alternating linear clobber

ryan hayward @ ualberta . ca

XChen TFolkersen KHasham ORandall

LSchultz EVandermeer

thank you organizers! MMüller!

CGTCV Jan 31 2025 Lisbon Portugal

clobber

- *intro to clobber* 2001 AGNW

  Albert Grossman Nowakowski Wolfe

  https://webdocs.cs.ualberta.ca/~hayward/papers/AGNW.pdf

  this talk:

  https://webdocs.cs.ualberta.ca/~hayward/talks/cgtcv.pdf

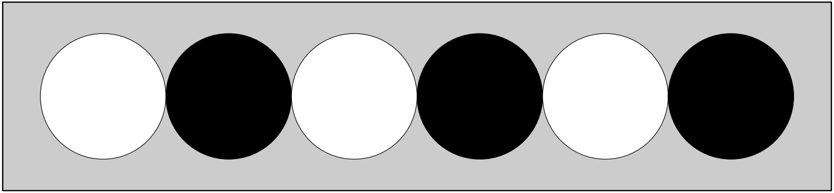
alternating linear clobber

- *linear clobber* clobber on a path
- *alternating linear clobber*

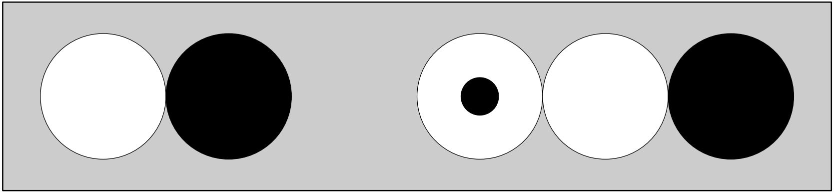
  linear clobber, starting from one of

  `ox, oxox, oxoxox, oxoxoxox, ...`

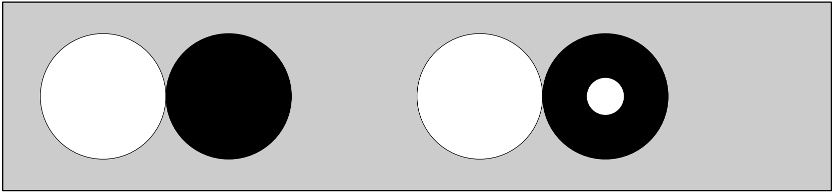
example ALC game



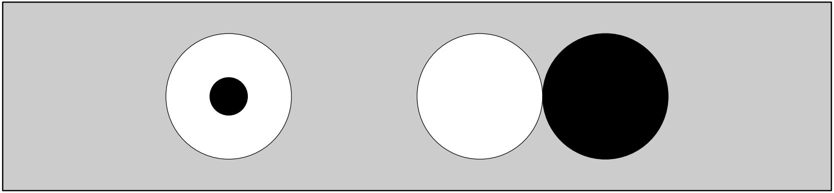
example ALC game



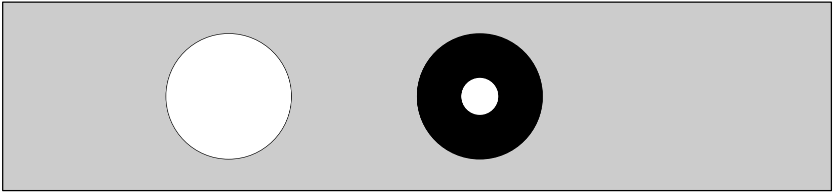
example ALC game



example ALC game



example ALC game



now White has no legal moves, White loses conjecture 3.2 AGNW 2001:

for every ALC start position except `oxoxox`,

first player can win

6 sets of parts (components)

$$\mathcal{A} = \{\text{ox, oxox, oxoxox, ...}\}$$

$$\mathcal{O} = \{\text{o, oxo, oxoxo, ...}\}$$

$$o\mathcal{A} = \{\text{o, oox, ooxox, ... }\}$$

$$o\mathcal{O} = \{\text{oo, ooxo, ooxoxo, ...}\}$$

$$o\mathcal{O}o = \{\text{ooo, ooxoo, ooxoxoo, ...}\}$$

$$o\mathcal{A}x = \{\text{ooxx, ooxoxx, ooxoxoxx, ...}\}$$

shorthand notation

$$\mathcal{A} = \{\text{a2, a4, a6, ...}\}$$

$$\mathcal{O} = \{\text{o, o3, o5, ...}\}$$

$$o\mathcal{A} = \{\text{o, oo3, oo5, ... }\}$$

$$o\mathcal{O} = \{\text{oo, oo4, oo6, ...}\}$$

$$o\mathcal{O}o = \{\text{ooo, oo5oo, oo7oo, ...}\}$$

$$o\mathcal{A}x = \{\text{ooxx, oo6xx, oo8xx, ...}\}$$

warmup

ox, oxo, ooxox, ooxoxoo $=$ $*$

oxx $= \uparrow$

oxox $= \pm\{\uparrow, 0\}$

ooxo, oxoxoxoxo $=$ $\uparrow *$

ooxoxx, oxoxoxoxoxox $=$ oxox $*$

oxoxo $= \{\uparrow \mid *\}$

theorem (easy)

theorem:

every part that arises in an ALC game,

or its negative,

is in one of $\mathcal{A}$, $\mathcal{O}$, $o\mathcal{A}$, $o\mathcal{O}$, $o\mathcal{O}o$, $o\mathcal{A}x$.

