

Domain-Independent Cost-Optimal Planning in ASP

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Abstract

We investigate the problem of cost-optimal planning in ASP. Current ASP planners can be trivially extended to a cost-optimal one by adding weak constraints, but only for a given makespan (number of steps). It is desirable to have a planner that guarantees global optimality. In this paper, we present two approaches to addressing this problem. First, we show how to engineer a cost-optimal planner composed of two ASP programs running in parallel. Using lessons learned from this, we then develop an entirely new approach to cost-optimal planning, *stepless planning*, which is completely free of makespan. Experiments to compare the two approaches with the only known cost-optimal planner in SAT reveal good potentials for stepless planning in ASP. The paper is under consideration for acceptance in TPLP.

KEYWORDS: Cost-Optimal Planning, Answer Set Programming, CORE-2 ASP Standard.

1 Introduction

Answer Set Programming (ASP) has been celebrated for its elegance and applicability to AI planning (Lifschitz 2002). A planning problem in this context is, given a collection of actions, each with pre-conditions and effects, and fluents which are properties over states, determine whether there is a sequence of actions from an initial state to a final state (Fikes and Nilsson 1971). State-of-the-art ASP planners like PLASP have been developed (Gebser et al. 2011; Dimopoulos et al. 2019).

In cost-optimal planning, each action is associated with a cost, and our objective is to find a plan which minimizes the sum cost of all actions. Any ASP planner can be trivially extended to a cost-optimal planner by adding weak constraints, but only for a given makespan. Such a planner does not guarantee global optimality. Eiter et al. (2003) present an approach to addressing this problem, but make the assumption that a polynomial upper bound on makespan exists and is known in advance.

In the related field of SAT, some work has been done on cost-optimal Partial-Weighted-MaxSat planning with regard to makespan (e.g., (Maratea 2012) and (Chen et al. 2008)). Again, readers should find this somewhat unsatisfactory. After all, finding a plan that is “cost-optimal with regard to makespan” is just a way to sidestep the complication the real problem presents. A makespan is just an internal artifact of the SAT approach to planning. A solution should not depend on the way in which the planner happens to order the actions. Ideally, we want an approach to planning which guarantees a globally optimal solution and makes no mention of makespan.

We are aware of only one existing logic-based approach that tackles this much more difficult problem for SAT planning, the paper by Robinson et al. (2010). Inspired by this work, we pursue a separate investigation into globally cost-optimal planning in ASP and develop a two-threaded planner, one thread being a regular planner and the other an *any-goal* planner. While the former

computes successive decreasing upper bounds of the optimal cost by iteratively increasing the makespan, the latter computes successive increasing lower bounds by planning in a modified environment where some amount of cheating is allowed. We achieve this by forcing the any-goal planner to “make progress” at each time-step. An optimal plan is obtained when the two bounds meet in the middle, i.e., they are guaranteed to agree at some point. However, unlike in Robinson’s approach, through the use of “make-progress” rules, we’re able to develop a planner which guarantees to eventually find an optimal solution (or report no solution) even when the problem contains actions with zero cost.

Using insights gained from this approach, we then develop a new approach to logic-based planning, *stepless planning*. The idea is to first engineer a planner which produces “partially ordered plans” - actions arranged into a graph of dependencies where stable model semantics ensures that the graph is acyclic; then we show how to express the problem in Σ_2^P of “making progress”; and finally, we show a critical component of the planner, the *suffix layer*, which determines how many occurrences of each of the actions and fluents we will need to produce an optimal plan.

We report experiments on our cost-optimal planners on benchmarks of (Robinson et al. 2010) and compare with Robinson’s SAT-based planner. We found that the stepless planner outperformed the other two planners in most domains and the two-threaded planner outperformed the SAT-based planner in most domains.

The paper is organized as follows. Section 2 describes ASP planning translated from SATPlan. Section 3 discusses the problem of no-solution detection and provides solutions. Section 4 extends no-solution detection to optimal planning, with Section 5 adding a *delete-free planner* as a suffix layer to improve the effectiveness. Section 6 gives the two-threaded planner and Section 7 is about the stepless planner. Section 8 reports experiments and Section 9 is on related work and future directions.

We assume that the reader is familiar with STRIPS planning. The ASP encodings in this paper are constructed to run on system CLINGO and generally follow the ASP-Core-2 Standard (Calimeri et al. 2015), except that we adopt two special features provided by CLINGO: (i) we will use `;` to separate rule body atoms since the conventional comma sign `,` is overloaded and has a different meaning in more complex rules, and (ii) the disjunctive head of a rule may be expressed conveniently by a conditional literal.

Parts of this paper have been moved to Appendices, including proofs, encodings, and some technical explorations. For more information, the reader may also want to consult the thesis written by the first author of this paper (Spies 2019).

2 Preliminaries: STRIPS Planning in ASP

We adopt a direct translation of 5 rules of SATPlan (Kautz 2004) into ASP and call the resulting planner ASPPlan.

```
rule 1. holds(F,K) :- goal(F); finalStep(K).
rule 2. happens(A,K-1) : add(A,F), validAct(A,K-1) :- holds(F,K); K > 0.
rule 3. holds(F,K) :- pre(A,F); happens(A,K); validFluent(F,K).
rule 4. :- mutexAct(A,B); happens(A,K); happens(B,K).
rule 5. :- mutex(F,G); holds(F,K); holds(G,K).
```

where $validAct(A, K)$ means that action A can occur at time K and $validFluent(F, K)$ means fluent F can be true at time K .¹ Time-steps used in constructing a plan are also called *layers*.

Rule 1 says that goals hold at the final layer. In rule 2, if a fluent holds at layer K , the disjunction of actions that have that fluent as an effect hold at layer $K - 1$. The next rule says that actions at each layer imply their preconditions. The last two rules are mutex constraints: in rule 4, actions with (directly) conflicting preconditions or effects are mutually exclusive, and in rule 5, the fluents that are inferred to be mutually exclusive are encoded as constraints.

Following SATPlan, we add to our plan “preserving” actions for each fluent. The goal is to simulate the frame axioms by using the existing machinery for having an action add a fluent that gets used some steps later. These preserving actions can be specified as:

```
action(preserve(F)) :- fluent(F).
pre(preserve(F), F) :- fluent(F).
add(preserve(F), F) :- fluent(F).
```

where each fluent F has a corresponding *preserving action* denoted by term $preserve(F)$. Preserving actions can be easily distinguished from regular actions. Now that an action occurs at time K indicates that its add-effect F will hold at time $K + 1$.

Note that the reason why rule 5 of ASPPlan prevents fluents from being deleted before they’re used is a bit subtle. In order for a fluent to hold, it must occur in conjunction with a preserving action at each time-step it’s held for. A preserving action has that fluent as a precondition and so would be mutex with any action that has it as a delete effect. This means that deleting actions cannot occur as long as that fluent is held (by rule 4).²

Like SATPlan, we run this planner by solving at some initial makespan K , where K is the first layer at which $validFluent(F, K)$ holds for all $goal(F)$, and if it is UNSAT, we increment *finalStep* by 1 until we find a plan.

