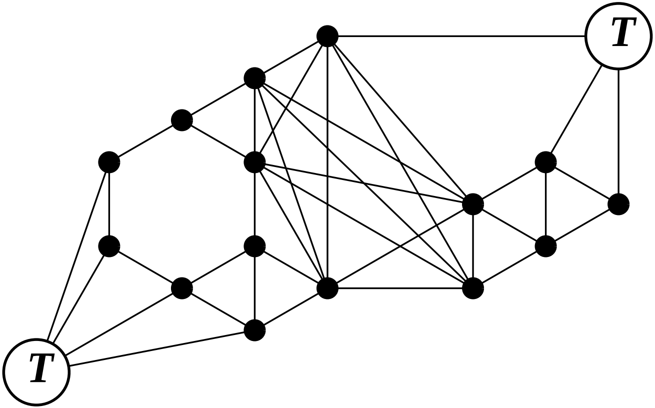
1951 Shannon machine



1951 Shannon machine



1951 Shannon machine



PROVABLY HARD

- 1975 Even & Tarjan — Shannon v-switching: PS-c
- 1981 Stefan Reisch — Hex: PS-c
- 2000 Clay Math Inst — P vs NP: $1 000 000

Humans

Solved openings

2001 Yang	17/49 7x7
2002 Yang	8x8
2003 Yang	9x9
2004 Noshita	7x7
2005 Noshita	8x8
2006 Mishima	8x8

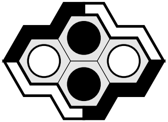
COMPUTERS

SOLVED OPENINGS

1995	Enderton		6x6
2000	van Rijswijck		6x6
2003	H Bjö Joh Kan Po vRij	5d	7x7
2007	Rasmussen et al.		7x7
2009	Arneson H Henderson	4d	8x8
2010	A H H	25d	some 9x9
2012	Pawlewicz H	110d x 24 thread	9x9
2013	Pawlewicz H	63d x 24 thread	centre 10x10

computers

COMPUTERS



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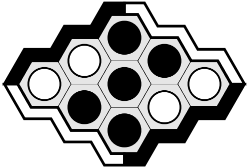

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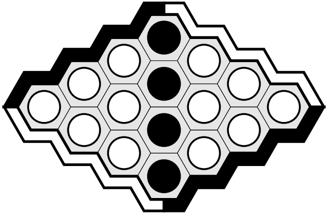

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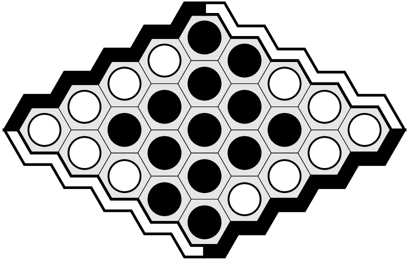

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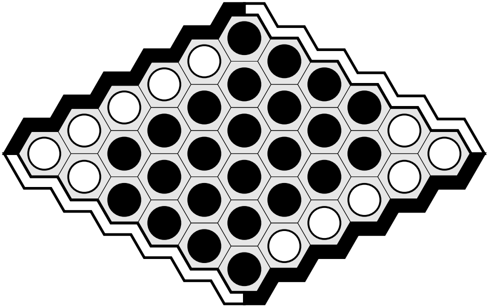

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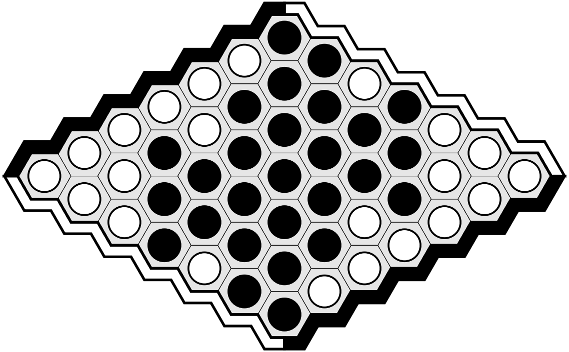

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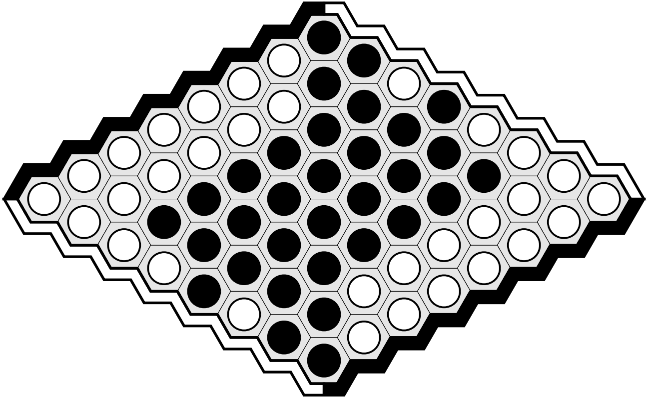

