Focused Depth-first Proof Number Search Using Convolutional Neural Networks for the Game of Hex

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Introduction: Game of Hex

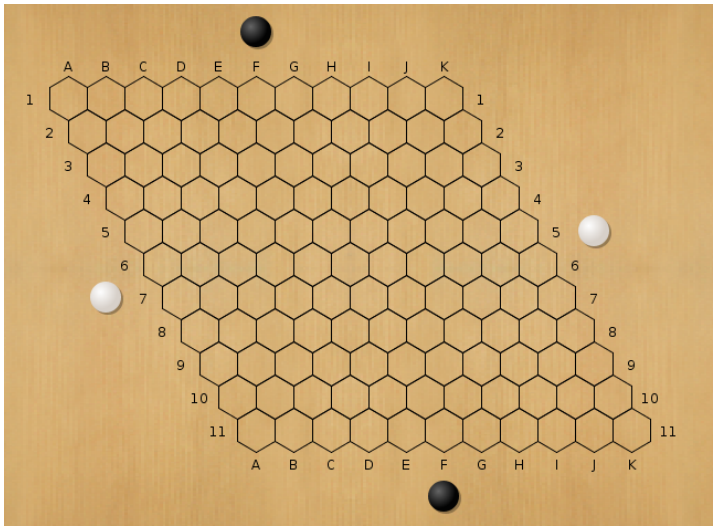


Figure: Hex is played on rhombus board.

- Invented in 1942, and in 1948 independently by John Nash.
- Played on rhombus board, 11×11 most popular.
- Two-player, perfect information, and zero-sum.
- No draws.

Introduction: Algorithmic Solving Hex

Facts we already know.

1. By strategy stealing, there is a wining strategy for first player from empty board.
2. The explicit strategy is unknown.
3. Solving arbitrary position is PSPACE-complete (Reisch 1981).

How to efficiently solve Hex positions by search?

Search Virtual Connections by Hierarchical Algebra (Anshelevich 2000), but

H-Search is incomplete.

Needs Tree Search.

Introduction: Algorithmic Solving Hex

Tree search to solve Hex,

- Inferior Cells Analysis can be helpful for pruning
- H-Search helps detect early win or lose

With these techniques + Depth-first search

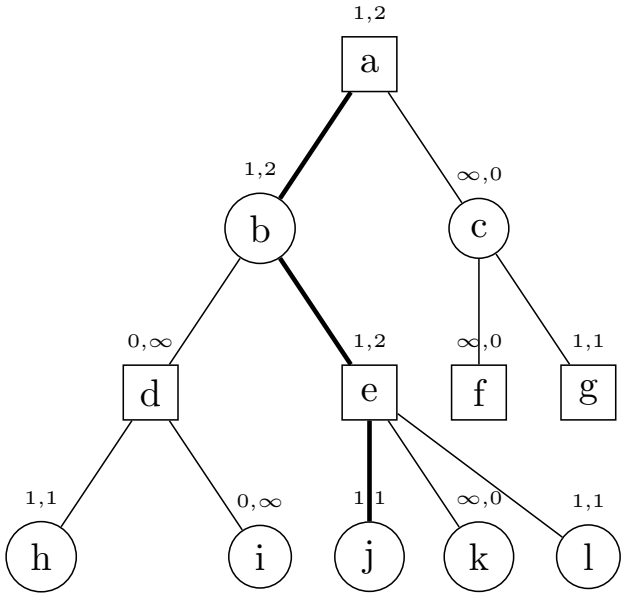
- 7x7 Hex is solved in 2004 (Hayward, Bjornsson, Johnson, Kan, Po and Rijswijck),
- 8x8 Hex is solved in 2008 (Henderson & Hayward).

Solving even larger board sizes is difficult,

Better tree search algorithms?

Review Proof Number Search

Proof number search (Allis et al. 1994).



$\phi(a) = \min\{\delta(b), \delta(c)\}$
$\delta(a) = \phi(b) + \phi(c)$
$\phi(s)/\delta(s)$ is the minimum number of leaf nodes to prove/disprove s.

Following ϕ, δ, there exists most promising node to expand. (requires the least effort to solve the root!)

PNS is best-first, works in an iterative fashion:

1. Select an MPN
2. Expand MPN, set children's proof and disproof number
3. Backup

Depth-first Proof number search

Problems of PNS

- Needs to save the whole tree
- Unnecessary traversal of the tree even MPN does not change

Alleviation: Depth-first PNS (Nagai, 2002):

- Use of bounds to avoid unnecessary traversal
- Use of fixed size transposition table (TT)

Is the algorithm still complete with fixed size TT?