This is a straightforward and unsurprising encoding in every respect, but has a somewhat surprising consequence as compared to SATPlan. Because ASP models are stable, for any fluent F , $holds(F, K)$ can only be true if there exists some action which requires its truth as per rule 3. Similarly for actions as per rule 2. Furthermore, since rule 2 is disjunctive at every step, the set of actions which occurs is a minimal set required to support the fluents at the subsequent step. This conforms exactly to the approach to planning in (Blum and Furst 1997): First build the planning graph, then start from the goal-state planning backwards, at each step selecting a minimal set of actions necessary to add all the preconditions for the current set of actions. That is, in this ASP translation, the neededness-analysis as carried out in (Robinson et al. 2008) is accomplished automatically during grounding or during the search for stable models.

Encoding Reduction: Rule 4 in ASPPlan can blow up in size when grounded because nearly any two actions acting on the same fluent can be considered directly conflicting. For example, imagine a planning problem in which there is a crane which we must use to load boxes onto

¹ Blum and Furst (1997) give a handy way to identify for each action and each fluent, what is the first layer at which this action/fluent might occur by building the *planning graph*. Note that $validAct/2$ and $validFluent/2$ as well as predicates $mutexAct/2$ and $mutex/2$ are all extracted from the planning graph.

² As a further note, when PDDL (planning domain definition language) without any extensions is defined, goals can only be positive and actions can only have positive preconditions. There is a `:negative-preconditions` extension to PDDL, but we didn’t use it. Any problem which uses `:negative-preconditions` can be trivially adapted to avoid using it by adding a fluent `:not-F` for every fluent `:F` and then adding a corresponding add-effect wherever there’s a delete-effect and vice versa.

freighters and there are many boxes and many freighters available but only one crane. Then we will have one such constraint for every two actions of the form, $load(Crate, Freighter)$, for any crate and any freighter. As there is already a quadratic number of actions in the problem description size, the number of mutex constraints over *pairs* of actions is *quartic* in the initial problem description size.

We would like to avoid such an explosion by introducing new predicates to keep the problem size down. We will only consider two actions to be mutex if one deletes the other’s precondition. But we will take extra steps to ensure that no add-effect is later used if the same fluent is also deleted at that step. Here is the revised encoding of rule 4.

```
used_preserved(F,K) :- happens(A,K); pre(A,F); not del(A,F).
deleted_unused(F,K) :- happens(A,K); del(A,F); not pre(A,F).
:- {used_preserved(F,K); deleted_unused(F,K);
    happens(A,K) : pre(A,F), del(A,F)} > 1; valid_at(F,K).

deleted(F,K) :- happens(A,K); del(A,F).
:- holds(F,K); deleted(F,K-1).
```

Effectively, we are splitting the ways in which we care that an action A can relate to a fluent F into three different cases: (i) A has F as a precondition, but not a delete-effect; (ii) A has F as a delete-effect, but not a precondition; and (iii) A has F as both a precondition and a delete-effect.

By explicitly creating two new predicates for properties (i) and (ii), we have packed this restriction into one big cardinality constraint. Further, we must account for conflicting effects, so we define one more predicate (`deleted/2`) which encapsulates the union of all actions from properties (ii) and (iii) (those that delete F) and assert that F cannot hold at this step if any of those actions occurred in the previous one.

Note that the formulation in (Blum and Furst 1997) will not allow two actions A and B to ever happen at the same time-step K if they have “conflicting effects” (one adds a fluent F and the other deletes the same fluent). Our encoding allows this, but only in cases where $holds(F,K)$ is false. Except for this technicality, the two are otherwise equivalent. A detailed justification is presented in Appendix B.

3 Planning with No-Solution Detection

From a theoretical standpoint, let us consider why cost-optimal planning is such a difficult problem. When the planner terminates with a plan of cost c , it is additionally asserting “I have proved there does not exist a plan of cost $c - 1$ ”. But here we immediately have a problem because all the planners we have written so far in ASP are not actually *planners* in the sense of (Blum and Furst 1997); they cannot identify when a problem has no solution. If there is no solution our ASP planners will simply march on forever searching for one until somebody kills the process. So before we can create a cost-optimal planner, we must first create a (normal) planner which can determine if a problem has no solution.

Planning is PSPACE-complete. What makes planning decidable is, of course, the finite state space. Any plan which goes on too long will eventually visit some state twice, so we only need to search for a plan among those that never revisit the same state. We can, of course, produce a naive upper bound on the number of possible states by taking $n = 2^{|fluents|}$ and then terminate the search after n steps, but let us try to do better. Indeed, a main technical innovation of this work

is the development of ASP-based decision procedures for planning, which can potentially prove the unsatisfiability of a planning problem much earlier than when taking the theoretical number of states as upper bound for the planning horizon.

First, instead of requiring goals to hold at final step (rule 1 of Section 2), let us say

```
{holds(F,K)} :- fluent(F); finalStep(K).
```

By using a choice rule here, the planner can now choose any goal it wants and then plan towards that goal. This makes our instance always satisfiable (just produce any valid sequence of actions, then take the end-state and claim that was your goal).

Now here comes the “we must make progress” rule. We’ll refer to this as the “layered make-progress” rule (which also includes its strengthening to be discussed in the next section) to distinguish it from the “stepless make-progress rule” for stepless planning.

```
:- not holds(F,K) : not holds(F,J), fluent(F); step(J); step(K); J < K.
```

In English: “For any time-step pair J and K where $J < K$, we cannot allow that every fluent F which does not hold at J also does not hold at K .” That is, any sequence containing time-steps (J, K) ($J < K$) where the state of K is a subset of that of J would fail this rule.

Thus, the rule guarantees that there exists a makespan n for which our planning instance is UNSAT. This is because we are now enforcing that the state must change at every time-step to take on some value which it did not have in any previous time-step. But if there are only m reachable states in our planning instance, then for all $n > m$ this is clearly impossible.

Hence, we can build a complete planner by running two separate computations in parallel. The first is our usual ASP planner which increases the makespan until it finds an instance for which the solver finds a plan. The second is our “any-goal” planner which increases the makespan until it finds an instance for which the solver returns UNSAT, at which point we record the previous makespan as *maxlength*. Once that is done, if the first instance manages to reach *maxlength* and report UNSAT, we can safely claim to have proved that no plan exists and terminate the solver.

3.1 Stronger Notions of Progress

Unfortunately, there exist problems that contain many independent variables which may be separately manipulated to generate a large easily-traversable state-space. For such problems, our solver above can produce long plans which idly “flip bits” to avoid repeating themselves.

To make this scenario more concrete, imagine that we take any unsolvable planning problem and adjoin to it a binary counter with one hundred two-state switches. In addition to the actions from the original problem, we also have two hundred actions which independently flip each of the switches in the counter (either from 0 to 1 or from 1 to 0). Even though this counter has no impact on the problem itself, it suffices to increase the length of the longest plan by a factor of 2^{100} because for every state in the longest path, we can flip through all possible arrangements of these switches before proceeding to the next state. This easily puts the possibility of solving the problem out of reach whenever there is no solution.

