

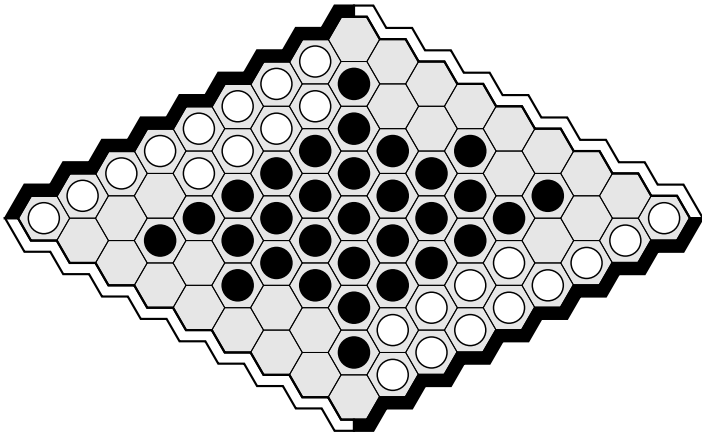
Solving Hex : Beyond Humans

Arneson Hayward Henderson

Comp Sci U of A
Edmonton Alberta Canada

Sept 2010

- 2010 ... our solver surpasses humans
- 9×9 Hex ... 53/81 openings



by hand ... early era

- 1942 Hein ... 4×4 ... easy-peasy
- 1942 Hein ... 5×5 ... lemon-squeezy
- 1957 Gardner ... 6×6 ?

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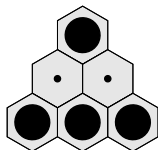
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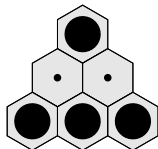
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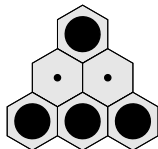
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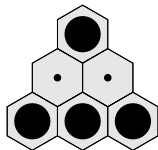
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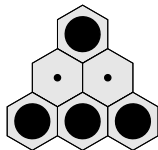
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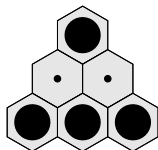
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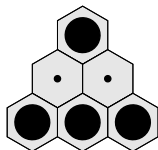
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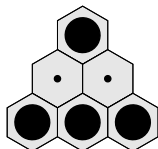
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- 8×8 64/64 : 300 hours
- 9×9 0/81 : 600+ hours

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- search ? + strategy-stealing symmetry check
- H-search ? + captured cells
- inferior cell analysis ?
- + permanently inferior
- + captured-reversible
- + star-decomposition domination
- DFS ? \implies F-DFPNS

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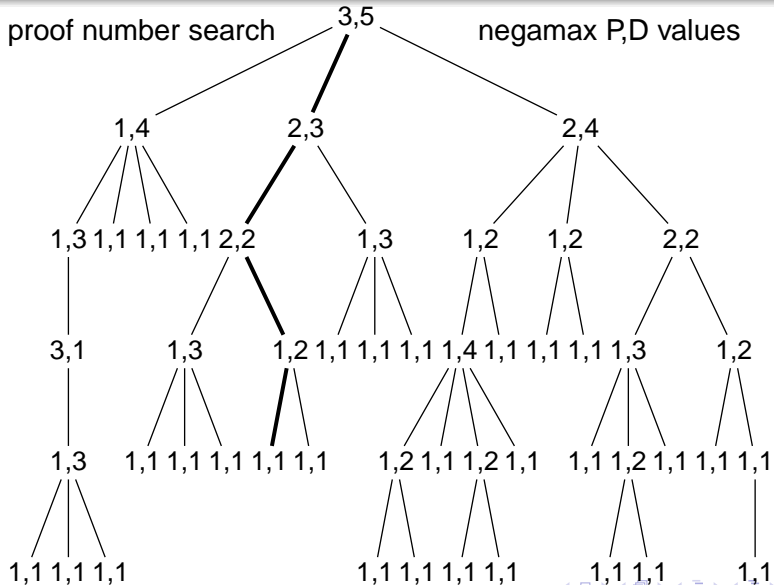
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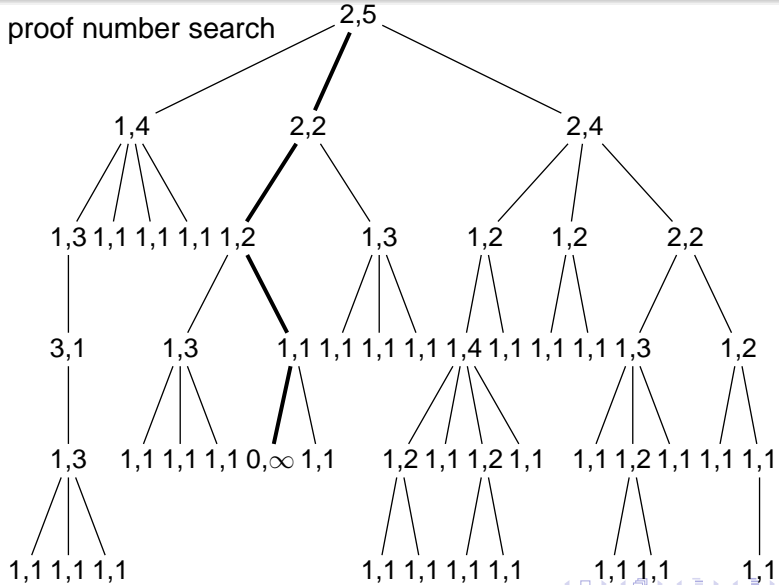
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proof number search