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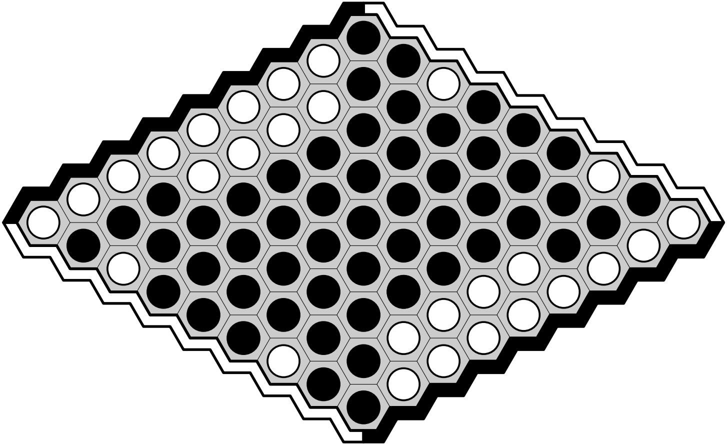
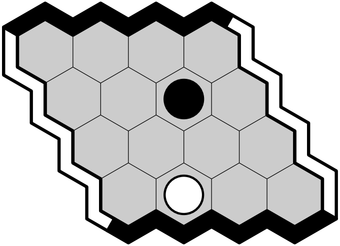


KNOWLEDGE

- virtual connections: combining rules, mustplay
- inferior cells: dead, captured, etc.

A VIRTUAL CONNECTION



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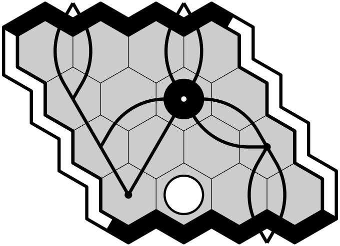
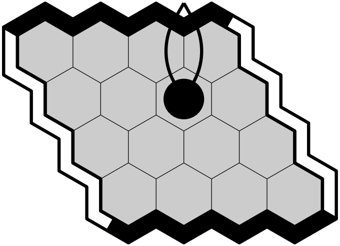


COMBINING RULE: AND (FULL)



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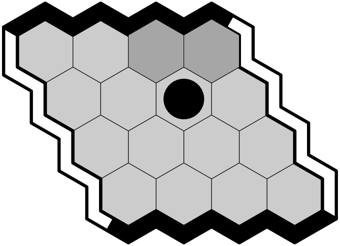

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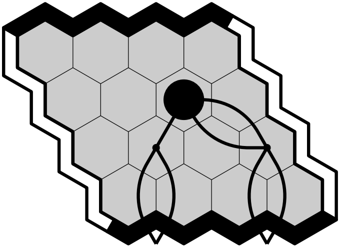

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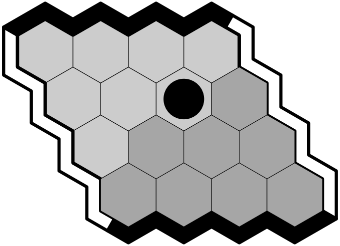

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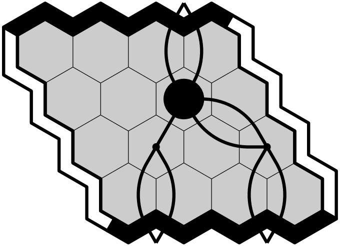

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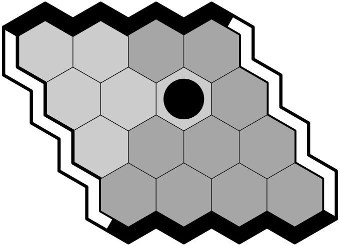
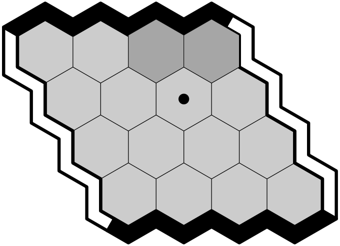


COMBINING RULE: AND (SEMI)



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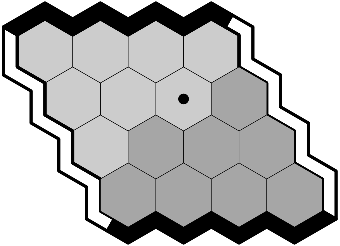