One way to deal with this is to somehow encode into our planner the knowledge that the longest possible time it can take to iterate over the possible states of two independent subproblems is the maximum of the respective longest times rather than the product. One attempt at this is given in Appendix A, where we reduce the length of the longest plan in the above example to 100.

We want to do better though and we can. We are able to formulate a stronger definition of

“make progress”, which we conjecture perfectly defeats the *independent parts* problem in all its forms. First, a definition.

Definition 3.1

A *partially-ordered plan* is a transitive directed acyclic graph G (equivalently, a partial ordering) of “action occurrences” such that all topological sorts of G are valid sequential plans.

Starting with any sequential plan S , we can generalize it to its canonical partially-ordered plan as follows. If a precedes b in S , then we will say $a \prec b$ for actions a and b (adding an edge from a to b) iff any of the following holds:

1. a adds some fluent which is used as a precondition for b
2. b deletes some fluent which is used as a precondition for a (and $a \neq b$)
3. a adds a fluent which b deletes
4. a deletes some fluent and b adds the same fluent
5. a and b are different instances of the same action³
6. There exists an action c such that $a \prec c$ and $c \prec b$

Note that these rules only apply to a - b pairs for which a precedes b in S (otherwise, the statements 3 and 4 above would appear to be contradictory).

If we add a source and sink s and t respectively to any partially-ordered plan such that for all actions $s \prec a \prec t$, we can consider any s - t cut x as a generalized “intermediate state” for this plan. To see this, take any ordering where the s -side actions all precede the t -side actions and look at what fluents hold after we have taken only the s -side actions. Let us call this state x -state.

Here comes the strongest possible (domain-independent) definition of “make progress” that we can think of. The idea is that for any set of actions identified as being sandwiched in between two cuts, they must make progress by turning some fluent to true.

Definition 3.2

A partially-ordered plan is *strongly minimal* iff, given any two s - t cuts x and y , if there exists any t -side action in x which is an s -side action in y , then there must be some fluent which is true in y -state but not in x -state. We similarly call a sequential plan strongly minimal if its canonical partially-ordered plan is.

An action is said to make progress if no two cuts exist on either side of the action without this property (that some new fluent occurs between them). In a strongly minimal plan, *all* actions make progress.

This beautifully handles the one hundred 3-state switch scenario by forcing us, for each switch w , to consider the generalized intermediate state where all w -flips happen before anything else. Thus if w is flipped to the same state twice we can produce the cuts at each of those states demonstrating that this plan is not minimal. The ASP encoding of this rule is complicated, and so will be relegated to Section 7 when we apply it to stepless planning.⁴ This is the “stepless make-progress rule”. In our two-threaded planner introduced next, we have chosen to implement the layered make-progress rule.

³ Actually, this rule is not strictly necessary as any *minimal* plan which satisfies the other five rules *will also* satisfy it. By keeping it, we don’t have to worry about discussing *action occurrences* until later when we actually start digging into the stepless planner.

⁴ The same technique *can* with some effort be encoded for layered planners as well. Essentially, it requires quite a bit of boilerplate in order to talk about *next* and *previous* occurrences of each fluent and action.

4 Extending No-Solution Detection to Cost-Optimality Detection

Now, let us add weak constraints for action costs to ASPPlan (called *MinASPPlan*⁵) as well as to the any-goal solver so that a plan with the least action cost can be identified by each solver for a given makespan. We run MinASPPlan and the any-goal solver in parallel.

Once the MinASPPlan solver produces a plan of cost C , we treat C as an upper-bound on the cost of an optimal plan. Then we tell the any-goal solver to only search for non-repeating plans of cost $< C$. When the any-goal solver terminates with UNSAT at some time-step n we claim that all non-repeating plans with cost $< C$ have a makespan $< n$ which means we can stop the MinASPPlan solver after finishing with makespan $n - 1$. Meanwhile, if the any-goal solver gives back a minimum-cost plan for layer n ,⁶ then that cost *is a lower bound* on the optimal cost of any plan with makespan at least n . Thus, by increasing makespan, MinASPPlan computes successive lower (i.e., non-increasing) upper bounds of optimal cost and the any-goal solver computes successive increasing lower bounds of the optimal cost; when the two meet in the middle, an optimal plan is identified.

Let us summarize what we have so far. We have two threads iteratively solve successively larger instances. We will name them I and II in deference to their similarity to Robinson’s (Robinson et al. 2010) Variant-I and Variant-II encodings.

- I is MinASPPlan; and
- II is the any-goal solver augmented by weak constraints by action costs. It is similar to I except for two major differences: (i) in place of the goal conditions, II is allowed to choose its own goal, and (ii) II is given some notion of progress together with the constraint that it *must make progress* at every time-step.

How we determine when to stop depends on which solver (I or II) lags behind. I’s result costs are monotonically nonincreasing while II’s result costs are monotonically nondecreasing. If I lags behind, then as soon as I’s lowest cost is \leq the II’s cost for the layer it is currently trying to solve, we can stop and report the solution at that layer as optimal. If II lags behind, then as soon as its cost for some layer is \geq the best known I-cost so far (at *any* layer), we can stop.

The asymmetry in deciding when to stop happens because of the types of bounds I and II produce. I will never produce a cost $D < C$ if C is the optimal cost, but II will continually increase its lower bound eventually marching straight past C and on to infinity (the point at which it returns no solution). This is why we must take into account the layer at which each lower bound was produced when determining if we are done, but we do not care what layer the upper bound was produced at.

5 Delete-Free Planning

The lower bounds produced by the Variant-II solver can be improved by adding a *suffix layer*, which is a *delete-free planner*. Delete-free planning (DFP) is a special case of planning which happens to be in NP. These are the planning problems without delete-effects. Surprisingly, DFP can be modeled as a graph problem.

Given a directed bipartite graph $G = (X, Y, E)$ with weights on X and a goal set $Y_F \subseteq Y$, find a minimum *acyclic* subgraph $G^* = (X^*, Y^*, E^*)$ such that

⁵ Besides ASPPlan discussed here, it also includes a smart encoding of mutex constraints (cf. Chapter 6 of (Spies 2019)).

⁶ Note that make-progress rules are crucial - without them, the minimum cost will always be zero.

1. $Y_F \subseteq Y^*$
2. If $x \in X^*$ and $(y, x) \in E$, then $y \in Y^*$ and $(y, x) \in E^*$
3. For all $y \in Y^*$, E^* contains at least one edge (x, y) (and $x \in X^*$).

Connection to DFP: X is the set of actions, and Y is the set of fluents, the (x, y) edges are add-effects and the (y, x) edges are preconditions. Y_F is the goal set and the initial set has been removed (together with all corresponding preconditions) from the graph. Rule 1 means the goal fluents must be true. Rule 2 means an action implies its preconditions. Rule 3 means every fluent must have a causing action. The graph must be acyclic to ensure the actions can occur in some order. This is possible because there is no incentive to ever take an action or cause a fluent more than once. As soon as any fluent is true, it is permanently true.