F-DFPNS

- PNS Allis et al
- DFPNS Nagai
- DFPNS in Hex ?
- ... requires non-incremental H-search :(
- ... uniform branching factor :(
- idea: move ordering + DFPNS = F-DFPNS

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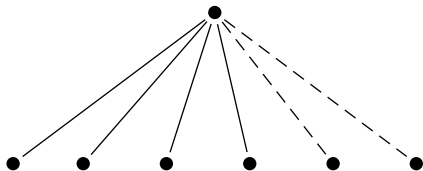
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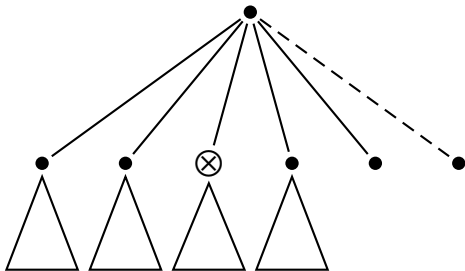
F-DFPNS (1)

- expand node
- consider first $b + \lceil f \times 6 \rceil = 4$ (of 6) live children



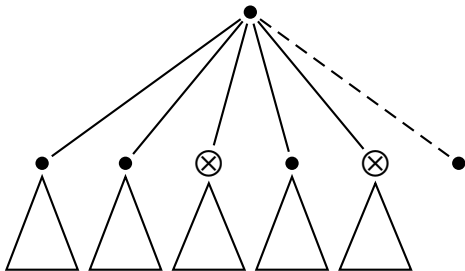
F-DFPNS (2)

- discover move 3 loses
- consider first $b + \lceil f \times 5 \rceil = 4$ (of 5) live children



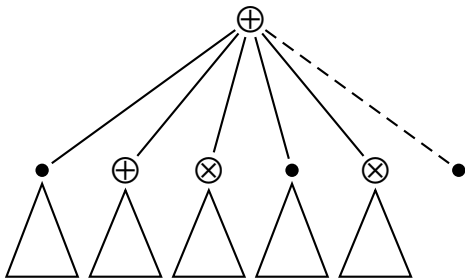
F-DFPNS (3)

- discover move 5 loses
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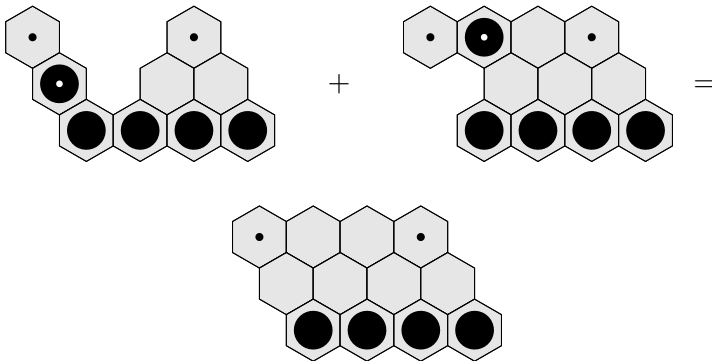
F-DFPNS (4)