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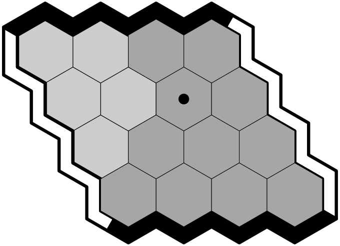

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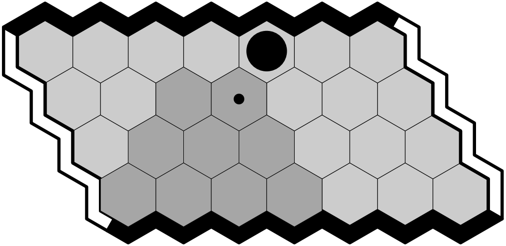

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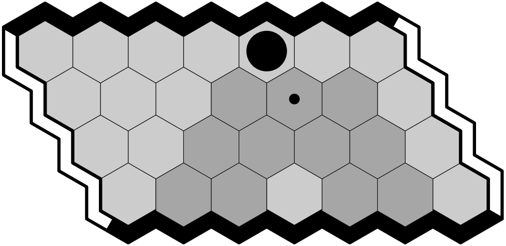

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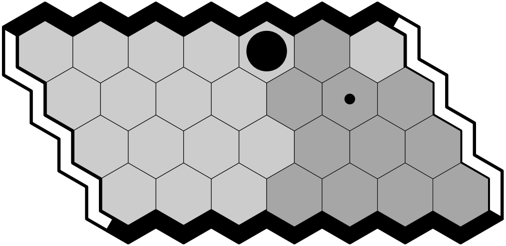

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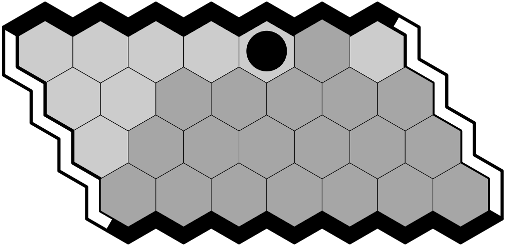
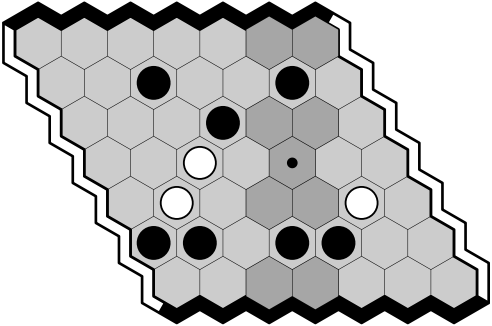


WHERE MUST WHITE PLAY?



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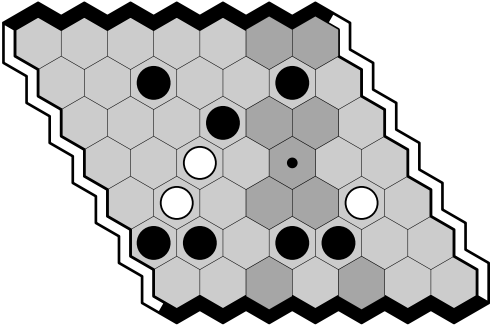

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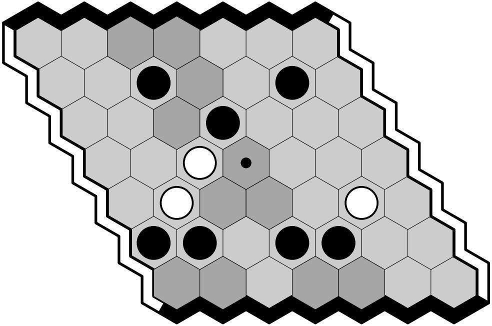