$\mathcal{A}'$ to $o\mathcal{O}o'$

$$
\begin{aligned}
\mathcal{A}' &= \mathcal{A} \setminus \{0, *, \texttt{oxox}\} \\
\mathcal{O}' &= \mathcal{O} \setminus \{0, *, \uparrow *\} \\
\mathcal{A}' &= \mathcal{A} \setminus \{0, \downarrow, *\} \\
o\mathcal{O}o' &= \mathcal{A} \setminus \{0, \texttt{oxox}*\}
\end{aligned}
$$

$\mathcal{A}'$ to $o\mathcal{O}o'$

$\mathcal{A}'$	{		a8	a10	oxox$*$	a14	...
$\mathcal{O}'$	{	o5	o7	o11	o13	o15	...
$o\mathcal{O}'$	{	oo6	oo8	oo10	oo12	oo14	...
$o\mathcal{A}'$	{	oo7	oo9	oo11	oo13	oo15	...
$o\mathcal{O}o'$	{	$\downarrow$	oo9oo	oo11oo	oo13oo	oo15oo	...

definition of set K

K: games g, each part in any of

$\mathcal{O}'$, $o\mathcal{O}'$, $o\mathcal{O}o'$, $\{\texttt{oxox}\}$, $\{*\}$, $\{\uparrow\}$, $\mathcal{A}'$, $o\mathcal{A}'$

define count vector $\mu(g)$

(a, b, c, d, e, f, y, z)

definition of set of exceptions Q

- $Q = \{$o5 $*$, o7 $+$ oxox, o15 $+$ oxox $*\}$
- thm: $\mathcal{O}'$ $\{$ o5 o7 o11 o13 $\ldots\}$ $\subseteq \mathcal{L}$
- thm: sum $\mathcal{O}' + * \setminus Q \subseteq \mathcal{L}$
- thm: sum $\mathcal{O}' +$ oxox $\setminus Q \subseteq \mathcal{L}$
- thm: sum $\mathcal{O}' +$ oxox$*$ $\setminus Q \subseteq \mathcal{L}$

recall set K

K: games g, each part in any of

$\mathcal{O}'$, $o\mathcal{O}'$, $o\mathcal{O}o'$, $\{\texttt{oxox}\}$, $\{*\}$, $\{\uparrow\}$, $\mathcal{A}'$, $o\mathcal{A}'$

define count vector $\mu(g)$

(a, b, c, d, e, f, y, z)

definition of sets $\mathcal{S}_0$, $\mathcal{S}_1$, $\mathcal{S}_2$

$\mathcal{S}$ games of $K \setminus Q$ with

$\mathcal{S}_0$: $a \geq c, \quad y \leq 1, \quad z = 0$ or

$a \geq c+1, \quad y = 0, \quad z = 1$

$\mathcal{S}_1$: $a \geq c+1$

$\mathcal{S}_2$: $a \geq c$, only ooxo from $o\mathcal{O}'$, $f \geq 1$ and

$b+d+e = 0$ or $b+d+e+f \geq 3$.

- $g \in \mathcal{S}_1 \cup \mathcal{S}_2$: each R-move stays in $\mathcal{S}_0$
- $g \in \mathcal{S}_0$: L can move into $\mathcal{S}_1 \cup \mathcal{S}_2 \cup \{0\}$
- $\mathcal{S}_0 \subset \mathcal{L} \cup \mathcal{N}$
- $(\mathcal{S}_1 \cup \mathcal{S}_2) \subset \mathcal{L}$
- $\mathcal{A} \setminus \{\texttt{oxoxox}\} \subset \mathcal{N}$

how Left wins on $\mathcal{A} - $ oxoxox: outline

- play $\mathcal{A}'$ to $\mathcal{O}'$
- play so that all parts are in $\mathcal{A}' \ldots o\mathcal{O}o'$
- ignore ox, ignore oxox
- never leave a game in Q

o5 + ox   o7 + oxox   oo15 + oxox + ox

where is the devil?

where is the devil?

in the details

definition of $\mathcal{S}_2$ restated

- sums of xxo, ox, oxox, ooxoxo
- k copies of xxo or
- at least 1 xxo, at least 3 parts

how can Right move on oxoxoxoxo?