No, but usually works fine if not #nodes $\gg$ TT.

Focused depth-first proof number search

Rethink of the definition of proof and disproof number:

- Shape of the tree matters, PNS exploits narrow and deep branches.

Hex has near "uniform branching factor".

Solution: FDFPN
focus on promising nodes (Henderson 2010).

- Resistance as move ordering
- Expands only $\mu \cdot \mathcal{A}(s)$ nodes.

Twice faster than DFPN in 8×8 Hex.

9×9 Hex openings are solved by parallel FDFPN (Pawlewicz & Hayward 2014), took $>$ 17 months.

Can we make FDFPN better?

Rethink the original FDFPN

Problems of the original FDFPN (Henderson 2010).

- Resistance often pathological
- "Artificial branching factor" is created in an uninformative manner.

Redefine Focused DFPN..
with the help of neural nets..

Redesign FDFPN

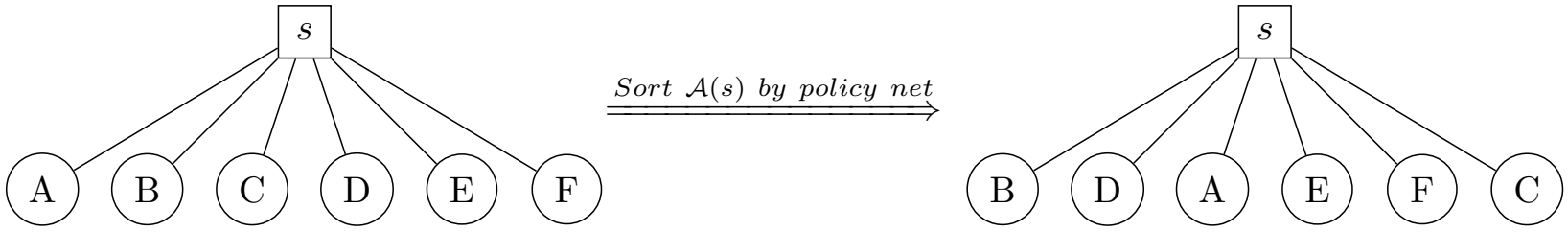
General idea:

- Use policy neural net for move ordering
- Use value neural net to create "artificial branching factor"

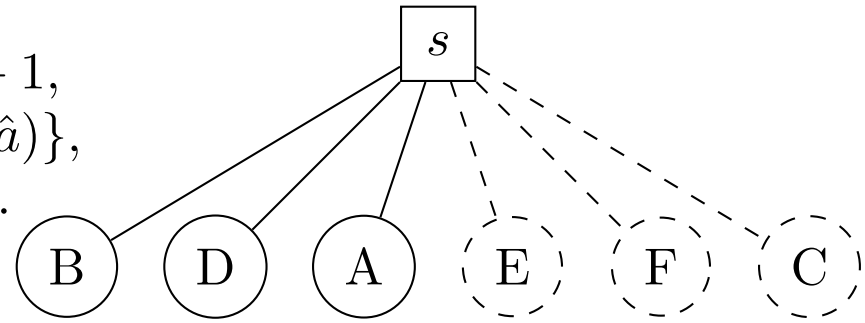
The motivation:

- policy neural net has high top-k prediction accuracy.
- value net provides reliable estimation of the optimal value of expanding node.

Redesign FDFPN with CNNs



$$\begin{cases} l = |\mathcal{A}(s)| \times \min\{\mu, 1 + v_\theta(s)\} + 1, \\ L = \{a \in \mathcal{A}(s) | P_\sigma(s,a) \geq P_\sigma(s,\hat{a})\}, \\ \text{where } \hat{a} \text{ is the } l\text{th move in } \mathcal{A}(s). \end{cases}$$



$v_\theta(s) \in [-1.0, 1.0]$ is the output of the value net.

μ is parameter.

