Evaluating Opening Moves in Y

with Parallel Decomposition

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- 2 players, black and white
- Triangular board, hexagonal tiling
- Cell colouring game, black goes first
- Goal: connect all three sides
- No draws



Opening Moves

- Choose a cell
- Suppose black's first move is in that cell
- If both players play perfectly, who wins?
- Want proof!
 - Constructive



Parallel Decomposition

- A method of finding (constructive) proofs
- Decompose goal into <u>subgoals</u>
- Subgoal depends on a set of cells
 - local pattern
- Subgoals must satisfy 2 requirements:
 - Parallel: All local patterns pairwise disjoint
 - Complete: Achieving all subgoals achieves the goal

Parallel Decomposition – Example

- White can play in <u>one</u> LP
- Black has a second player win on each LP
- => Black can eventually win all LPs => achieve all subgoals => win
- => Black has a second player winning strategy



Prior Work & Motivation

- Applied to Hex by Yang et al. (2001, 2003)
- Proves win/loss status of several opening moves
- My work: attempt to apply this method of proof to Y
 - Exact same goal for both players
 - Three sides to connect
 - Can PD work for Y?



Results

 Base-7 Y: all openings solved



 Base-8 Y: one opening solved (three with symmetry)



Base-7 Y

- There are 7 opening moves in Base-7 Y
 - Symmetry
- I claim:
 - G and F win
 - A, B, C, D, and E lose



- Easy opening to start with
- Black plays at G, black to win
- Every response by white is a *case*
 - Suggest next move
 - Find a PD
 - Prove a 2PW each LP
- Symmetry helps!



- To make the proof faster, can often group cases
 - Assume the opponent gets *all* of those cells
- Only one case here



7Y: Opening G, Case G1

- Assume white has all the G1 cells
- Suggest a response
 - (2,5)
- Find a PD
- Prove 2PW on each LP



7Y: Opening G, Case G1

- Subgoals:
 - Connect center to left (Δ)
 - Connect center to top and right (□)
 - "Center/Corner ring" (very, very useful)
- Note subgoals may involve connecting multiple pieces, sides, etc.



7Y: Opening G, Case G1

- Black has 2PW on each LP
- All cases proven
- G is a winning opening



- A more balanced opening
- Balanced positions are hardest to prove
- C has the longest proof overall



- White can win by playing in the center
- 11 cases let's analyze a strong response by black



- Case C11: black \rightarrow (3,2)
- White \rightarrow (2,3)
- No obvious solution, so we play one more turn
- Every move black could make is a *subcase*
 - Suggest a move
 - Parallel decomposition
 - 2PW on each LP



- Subcase (d): black \rightarrow (4,3)
- White \rightarrow (4,2)
- Now we find a parallel decomposition



- Subcase (d): black \rightarrow (4,3)
- White \rightarrow (4,2)
- Now we find a parallel decomposition
- "When in doubt, play another turn."
- Opening C: 11 cases, 23 subcases, 4 subsubcases
- Most out of any opening



Wrapping up Base-7 Y

- PD tends to create long proofs with many cases
- Ideas seen so far are enough to understand and create proofs for base-7 Y



Wrapping up Base-7 Y

- All openings in base-7 Y are solved by parallel decomposition
- Proofs are constructive
- Result: second player winning strategy for base-7 Y with swap rule



Base-8 Y

- Y is known to be a first player win (any size), but the proof of this is not constructive
- Use parallel decomposition to find a winning strategy for base-8 Y



Base-8 Y

- Black's first move is the center
- Symmetry helps (again)
- Eight cases for where white could move
 - Base-7 had only one case for the center!
- Let's look at case B2



- Connecting to the left is easy
- Connecting to the top and right is a single subgoal
 - 2PW on that LP is not obvious



Local Pattern Analysis

- Local patterns can be analyzed with parallel decomposition
- Sometimes, the fastest way to find a proof is to play one more turn



Local Pattern Analysis

 Just like in base-7: many cases, easy to verify







Base-8 Y: Cases B1, B3

- Cases B1, B3 have the same basic structure
- Same local pattern appears in all three (hence the similar labels)





- Strongest response for white
- Top LP not obvious
- Could use subcases, ect.
- Must-play analysis
 - Faster in this case



- IMPORTANT: analysis of this local pattern must not leave that local pattern
 - cannot make <u>any</u> assumptions about other cells
- Black has two SCs
- White must play in the intersection, or black achieves its goal







- Case D is proven
- I've shown you all the tools you need to analyze Y positions with parallel decomposition
 - (with some help from semi-connections and must-play analysis)



Base-8 Y

- Center is a win
 - Strategy constructed



Future Work: Beyond Base-8

- Want to find proofs on
 - Larger boards
 - Balanced positions
- Parallel decomposition
 still works, but more cases
- Manual proof finding quickly becomes tedious



Future Work: Beyond Base-8

- "Machine assisted proofs"
 - Sufficiently strong players could suggest responses, find virtual connections, etc.
- Hard part: finding a decomposition, or choosing to play another turn (subcases)



Conclusion

- Parallel decomposition: a method of finding proofs
- Evaluated all openings on base-7 Y
 - 2PW strategy with swap rule
- Evaluated one opening on base-8 Y
 - 1PW strategy without swap rule
- Parallel decomposition is useful for analyzing Y

The End