can assume R clobbers to right

(oxoxoxoxo has left-right symmetry)

oxoxoxoxo $\rightarrow$ _ ooxoxoxo

oxoxoxoxo $\rightarrow$ ox _ ooxoxo

oxoxoxoxo $\rightarrow$ oxox _ ooxo

oxoxoxoxo $\rightarrow$ oxoxox _ oo

how L plays: part 1

rule	{game} or subgame	result
1.	$\{$ a8 *$\}$	a4 $\uparrow *$ (avoid 5 $*$)
	$\{$a10 a4$\}$	o5 a4 $\downarrow *$ (avoid 7 a4)
	$\{$a18 a4 *$\}$	a14 a4 $\uparrow *$ (avoid 15 a4 $*$)
	a8, a10, a14, ...	o5, o7, o11, ...
2.	oo7, oo9, ...	oo4, oo6, ...
3.	$\{$o5 $\downarrow\}$	$\uparrow\downarrow$
	o5 a4 $\downarrow *$	o5 a4 $* *$
	a4 $\downarrow$	$\uparrow\downarrow$
	$\downarrow$	$*$

how L plays: part 1

4.	oo9oo, oo11oo, . . .	oo4, oo6, . . .

how L plays: part 1

5.	$\{oo10\ *\}$	$7*\ *$ (avoid $o5\ *$)
	$\{oo12\ a4\}$	$a4\ \uparrow*$ (avoid $o7\ a4$)
	oo10	o5
	oo12	o7
	oo14	$o11*$
	oo16	o11
	oo18	o13 (avoid $o15\ a4\ *$)
	oo20	$o17*$ (avoid $o15\ a4\ *$)
	oo22, oo24, . . .	o17, o19, . . .

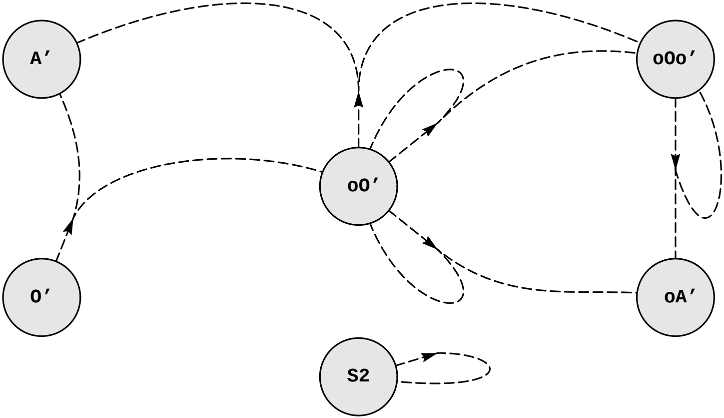
how L plays: part 2

6a.	oo8	$\uparrow\uparrow*$
	oo6 or a4	$\uparrow$ case a
	$\uparrow$ or $*$	0 case a
	a4 $*$	$**$ case b
	a4	$\uparrow$ case b
	oo6 oo6	oo6 $\uparrow*$
	oo6 a4	$\uparrow*$
	oo6 $*$	$\uparrow**$
	oo6 or a4	$\uparrow\uparrow$

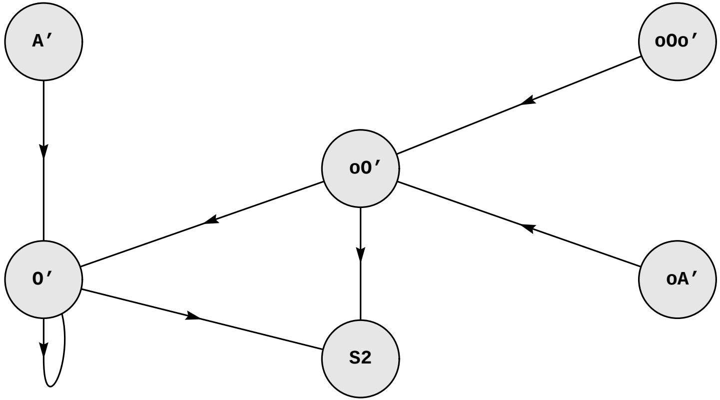
how L plays: part 2

6b.	$\uparrow *$	$\uparrow$
	oo6 $*$?	$\uparrow *$?
	oo6 ?	$\uparrow *$?
	a4	$\uparrow$
	$\uparrow \cdot k*$	$\uparrow \cdot k$
	$\uparrow \cdot k$	$\uparrow \cdot (k-1)$
7.	o5, o7, o11	$\uparrow$, o5, o7$\uparrow$
	o13, o15, ...	o11, o13 ...

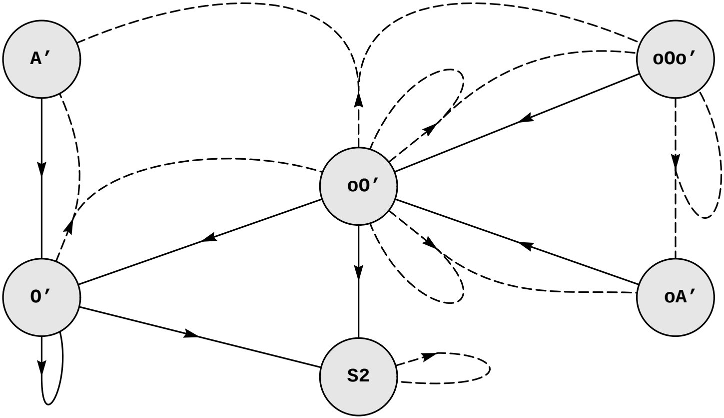
how Right can move



how Left plays to win



how Left and White play



thank you

questions? ask ryan :)

email hayward@ualberta.ca