We now can encode DFP in ASP as solving the above graph problem *independent of makespan*. Its encoding in ASP can be found in Appendix C.

The key takeaway is that the encoding is an efficient “one-shot” encoding in ASP. Rather than structuring the problem into layers and then iteratively increasing the makespan until a solution is found, we eschew layers entirely and encode the problem as a single ASP instance. This is similar to how (unlike with SAT encodings) Hamiltonian Path can be encoded one-shot in ASP (Spies 2019) without needing numeric fluents or layers or quadratic space when grounded. The problem of ensuring that an acyclic structure exists is solved by the stable-model semantics.

This may be considered the most novel contribution of this paper. Besides for delete-free planning (and the suffix layer in the next section), the same trick will also be used later for “Stepless Planning” where we arrange actions and fluents into a graph and rely on stable-model semantics to ensure that the graph is acyclic. Without the ability to do this, other encodings of planning problems are forced to rely on either using numeric fluents (not compatible with SAT-based techniques) or structuring the problem into layers (which multiplies the grounded size of the problem by the number of layers needed).

6 A Two-Threaded Cost-Optimal Planner with Suffix Layer

As with A^* -search, we can generate successively better lower bounds by planning normally from the starting state Q to some intermediate state S chosen by the planner and then finding the minimum-cost solution to the delete relaxation for the planning problem from S to the goal state.⁷ This suggests a natural way to modify our Variant-II encoding in Section 4 to find better lower bounds. We append a “suffix layer” at the end, which must generate a plan in the delete relaxation of the problem from the chosen any-goal state to the actual goal state. The costs for any actions taken in the suffix layer must be added to the total cost of our plan. Indeed, in many cases this produces a remarkable lower bound.

We now give a complete description of our two-threaded planner. We have two ASP programs running in parallel. One is the Variant-I standard ASPPlan solver with weak constraints for action costs. The other is the Variant-II solver with a progress rule and appended a suffix layer.

- Both solvers independently run successively on makespan 0, 1, 2 etc. until we kill them.
- Each time the Variant-I solver begins solving a new makespan, we update the current makespan being solved for.

⁷ More precisely, we encode in ASP the problem of finding the minimum *total* cost across all possible subgoal states of $cost(normalplan) + cost(suffixplan)$ (given that the normal plan respects whichever progress rule we choose to employ)

- When the Variant-I solver finds a plan, we record the plan and its cost if this is the lowest-cost plan found so far.
- When the Variant-II solver finds an optimal plan for some makespan using the suffix layer, we record the optimal cost as a lower bound for that makespan (as well as all larger makespans).
- If the Variant-II solver ever finds an optimal plan which *doesn't* use the suffix layer, then that plan is globally optimal. We can return it as a solution and entirely ignore the Variant-I solver (this only happened twice in all of our experiments and seems to be fairly unlikely).
- If the Variant-II solver obtains UNSAT for a layer, we can stop running it and record the cost of that and all future layers as ∞ .
- Any time the best-cost plan found so far (by the Variant-I solver) is no greater than the Variant-II lower bound for the currently-solving layer or *any* earlier layer, we can stop both solvers and report that plan as an optimal solution.
- Any time the Variant-I solver is solving for a makespan whose Variant-II lower bound is ∞ , we stop the solver and return the best-cost plan found so far or “no solution” if no plan has been found.

The correctness of our two-threaded planner depends on the correctness of two component solvers. While this is straightforward for Variant-I solver, we have the following claim for Variant-II solver.

Theorem 6.1

Let Q be a planning problem. Assume that we are using the layered ‘make-progress’ rule.

- (Variant-II Soundness) If Q has an optimal solution P with cost C and makespan n , then for any $k \leq n$, at makespan k the Variant-II solver will find a relaxed plan with cost $\leq C$.
- (Variant-II Completeness) If Q has no solution, then the Variant-II solver will eventually produce an UNSAT instance.

Intuitively, the completeness is due to non-repetition of states enforced by the ‘make-progress’ rule, and the soundness follows from the fact that if there exists a plan, then it can be “reduced to a plan which satisfies the ‘make progress’ rule. Furthermore, it can be truncated at any lower makespan to a partial plan which satisfies the ‘make progress’ rule. A more detailed argument (and all proofs of the claims of this paper) can be found in Appendix B.

7 Planning without Layers: Stepless Planning

Besides the delete-free planner above (which is only useful for delete-free planning problems), all the planners so far in this paper (and indeed, all SAT/ASP planners that we have encountered) have used layers to order the actions and fluents that occur within a planning problem. But let us consider the notion of *partially ordered plan* from Definition 3.1, where no layers are specified. Any topological sort of this graph corresponds to a valid plan. Perhaps we could avoid layers entirely and embed action-dependencies directly. The idea here is that, just as with delete-free planning, we can create a plan by specifying only which actions and which fluents hold, and we will rely on stable model semantics to ensure that the resulting solution graph is acyclic.

As was mentioned in Section 5 this is the most novel contribution of this paper. We use stable-model semantics, rather than layers, to produce an acyclic plan.

There is a key difference between delete-free planning and full stepless planning though, which

accounts for the distinction in computational complexity. In the case of delete-free planning, no fluent holds more than once and no action occurs more than once. In stepless planning, it is possible for an action to occur multiple times. As such, we will have to have separate atoms in our encoding representing each *occurrence* of an action. But prior to solving, we don't know how many occurrences of each action or fluent will be needed.

Here, we will first present a solver that *assumes* it has enough occurrences and then we will come back to the issue of figuring out how many of each are needed in order to produce an optimal plan. The stepless planner is *significantly* more complicated than anything else done in this paper so we put more care into explaining what each line of ASP code does, but we will have to do it in an appendix (Appendix D). Here we provide an outline of the planner. Additionally, since no planner like this has ever been built before, we will take more care to try and bridge the gap between the standard approach to planning and the approach being presented here.

7.1 Stepless Planner Encoding

To avoid an $O(|\text{actions}|^2)$ -size encoding, we don't directly encode dependencies between actions. Instead we use the fluents as intermediate nodes in the solution graph.

An *occurrence* of a fluent F will be encoded as an object in an atom, $\text{fluentOcc}(F, M)$, where M is a sequentially-ordered index. $M = 0$ is reserved for the initial fluents. All others start at $M = 1$ (when caused by some action). Similarly, the object $\text{actOcc}(A, N)$ indicates an occurrence of action A .⁸ In stepless planning, there are no preserving actions since there are no layers to preserve things across, and we don't utilize mutex relationships between objects. Whereas in our previous encodings the causes and destroyers of each fluent were implicit, here we must explicitly give which occurrence of which action AO causes which fluent occurrence FO to hold ($\text{causes}(AO, FO)$) and which occurrence of which fluent FO is used as a precondition for which action occurrence AO ($\text{permits}(FO, AO)$). Additionally, we need an atom for each deleted fluent occurrence FO which action occurrence AO has as a precondition and deletes ($\text{deletes}(AO, FO)$) and also one in the rare case that an action has a fluent as a delete-effect, but not a precondition, for which occurrence of the fluent F the action occurrence AO follows ($\text{follows}(AO, FO)$). (Refer to Appendix D under the subtitle *Problem Description*.)

