Playing and Solving the Game of Hex

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Overview

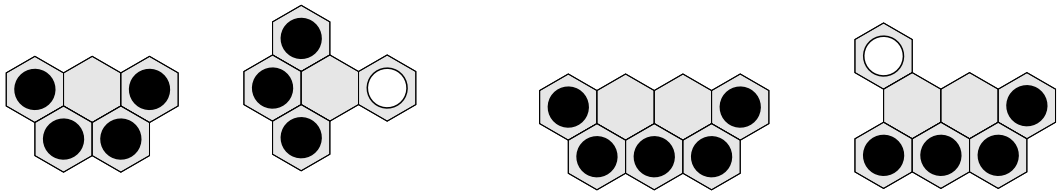
Game of Hex:

- Classical PSPACE-complete problem
- Graph theory, combinatorial game theory, artificial intelligence

Thesis objectives:

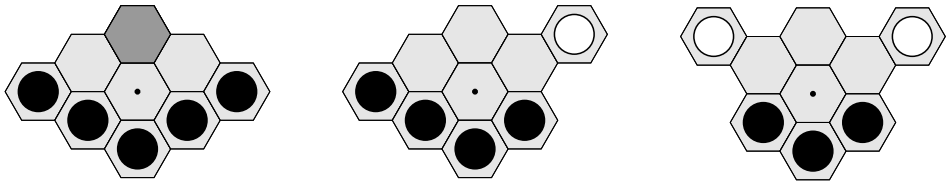
- expand on the mathematical and algorithmic knowledge for the game of Hex, and
- apply and adapt artificial intelligence techniques to make use of such knowledge

Previous Fillin



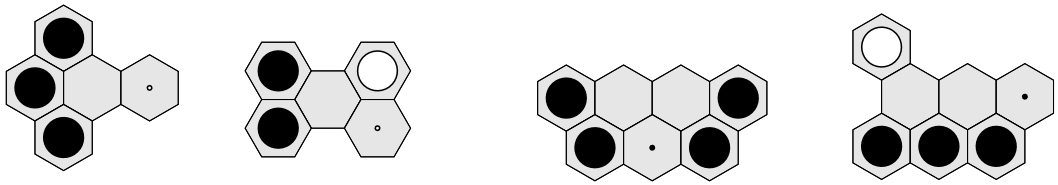
- Dead
- Captured

New Fillin



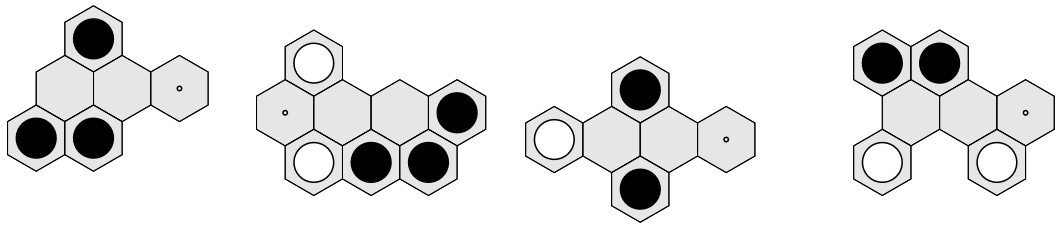
- Permanently inferior

Previous Pruning



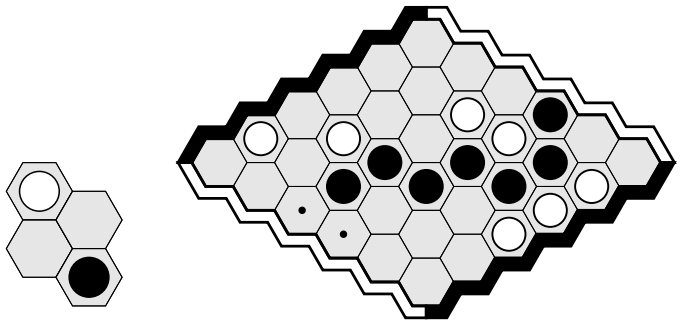
- Vulnerable (dead-reversible)
- Capture domination