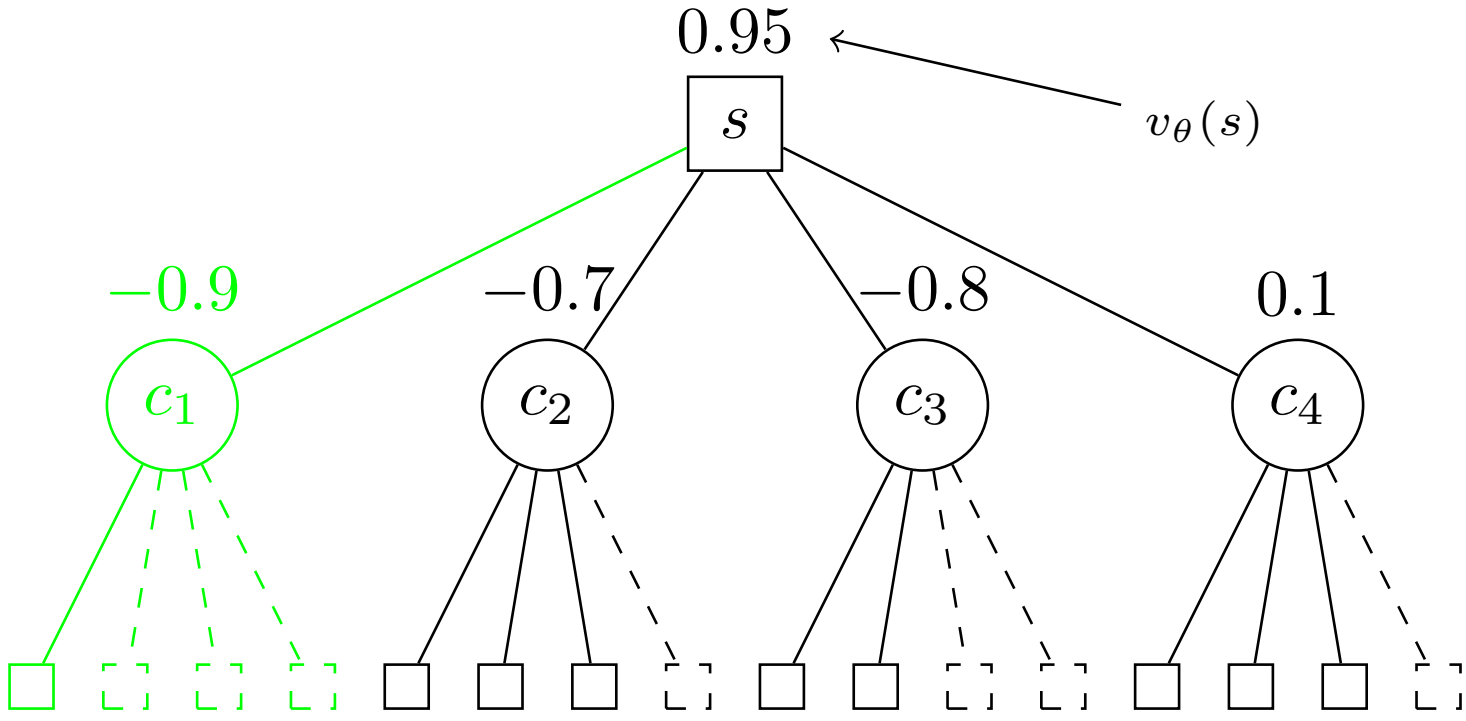
$P_\sigma(s,a)$ is the prior probability from the policy net for state-action (s,a).

Smaller $v_\theta(s)$, smaller search window!

Redesign FDFPN with CNNs

Why smaller window when v_θ is small?