From this we can structure the graph and assert that it is acyclic. For each action occurrence we have an "event"; additionally there is an event for the start and end of each occurrence of each fluent. There is also an event "goal" which corresponds to the goal state being reached.

Events are grouped into *vertices* in our graph each of which contains at most one action occurrence. When an action occurrence *causes* a fluent, the action and the start of that fluent belong to the same vertex. Similarly when it *deletes* a fluent, the action and the end of the fluent belong to the same vertex. To encode this we use the predicate $\text{inVertex}/2$ which indicates that its first argument belongs to the vertex named by the second argument. (Refer to Appendix D under the subtitle *Plan Event Graph*.⁹)

⁸ We index action and fluent occurrences with numbers N and M and have symmetry-breaking rules ensuring that the occurrences happen in numerical order for a given action or fluent, but it is important to understand that these numbers are *not* layers. There's no global *step* of any kind to which they correspond. A fluent occurrence can be used as a precondition for an appropriate action occurrence regardless of what their indices are or how they relate to each other. The same goes for an action *causing* a fluent. The indices are simply to be able to distinguish between multiple occurrences of the same object; they have no global significance or relation to any other object.

⁹ We have removed the goal event from the encoding in the appendix since it is incompatible with the suffix layer. To see the original encoding refer to (Spies 2019).

7.2 Making Stepless Progress with a Suffix Layer

Now we need a way to assert that the action occurrences of a given stepless plan “make progress”. With no layers to make assertions about, the only notion of progress we are left with is the definition (Def. 3.2) of a plan which is *strongly minimal*. This definition logically takes the form of “there does not exist a pair of (s,t) -cuts such that ...” This means that *given* a particular plan, determining whether it is strongly minimal is likely co-NP-complete (membership is straightforward but the hardness is an open conjecture), and then the problem of *determining the existence of* such a plan for a given collection of atoms and fluents could possibly be Σ_2^P -complete. Luckily, ASP gives us a way to encode problems in Σ_2^P through the use of *disjunctive rules* (Baral 2003). The code for this can be found in Appendix D under the subtitle *Strong Minimality*.

If there aren’t enough occurrences of a fluent or action, we can tack on a suffix layer in the same way we did with the stepped-cost-optimal planner. In the code in Appendix D under the subtitle *Suffix Layer*, we replace all uses of *goal* with a *subgoal* which is the entry-point into the suffix layer. The coding is similar to the suffix layer used in the two-threaded planner, but there are a few key differences. First, if the suffix layer is used at all, we use an atom *useSuffix* to indicate that this is true. There is a cost of 1 at level -1 for *useSuffix* so among plans of equal cost, the solver will prefer one which *doesn’t* use the suffix to one which does. If an optimal solution doesn’t use the suffix, then it must be globally optimal with respect to cost. (Refer to Appendix D under subtitle *Suffix Layer*.)

Finally, we will add rules to enforce the use of action and fluent occurrences from our bag so that the planner resorts to the suffix layer only when it “runs out” of something. With this we know how to expand our bag of occurrences. Each time we get back a plan making use of the suffix layer, look at all the fluents or actions which were saturated by that plan and add another occurrence of each one. (Refer to Appendix D under subtitle *Saturated*.)

This, coupled with our definition of making progress, is what guarantees that it will eventually find a plan or determine that none exists. The suffix layer is only used because the planner ran out of something it needed and needs to request more of that item from the controlling program (in particular, not as a way to save on plan cost).

Appendix E provides a detailed example of running the stepless planner, and Appendix B gives a proof of the following theorem.

Theorem 7.1

- (Stepless Soundness) All plan costs produced by the stepless planner are lower bounds on the cost of the true optimal plan.
- (Stepless Completeness) The stepless planner will eventually find the solution if it exists or produce an UNSAT instance if it doesn’t.

8 Experiments

We ran our cost-optimal two-threaded solver and stepless solver on most of the same instances as Robinson (Robinson et al. 2010) and here report results¹⁰.

Experiments were run on a cluster of *c3.large* Amazon EC2 instances each with two Intel

¹⁰ excluding *satellites* since our planner doesn’t support the *:equality* extension to PDDL and *miconic* since we couldn’t find the problem files for it. Robinson was kind enough to send us the instances from his constructed *ftb* domain so we can report performance on that as well

Table 1. Experiments with two-threaded planner and stepless planner

Problem	C^*	n	n_*	t_π	t_*	$t_{2\text{-threaded}}$	n_s	t_{stepless}	t_s	t_{Robinson}
block-12	20	20	17	0.5	1203.4	1203.9	-	-	-	?
block-15	16	16	12	0.4	113.4	113.8	7	89.4	33.7	?
block-18	26	26	16	0.9	256.8	257.7	-	-	-	3.2
block-23	30	-	-	-	-	-	-	-	-	29.8
block-25	34	-	-	-	-	-	-	-	-	27.4
depots-2	15	-	12	-	771.5	771.5	2	9.7	4.2	-
depots-13	25	-	-	-	-	-	3	475.9	137.2	-
driverlog-2	19	-	-	-	-	-	20	215.5	44.4	-
driverlog-3	12	7	3	0.1	0.9	1.0	1	0.4	0.4	450.2
driverlog-11	19	-	-	-	-	-	1	13.5	13.5	-
elevators-2	26	3	0	0.4	1.7	2.1	1	2.7	2.7	13.0
freecell-3	18	-	-	-	-	-	2	420.5	344.0	-
ftb-30	1001	25	0	1.8	0.3	2.1	1	5.5	5.5	1.8
ftb-38	601	33	0	2.7	0.2	2.9	1	3.2	3.2	1.5
ftb-39	801	33	0	3.9	0.3	4.2	1	5.6	5.6	2.2
ftb-40	1001	33	0	3.9	0.4	4.3	1	8.2	8.2	?
gripper-1	11	7	4	0.1	0.4	0.5	2	0.4	0.2	14.6
gripper-2	17	11	8	0.6	312.4	313.0	7	23.5	9.7	-
pegsol-9	5	15	11	3.9	35.9	39.8	5	131.5	46.6	386.8
pegsol-16	8	21	17	48.3	1029.0	1509.3	10	910.2	280.8	-
pegsol-18	7	-	-	-	-	-	7	1548.0	537.1	-
rovers-3	11	7	4	0.1	0.2	0.3	1	0.1	0.1	49.4
rovers-4	8	4	0	0.0	0.0	0.0	1	0.1	0.1	?
rovers-6	36	-	-	-	-	-	48	1354.3	391.0	-
rovers-9	31	-	-	-	-	-	53	1040.6	101.4	-
rovers-14	28	-	-	-	-	-	72	900.7	55.9	-
storage-7	14	14	11	0.6	42.9	43.5	10	89.2	42.4	1.1
storage-8	13	-	-	-	-	-	15	799.1	239.5	-
storage-9	11	-	-	-	-	-	9	181.0	46.0	-
storage-13	18	-	-	-	-	-	-	-	-	244.0
TPP-5	19	7	2	0.1	0.2	0.3	2	0.5	0.3	-
TPP-7	34	-	-	-	-	-	13	189.6	32.4	-
transport-1	54	5	0	0.1	0.1	0.2	2	0.5	0.3	0.2
transport-2	131	12	4	74.3	55.1	129.4	2	111.6	106.3	-
transport-11	456	9	3	0.3	1.6	1.9	2	163.4	151.4	-
transport-21	478	7	1	0.2	0.6	0.8	2	5.2	3.5	-
zenotravel-4	8	7	3	0.5	2.8	3.3	3	14.1	6.9	783.4
zenotravel-6	11	7	0	7.2	6.5	13.7	1	2.1	2.1	-
zenotravel-10	22	-	-	-	-	-	1	1387.1	1387.1	-