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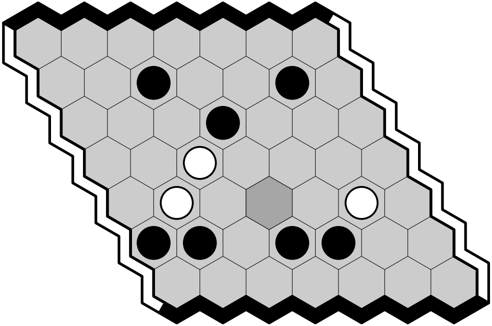
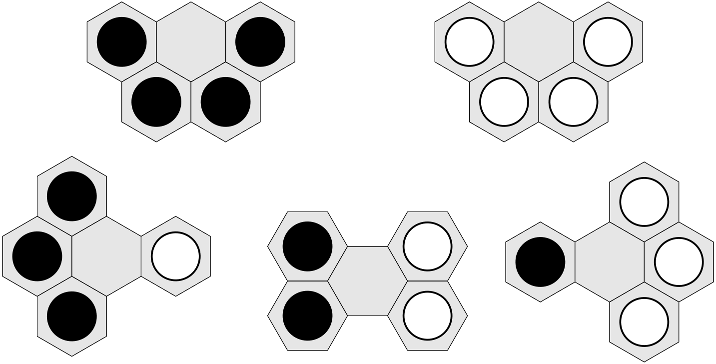
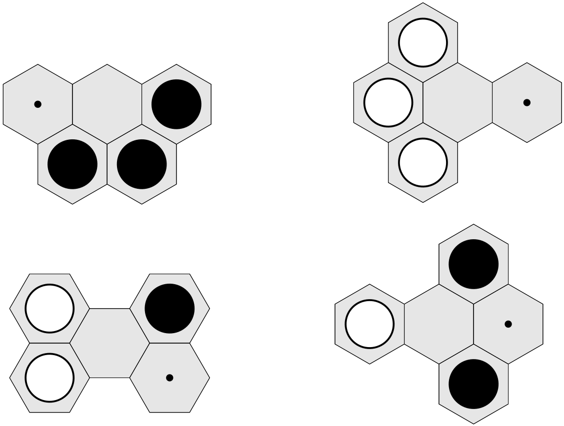


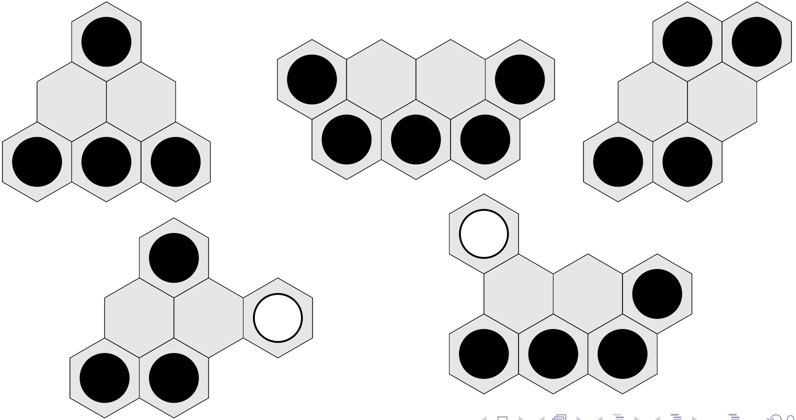
DEAD



BLACK-DOMINATED (DOT SUPERIOR)



BLACK-CAPTURED



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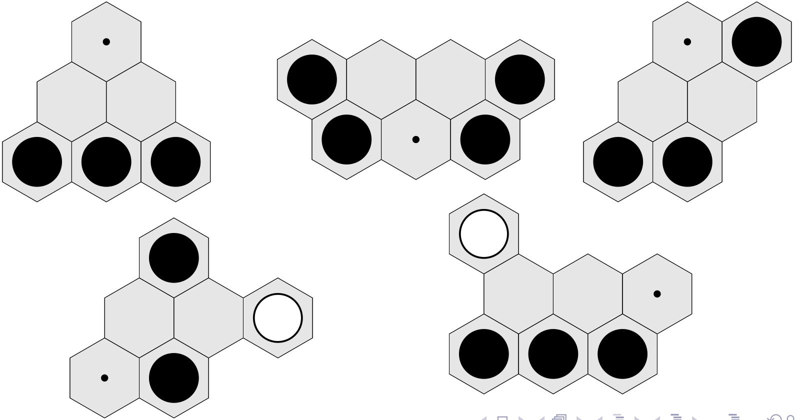
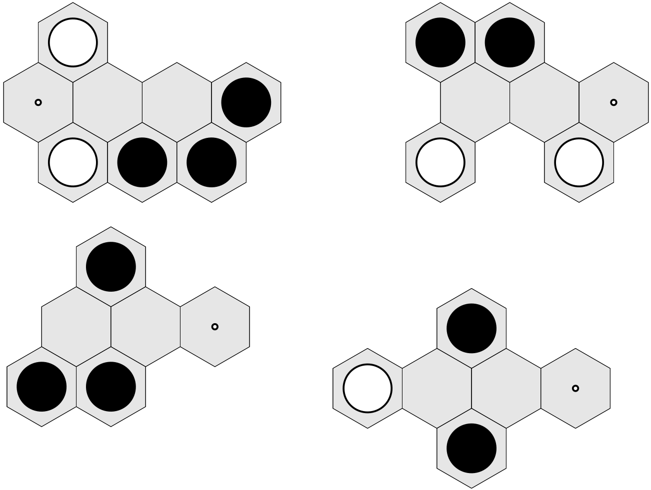
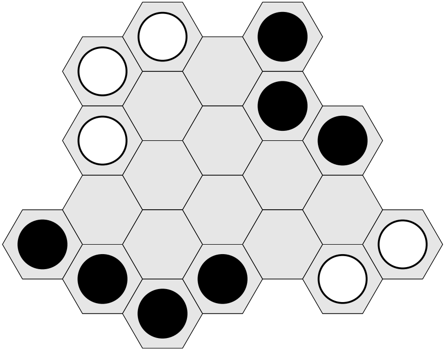