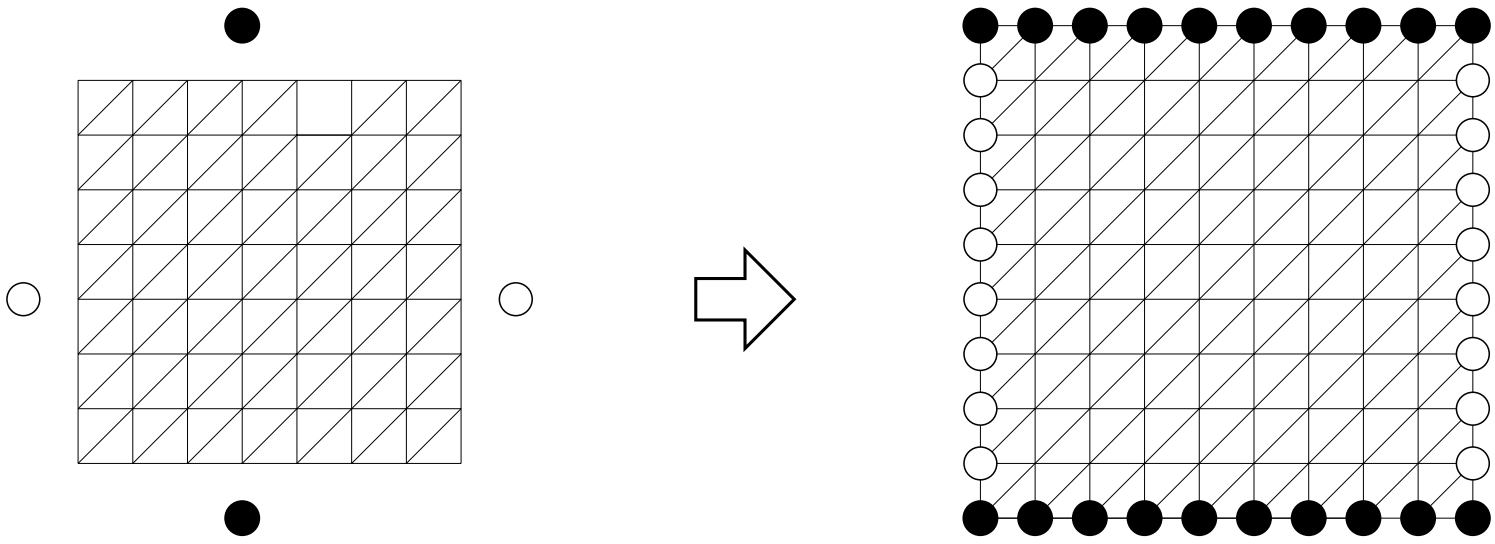
PNS favours "narrow" branches.



Smaller window forces PNS prefer node c_1 (Smaller proof/disproof number).

Experiment: Game studied

8×8 Hex openings.
Tractable but non-trivial.
A Hex-board can be turn into an square board.