Xeon 2.8 GHz CPU cores and 3.75 GB of memory. We used GNU Parallel (Tange 2018) to distribute the work of running multiple instances.

For comparison, we include Robinson’s reported results scaled down by a factor of $\frac{2.6}{2.8}$ to account for the difference in processor speeds.

For each domain, we report the largest instance solved by each of the two-threaded planner, the stepless planner, and Robinson’s planner where largest is measured by the amount of time it took that planner to solve the instance. Where it differs, we also report the largest-indexed instance solved by each of the two-thread and stepless planners.

Every plan produced by either planner was validated by the Strathclyde Planning Group plan verifier VAL (Howey et al. 2004).

The column C^* is the optimal cost found for each instance. In all cases the optimal cost for the two-thread planner agrees with the optimal cost reported by Robinson (Robinson et al. 2010) where applicable (Robinson compares his results against a *non*-SAT-based planner and our optimal costs agrees with that as well).

The column n is the lowest makespan at which the problem has a C^* plan (according to our

Variant-I solver). Our value for the makespan n agreed with all of Robinson’s reported results except for Rovers-3 where we found we only needed a makespan of 7 to produce the optimal plan while Robinson reported a required makespan of 8. We suspect this is because the definition of mutex of (Blum and Furst 1997) is overly restrictive for actions (cf. the footnote on page 3).

n_* is the makespan at which our Variant-II suffix solver proves C^* is optimal. Interestingly, for many of instances this value was 0 which indicates that the optimal plan in the delete-free reduction of the problem has the same cost as the true optimal plan.

t_π is the time required to find the plan (by our Variant-I solver); t_* is the time required to prove optimality (by our Variant-II suffix solver); and $t_{2-threaded}$ is the sum of these two numbers (can be thought of as “total solve time” although the algorithm necessitates that they run in parallel, so the actual wall-clock time required to run them was the maximum, not the sum, but with two CPU cores rather than one). All reported times are measured in seconds.

n_s is the number of times the stepless solver was run for this instance (each time adding more items to its bag of fluents and actions based on what was saturated in the previous rounds). On the last of these runs it produced an optimal solution which doesn’t use the suffix layer and hence is globally cost-optimal. t_s is the total time running the stepless solver across all runs.

One important distinction between the experiments run with the two-threaded solver and those run with the stepless solver is that the two-threaded solver took advantage of “iterative” solving. That is, CLINGO provides an API for interacting with it programatically. Rather than restarting from scratch each time there is a new instance to be solved, we can after observing the solution at makespan k , make some adjustments to the instance so that it now represents the program for makespan $k + 1$ and then ask CLINGO to *continue* solving from this point while maintaining any learnt clauses which are still relevant.

This is incredibly powerful and resulted in a major speedup in the two-threaded solver.

For the stepless solver this was not possible since it relies heavily on full-program-spanning loop constraints to give correct results, but CLINGO doesn’t support having loop constraints cross multiple iterative stages. Thus, every time new fluents or actions are added to the stepless solver’s bag, it starts solving from scratch. In the future we hope CLINGO can support this, but they have no plans to do so at this time.

l_s is the total time required for the last iteration of the stepless solver to run. This one run by itself is sufficient to both find the globally optimal solution *and* prove its optimality. However we know of no more efficient way to find the right bag of actions and fluents in order to guarantee the optimal solution won’t use the suffix layer. This number is still interesting in that it provides a lower bound on the time it would take to solve the instance if CLINGO supported loop constraints crossing program section boundaries (so that we could add more occurrences and continue solving rather than having to restart). It gives us some idea of what savings such a modification to CLINGO might provide.

$t_{Robinson}$ is the total time reported by (Robinson et al. 2010) to find the optimal solution scaled by a factor of $\frac{2.6}{2.8}$. A question mark ? in this column indicates the time is unknown since it’s not reported in (Robinson et al. 2010). If the solver for which this row is maximal successfully solved the largest instance reported by Robinson in this domain and found this instance to be larger, we fill with a dash mark – rather than ? in this column (our best guess as to whether Robinson’s planner solved it). A – in any other column indicates the relevant planner failed to solve the instance in less than 1671.4 seconds (30 minutes scaled down by $\frac{2.6}{2.8}$). In the case of depots-2, the Variant-II suffix solver reached layer 12 before the Variant-I MinASPPlan solver and so it found an optimal no-suffix solution by itself.

All instances were solved with CLINGO version 5.2.3. The controller logic for both the two-threaded solver (handling of incremental solving, coordinating the two solvers, and figuring out when to terminate the search) and the stepless solver (figuring out which occurrences to include and topologically sorting the output) was written in Haskell using the `clingo-Haskell` bindings written by `tsahyt` (GitHub alias) to communicate with CLINGO .

We used the default configuration and options for CLINGO except that the stepless planner used the `-opt-usc` option which finds optimal solutions by expanding an unsatisfiable core (Definition 2 in (Alviano et al. 2015)).

All planners presented here are available on GitHub at <https://github.com/davidspies/asplan2>. Feel free to contact the repo owner (the first author of this paper) for any help with reproducing these results.

In all domains except for *blocks*, *ftb*, and *storage*, our two-threaded solver outperformed Robinson’s SAT-based solver and our stepless solver outperformed both (in terms of number of instances solved). In the case of *storage*, the stepless solver and Robinson’s solver each solved an instance which the other failed to solve which seems to point to the possibility that the stepless solver encounters different difficulties from a more traditional approach. One more piece of evidence favoring this conclusion is that the toy example bridge-crossing problem from Appendix E required a full 30 seconds to solve (whereas the 2-threaded solver solves it in 2 seconds) and in general we found that on small/toy problems the stepless solver’s performance is abysmal compared with other approaches we tried but scales better with larger instances.

Prior to running the full suite of experiments, the above observation gave us the mistaken impression that the stepless solver was interesting as a theoretical oddity, but fails to produce decent results in practice, since for every example we ran it on while tuning it, it seemed to run slower than the two-threaded solver. It was a pleasant surprise to discover when officially running the experiments that in fact the inverse was true.