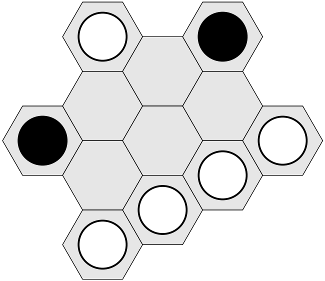
BLACK-CAPTURE-REVERSIBLE (TO WHITE DOT)



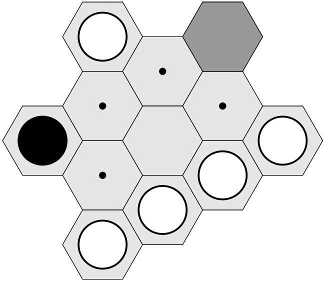
BLACK FILL DECOMPOSITION



STAR DECOMPOSITION

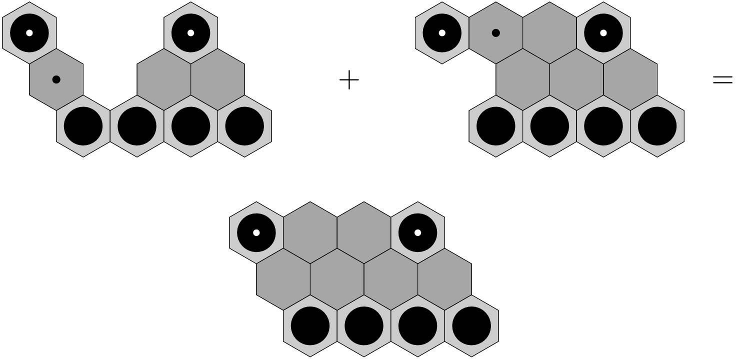


BLACK STAR DECOMP DOMINATION



modify H-search

- and/or combining rules + capture



SOME ANSWERS

FAIR ?

FAIR ?

- $n \times n$, $n \geq 2$, most win psns have losing moves
- $n \times n$, random play, n even: Prob(1pw) $=$ .5
- $n \times n$, random play, n odd: Prob(1pw) $\rightarrow$ .5 (?)

HOW LONG UNTIL 11X11 ?

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yr	size	states (approx)	center cell: solver fn calls
42	2x2	9.0 e 0	0
42	3x3	5.5 e 1	0
42	4x4	7.6 e 5	0
42	5x5	4.0 e 9	0
42-95	6x6	4.0 e 14	2
01-03	7x7	1.5 e 20	68
02-09	8x8	1.0 e 27	19 554
03-12	9x9	2.7 e 34	912 352
13-	10x10	1.2 e 43	5 821 097 789
	11x11	2.2 e 52	??? ??? ??? ??? ???

11x11 HANDICAP: WAGER ?

11x11 HANDICAP: WAGER ?

- exists simple strategy that wins
  - up to 5×5 with 1 stone
  - up to 11×11 with 2 stones
  - up to 17×17 with 3 stones
  - ...

webdocs.cs.ualberta.ca/~hayward/talks/hex.handicap.pdf

8 hours to code player : algorithms ?

8 HOURS TO CODE PLAYER : ALGORITHMS ?

basics

- search ? flat monte-carlo (random simulations, keep stats)
- detect wins ? union-find
- stats: all-moves-as-first (each winning stone gets bonus)
- move selection: highest AMAF score

improvements

- in simulations, save bridges
- monte carlo tree search
- code sample: `https://github.com/ryanbhayward/miowy`

`webdocs.cs.ualberta.ca/~hayward/670gga/jem/gga.html`

THANK YOU

- invitation Prof Mark Kayll UMontana
- solving 10×10 Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
- Natural Sciences and Engineering Research Council of Canada