Experiments: Neural net architecture

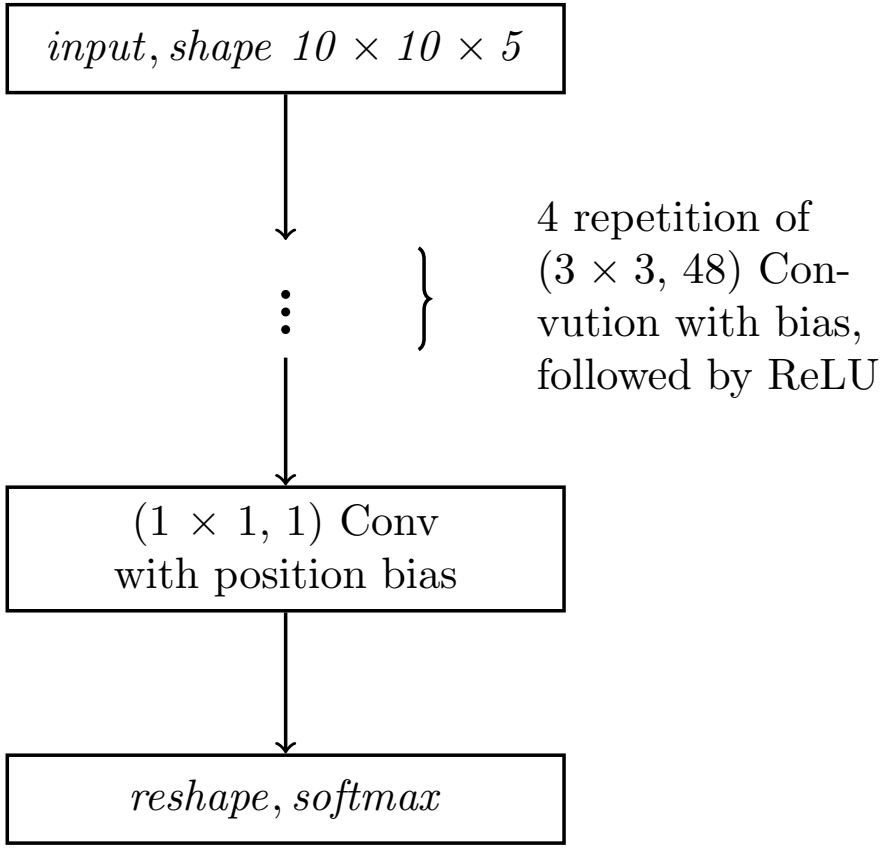


Figure: Architecture for policy net

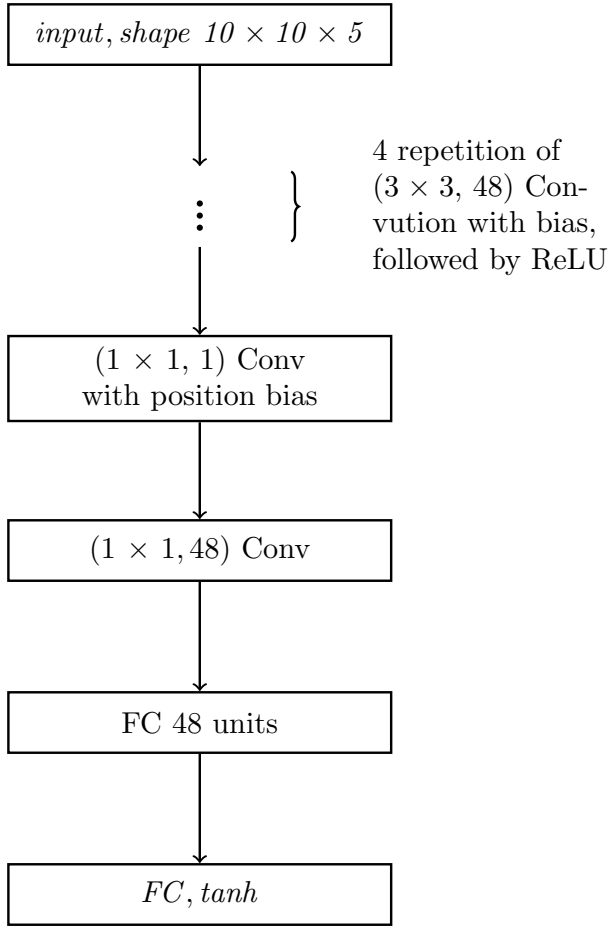


Figure: Architecture for value net

Experiments: policy and value net

Data: 6.5×10^5 state-action or state-value pairs.