9 Related Work and Final Remarks

Our 2-threaded solver algorithm is inspired by the approach of (Robinson et al. 2010). The better performance of our planner, besides solver technologies, seems partly due to the grounding size and search space pruning under stable model semantics as commented in Section 2, and partly due to less clustered encoding in ASP than in SAT, plus the smart encoding of mutex constraints. But note that their approach works only if we assume the problem is solvable (which he does) and all actions have positive (nonzero) costs (which he also does).

Eiter et al. (2003) propose an approach to finding globally cost-optimal plans in ASP with action costs, but confine their discussion to the planning problems which look for *polynomial length plans*. They make the assumption that the planning domain has some polynomial upper bound on plan lengths which is known in advance. In contrast, we do not make such an assumption. It is interesting to note that through some key technical innovations, we are able to show that the current ASP techniques are capable of encoding cost-optimal planners without this assumption.

Partially-ordered plans have been explored elsewhere. One is the CPT planner (Vidal and Geffner 2006) for optimal temporal planning using Constraint Programming (CP), where actions have *durations* and *makespan* refers to total duration which corresponds to the cost of a plan in our setting. Optimality here means minimal duration. CPT consists of pre-processing that induces lower bounds to be used in starting makespan and in formulation of constraints, a branching scheme, and a CP-based branch and bound search. The branching scheme is specifically designed for temporal

planning aiming for increased reasoning efficiency. The current makespan B increases by 1 if no plan is found. Thus, the first plan found is guaranteed to be duration minimal. CPT dynamically generates *action tokens* from *action types* achieving the similar goal of the suffix layer of our stepless planner. Thus, a main difference between CPT and our stepless planner is no-solution detection in our case and the lack of it in CPT, i.e., the current CPT does not terminate on its own for UNSAT instances.

Delete-free planning has been investigated as a stand-alone topic, including a CP solution (Barták et al. 2012). To the best of our knowledge, our modeling of delete-free planning as a graph problem is original, and it leads to a five-line ASP program which does everything (cf. Appendix C).

The *Madagascar Planner* is a family of efficient implementations of the SAT based techniques for planning. The main idea is, instead of using the standard decision heuristics such as VSIDS, planning-specific variable selection heuristics are applied (Rintanen 2012). One would expect that the same idea can work for ASP-based planning, and in this case, our 2-threaded cost-optimal planner can benefit from it directly.

Cost-optimal planners can also be built on the SAS^+ platform. An SAS^+ planner based on greedy selection of a subset of heuristics for guiding A^* search (Lelis et al. 2016) has made to the top tier in IPC-2018. SAS^+ planning can be encoded in SAT and ASP as well, but the most critical component, the selection algorithm, needs to be implemented by an external program.

Though the goal of this paper is limited to ASP-based cost-optimal planning, there is always a question of whether such a planner is competitive in efficiency (in terms of solving time) with state-of-the-art planners, e.g., the top planners from IPC. Further investigation and experimentation are needed to address such questions. A major advantage of ASP-based planners is the succinctness and elegance of the encoding. An expressive KR language like ASP provides some unique advantages, e.g., determining the existence of a plan that satisfies the stepless make-progress rule is likely Σ_2^P -hard, which can be a challenge for other KR languages.

Stepless planning is a brand new approach to logic-based planning and brings with it a lot of unknowns and potentials for future directions. One issue is that the lack of any notion of simultaneity makes certain standard optimizations difficult, such as incorporating mutex constraints and supporting *conditional-effects* (an extension to STRIPS planning). The latter extension has been realized in SAT-based planning (Rintanen 2011), but incorporating it to stepless planning appears to be non-trivial. Our stepless planner is a nontrivial application that requires support-edges cycles to extend across different program sections, it would be nice if CLINGO supported iterative solving with this.

More recently, no-solution detection for planners has become an interesting topic, along with the competition called Unsolvability IPC, which aims to test classical automated planners to detect when a planning task has no solution. Our no-solution techniques presented in this paper may be relevant. This is one interesting future direction.

Finally, *property directed reachability* (PDR), a promising method for deciding reachability in symbolically represented transition systems, which was originally conceived as a model checking algorithm for hardware circuits, has recently been related to planning (Suda 2014). The relationship with our stepless planner deserves a further study; in particular, an interesting question is whether and how PDR-based planners can be strengthened to become cost-optimal.

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Appendix A Actions Happen As Soon As Possible

Here, we give the rule that says that actions always happen as soon as it is possible, but we must be careful. There are quite a few things which might prevent an action from occurring any sooner. If we leave any out, we risk rendering the problem unsolvable. For an action to be able to occur at the previous time-step, its preconditions must hold at the previous time-step, its delete-effects should not be used at the previous time-step, and its used add-effects should not be deleted at the previous time-step. There are a few other conditions which at first appear to be necessary (such as its preconditions must not be deleted at the previous time-step), but upon further consideration you may notice that all of these are redundant if our goal is specifically to prevent the action from occurring *at the current time-step*. We must borrow our definition of `deleted/2` from the modified encoding of rule 4 in Section 2 of the paper (see **Encoding Reduction** of that section) and additionally add a similar definition for `used/2`.

```
deleted(F,K) :- happens(A,K); del(A F).
used(F,K) :- happens(A,K); pre(A,F); not preserving(A).

:- happens(A,K); K > 0; not preserving(A);
   holds(F,K-1) : pre(A,F); not used(F,K-1) : del(A,F);
   not deleted(F,K-1) : add(A,F), holds(F,K).
```

How does this defeat the 100-switch-scenario? Remember that the 100 switches exponentially increased the plan length because the planner may choose to flip some switches but not others to achieve one state, but then flip those other switches later to achieve an alternative state.

For every switch this rule boils down to, “if we want to flip switch i at time t , then we must also flip switch i at time $t - 1$ as well”. Otherwise the solution fails this rule since the switch flip *could have occurred* one action sooner.

Under this rule, we have made it impossible to achieve more than 100 unnecessary states within the 100-switch problem. At each step where we do not make progress somewhere else, we must choose at least one switch to stop flipping (if we toggle the exact same set of switches as in the last step, we revert to the same overall state as two steps earlier which is forbidden by the layered ‘make-progress’ rule).

More generally, one can see that wherever a planning problem has multiple independent parts, this rule forces all the parts to proceed independently and not stall needlessly. However, the rule still has some gaps.

- Even adding a linear number of unnecessary steps is suboptimal. All the switches are independent so we really should not be adding more than one layer regardless of how many switches there are.
- The switch scenario is contrived to make our solution look better than it is. One can easily see that by using three-state switches rather than two-state switches (where each state is reachable from the other two), it is still possible to construct exponential-length plans even with this restriction in place. This is because we can still reach an exponential number of states while continually changing every switch at every time-step.

The above issues are addressed by Definition 3.2 of the main paper where we give the ‘make-progress’ notion which is used by the stepless planner.