- discover move 2 wins, so ...
- ... root solved without exploring 6th move



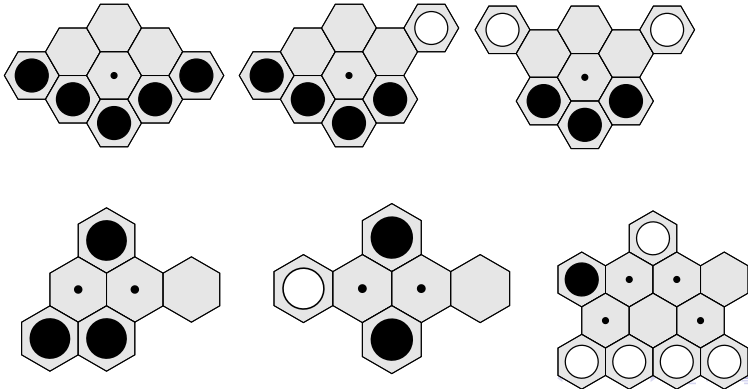
modify H-search

- and/or combining rules + capture



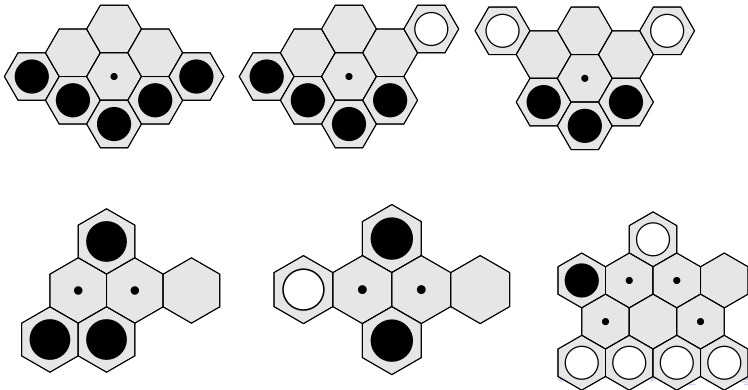
more inferior cell analysis

- permanently inferior cells
- captured reversible cells
- star decomposition domination



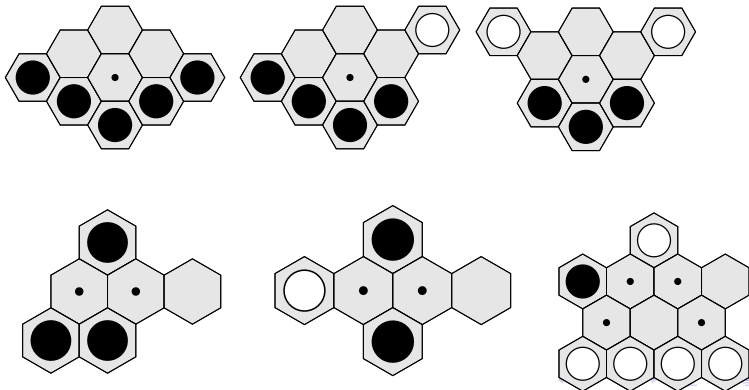
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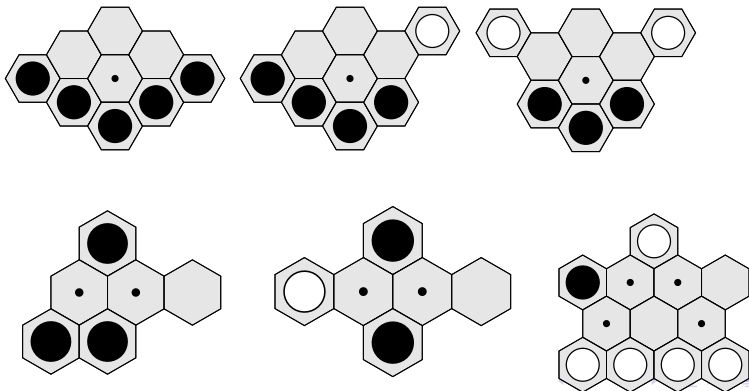
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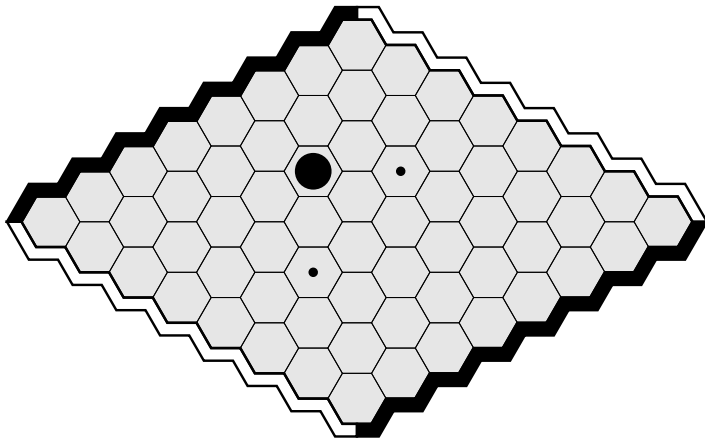
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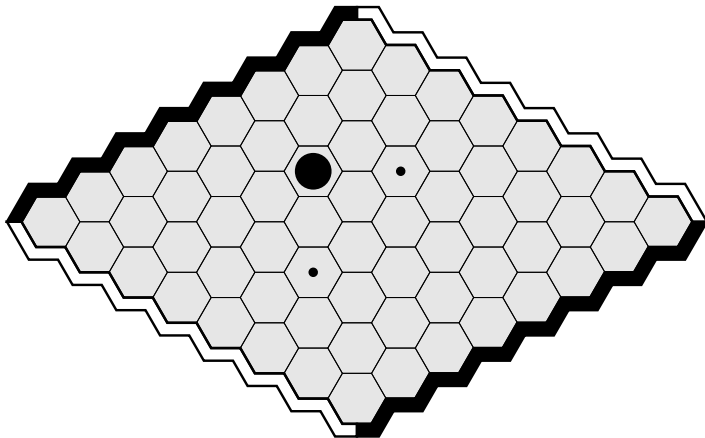
strategy stealing symmetry pruning