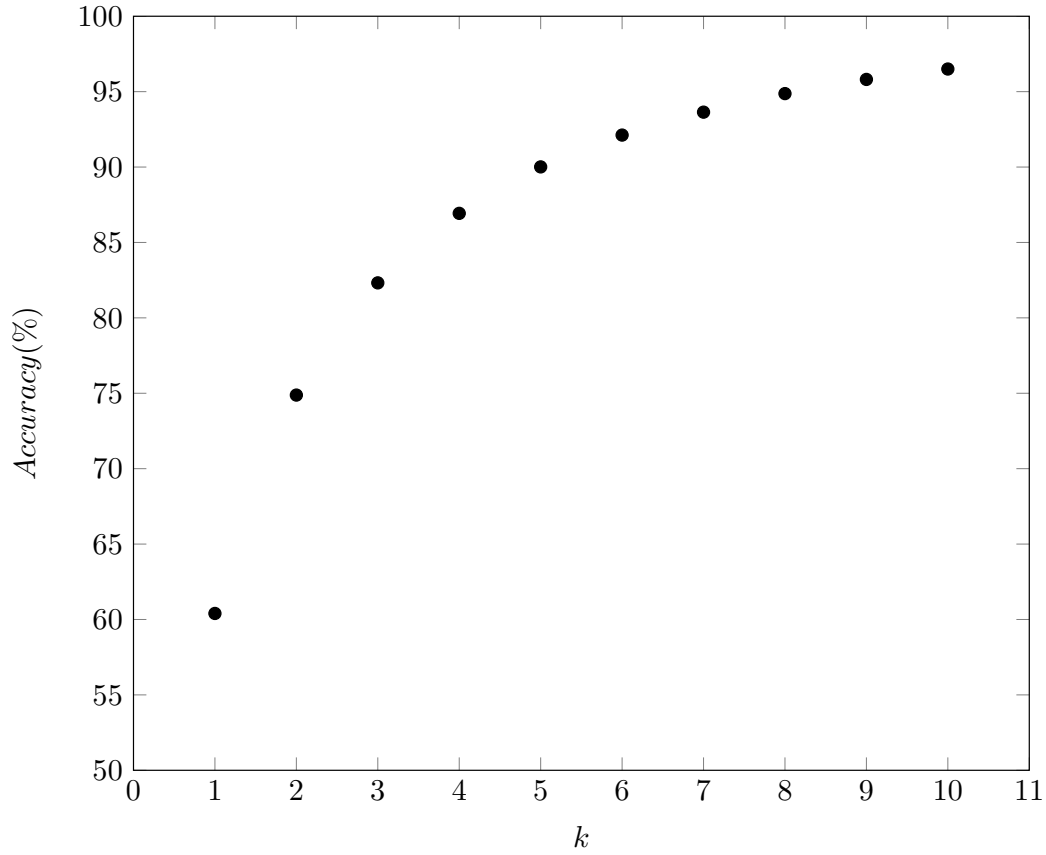
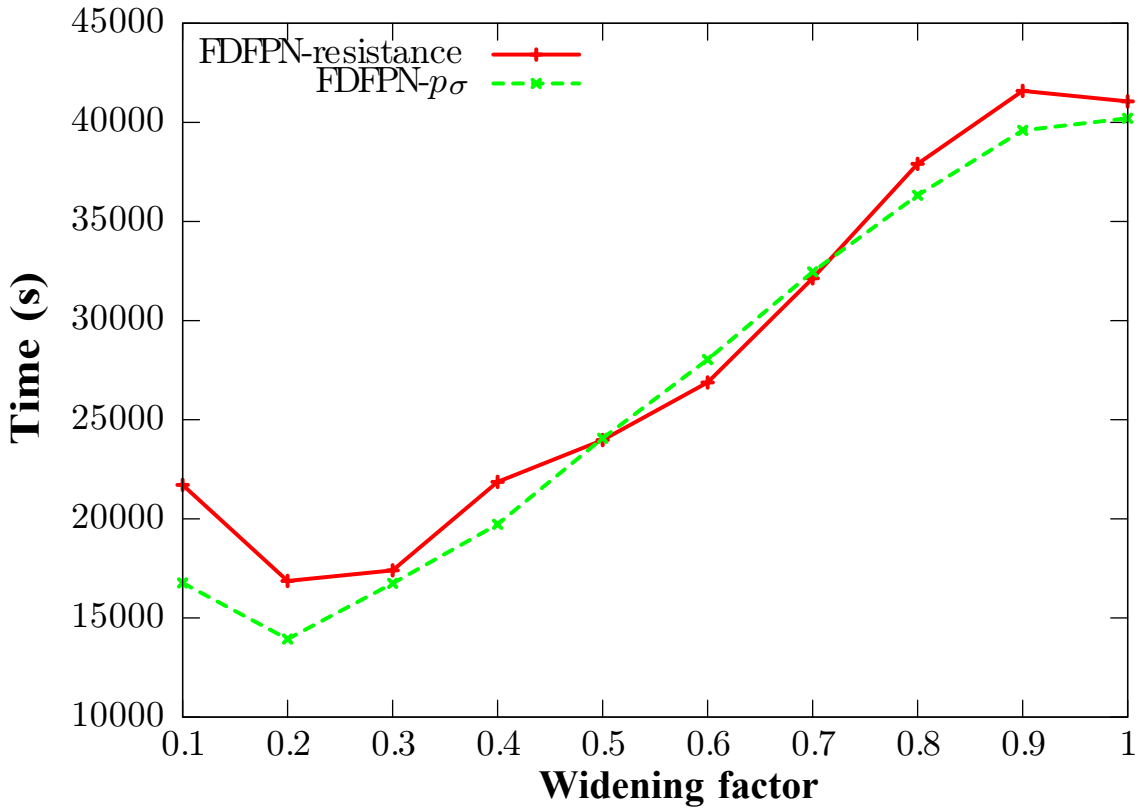


Figure: Top k prediction accuracy on the test data

MSE of value net: 0.083.

Experiments: compare policy net and Resistance

Could FDFPN perform better by just replacing its move ordering function with policy net?



- policy net is better at small winder factor.
- widening factor is close to 1, both recede to normal DFPN.

policy net selects better moves than Resistance.

Experiments: FDFPN-CNN

Solving all 8x8 opening positions:
Improvement:

- #expansion: 46.7%
- #time: 40% (with Tensorflow 1.0)

Experiments: further investigation of the value net

In the discovered solution graph, for each node, compare the estimated value with ground truth.
A simple classifier:

$$f(s) = \begin{cases} 1, & \text{if } v_\theta(s) > 0 \\ 0, & \text{if } v_\theta(s) \leq 0 \end{cases}$$

What is the **error rate** of f?
about 14.3%.

Conclusions

Conclusion:

- FDFPN-CNN is stronger than FDFPN

Reason:

- Better move selection using policy net
- Creating “artificial” non-uniform branching factor with value net.

Further work

Further improvement:

1. Better neural network design, perhaps residual neural net, and more regularization..
2. Dealing with the imperfectness of the training data.
3. Exploiting AND/OR structure to improve the value estimation accuracy.
4. Modify the calculation/initialization of proof and disproof number with value net.