New Pruning



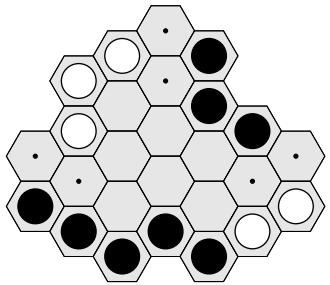
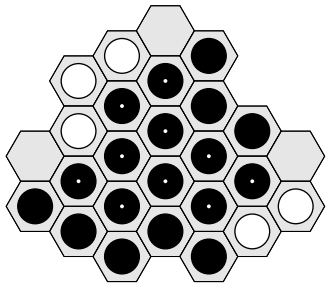
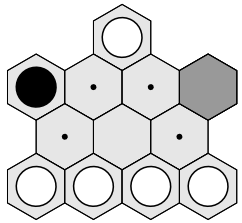
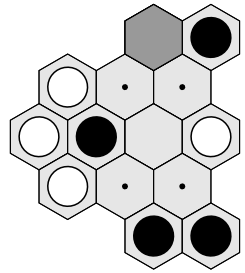
- Captured-reversible
- Neighbourhood domination
- Induced path domination

Generalized Decomposition



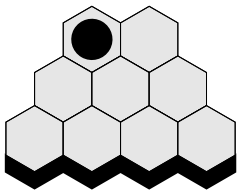
- Opposite colour bridge
- Split decomposition

Cyclic Decomposition

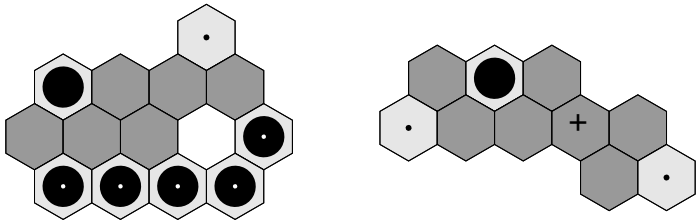
- Dead decomposition
- Captured decomposition
- Star decomposition domination

Applications



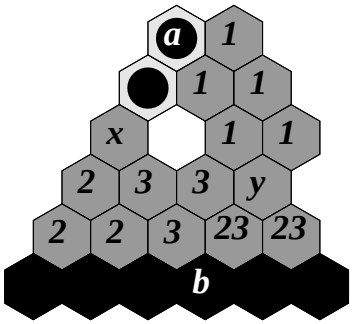
- Probing 4-3-2 connection strategy
- Acute corner domination
- Handicap strategy

H-search Algorithm



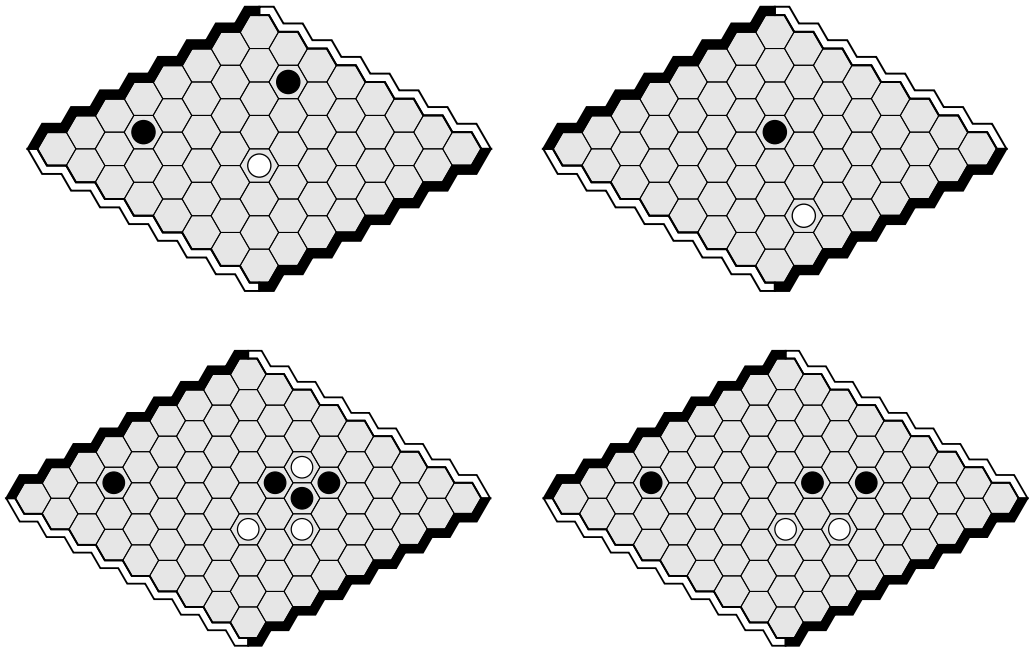
- Use known connection strategies to deduce more connection strategies
- Algorithm is not complete