- 1st player wins on color-symmetric board



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feature contributions (8×8)

feature f turned off	time (s)	time ratio
captured-cell H-search	196,227	1.75
inferior cell analysis improvements	126,201	1.13
strategy-stealing pruning	118,010	1.05
—	112,121	1.00

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solver scaling

- all $n \times n$ openings \approx one $(n + 1) \times (n + 1)$
- estimate: one $10 \times 10 \approx 870$ days
- estimate: all $10 \times 10 \approx 750 \times 870$ days ≈ 1800 years

board size	fastest opening	all openings
7×7	0.5	384
8×8	155	112,121
9×9	96,168	? ? ? ? ? ? ?

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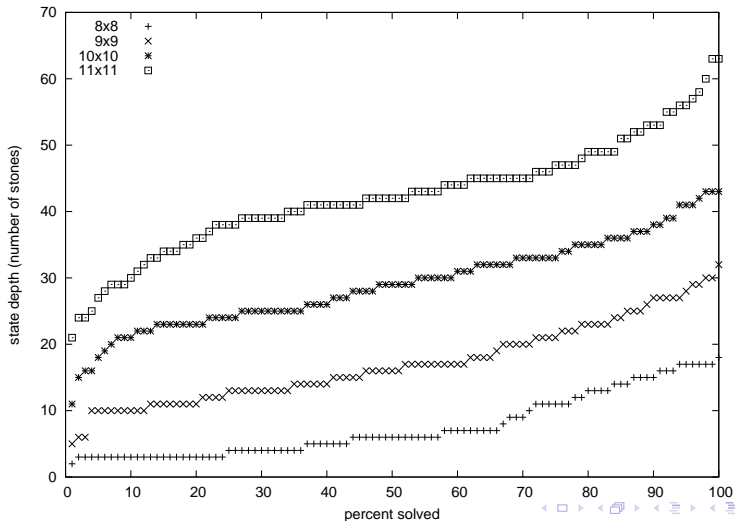
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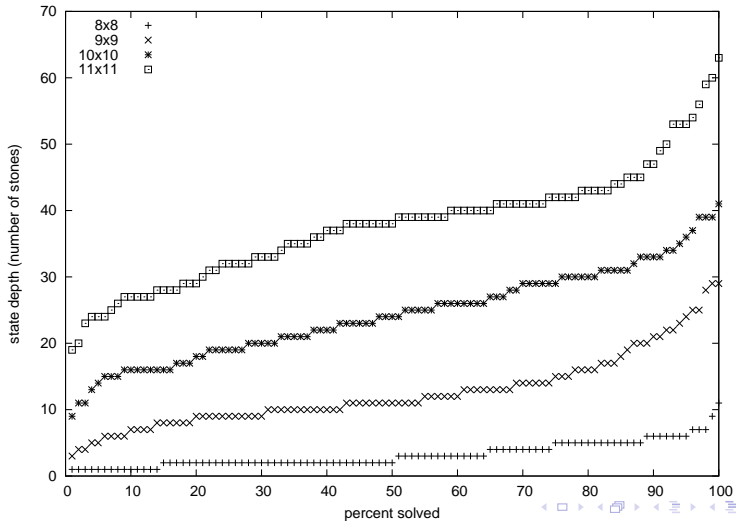
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10 sec solver data: stones vs % solved



60 sec solver data: stones vs % solved



11×11

- players can use 1-thread for solver
- ~ 60 s/move, so many/most solved by move 35/45

by 2020 ?

- ... compute power: $32\times$
- ... $6\times$ solving time ≈ 4 stones
- ... Hex stones: $+ 32/6 * 4 \approx + 21$
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- NSERC Alberta Ingenuity
- UofA GAMES UofA Hex
- M Mueller J Schaeffer