H-search Augmentations



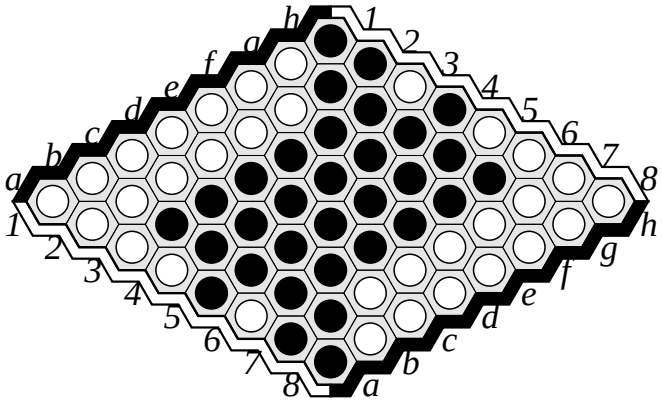
- Crossing rule
- Carrier intersection on captured sets
- Common miai substrategy

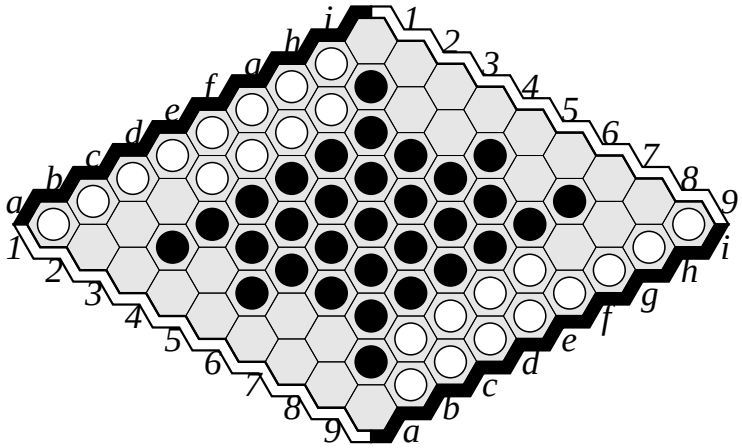
Position Value Deductions



- Winning carrier reduction and pruning
- Strategy stealing argument
- Player exchange deduction
- Unique probe deduction

Solved Openings





- Focused DFPN + Hex tools
- All 8x8 openings: 31 hours total
- Most 9x9 openings: 1-25 days each

Wolve and MoHex

- Wolve: alpha-beta player
- MoHex: monte carlo tree search player
- Both gold and silver medals in 2008 and 2009

Update/Errata

Page 74 (missing data):

Feature f off	**% time**	**% MID**	**% knowledge**
Focused DFPN	214.4	165.2	217.2
H-search border templates	186.1	444.2	458.1

Table: FDFPN solver feature contributions for one 9×9 Hex opening.

Page 79 (data error):

Wolve variant	**Win %**	**Time per game (avg, max)**
4-ply border midpoints	82.2 ± 2.5	2238.5, 6900.8

Table: Wolve variants: performance against Six.