Appendix B Proofs

B.1 Correctness of Mutex Action Rules

We show that the following handle all mutex action constraints that we care about strictly via unit propagation (labels are added for reference):

1. $\text{used_preserved}(F,K) :- \text{happens}(A,K); \text{pre}(A,F); \text{not del}(A,F) .$
2. $\text{deleted_unused}(F,K) :- \text{happens}(A,K); \text{del}(A,F); \text{not pre}(A,F) .$
3. $:- \{ \text{used_preserved}(F,K); \text{deleted_unused}(F,K);$
 $\quad \text{happens}(A,K) : \text{pre}(A,F), \text{del}(A,F) \} > 1; \text{valid_at}(F,K) .$
4. $\text{deleted}(F,K) :- \text{happens}(A,K); \text{del}(A,F) .$
5. $:- \text{holds}(F,K); \text{deleted}(F,K-1) .$

given the existing rules:

6. $\text{holds}(F,K) :- \text{happens}(A,K); \text{pre}(A,F) .$
7. $:- \text{holds}(F,K); \text{holds}(G,K); \text{mutex}(F,G) .$

Proof

Suppose A and B are mutex actions because A deletes fluent F and B has F as a precondition. The proof is based on a case analysis.

Case 1. A does not have F as a precondition and B does not delete F . If we select $\text{happens}(A,K)$ then by unit propagation we have $\text{deleted_unused}(F,K)$ (2) and then $\neg \text{used_preserved}(F,K)$ (3) and then $\neg \text{happens}(B,K)$ (1). We can write this as:

$$\text{happens}(A,K) \Rightarrow_2 \text{deleted_unused}(F,K) \Rightarrow_3 \neg \text{used_preserved}(F,K) \Rightarrow_1 \neg \text{happens}(B,K)$$

Case 2. A has F as a precondition and B does not delete F . Then,

$$\text{happens}(A,K) \Rightarrow_3 \neg \text{used_preserved}(F,K) \Rightarrow_1 \neg \text{happens}(B,K)$$

Case 3. A does not have F as a precondition, B deletes F . Then,

$$\text{happens}(A,K) \Rightarrow_2 \text{deleted_unused}(F,K) \Rightarrow_3 \neg \text{happens}(B,K)$$

Case 4. A has F as a precondition, B deletes F . Then,

$$\text{happens}(A,K) \Rightarrow_3 \neg \text{happens}(B,K)$$

In the case of conflicting effects (A adds F , B deletes F), there's only a conflict when the conflicted fluent "holds". So in fact we actually only care about a unit propagation mutex between $\text{holds}(F,K+1)$ and $\text{happens}(B,K)$. A is not relevant (as mentioned in the Section 2 of the main paper, this is where our mutex rules differ from those of (Blum and Furst 1997)). So

$$\text{holds}(F,K+1) \Rightarrow_5 \neg \text{deleted}(F,K) \Rightarrow_4 \neg \text{happens}(B,K)$$

Conversely:

$$\text{happens}(B,K) \Rightarrow_4 \text{deleted}(F,K) \Rightarrow_5 \neg \text{holds}(F,K+1)$$

Finally, suppose A and B have mutex preconditions ($\text{pre}(A,F); \text{pre}(B,G); \text{mutex}(F,G)$). Then (again by unit propagation),

$$\text{happens}(A,K) \Rightarrow_6 \text{holds}(F,K) \Rightarrow_7 \neg \text{holds}(G,K) \Rightarrow_6 \neg \text{happens}(B,K)$$

Thus no explicit mutex action rules are needed beyond this. Thanks to unit propagation, the effect is the same. \square

B.2 Soundness and Completeness of Variant-II Solver

The make-progress rule here refers to the layered ‘make-progress’ rule.

Lemma Appendix B.1

Given a planning problem solution (plan) P , we can find a solution P^* which satisfies the ‘make-progress’ rule such that $C(P^*) \leq C(P)$ ($C(P)$ is the sum action cost of plan P) and $n(P^*) \leq n(P)$ ($n(P)$ is makespan of plan P).

Proof

If P satisfies the ‘make-progress’ rule, then $P^* = P$ and we’re done. Otherwise there exists a pair of layers l_1 and l_2 such that $S_P(l_2) \subset S_P(l_1)$ ($S_P(l)$ is the set of fluents which hold at layer l in P). We can reduce P by “removing” all the layers between l_1 (exclusive) and l_2 (inclusive) along with any actions that occur on those layers. Repeat this process until no such pair of layers exists. Since every iteration removes at least one layer and P has a finite number of layers, it follows that this will eventually terminate and the resulting plan P^* will have no such pair and thus satisfy the ‘make-progress’ rule. \square

Corollary Appendix B.1

If a planning problem Q is solvable, then it has an optimal solution which “makes progress” according to the rule.

Lemma Appendix B.2

Given a plan P^* which satisfies the ‘make-progress’ rule, any prefix of that plan $P_{..k}$ also satisfies the ‘make-progress’ rule.

Proof

This is trivial: If there exists no pair of layers in P^* with some property, then of course there exists no pair of layers with that property in any prefix of P^* . \square

Lemma Appendix B.3

Given a planning problem Q with solution P , the delete-free relaxation of Q has a solution P^* , the set of actions that occur in P (ordered according to the first time they are taken in P). It follows that $C(P^*) \leq C(P)$ (note: this is not strictly equal since actions in P may be taken more than once and incur their cost every time).

Proof

Also trivial: If a precondition or goal is satisfied at some layer in P , then it must also be satisfied by that time in the delete-free relaxation since all the same actions have occurred. \square

Theorem Appendix B.2

(Variant-II Soundness Theorem) If Q has an optimal solution P with cost C and makespan n , then for any $k \leq n$, at makespan k the Variant-II solver will find a relaxed plan with cost $\leq C$.

Proof

This can be established by constructing a solution at makespan k with cost $C(P, k) \leq C$. By Corollary Appendix B.1, we may assume WLOG that P makes progress. First, set the ‘subgoal’ fluents to be $S_P(k)$. The fluents and actions in the normal part of the program match P exactly. By Lemma Appendix B.2 these will satisfy the ‘make-progress’ rule. Finally, the suffix layer is solved by the set of actions that occur in the suffix $P_{k..}$ which solves the delete-free relaxation by Lemma Appendix B.3. $C(P, k)$ here is the sum of two parts; the solution to the prefix $P_{..k}$ which is the same as $P_{..k}$ (and therefore has the same cost as $P_{..k}$), and the relaxed solution to the suffix $P_{k..}$, which by Lemma Appendix B.3 is no greater than in $C(P_{k..})$ so $C(P, k) \leq C$. It follows that the *optimal solution* at makespan k is at most $C(P, k)$ (and so is transitively $\leq C$). \square

Theorem Appendix B.3

(Variant-II Completeness Theorem) If a planning problem Q has no solution, the Variant-II solver will eventually produce an UNSAT instance.

Proof

A plan which makes progress cannot encounter the same state twice and there are a finite number of possible states. This means that the length of a plan which makes progress is bounded by the number of possible states. Thus, for a sufficiently-large makespan, the make-progress rule is unsatisfiable. \square

B.3 Soundness and Completeness of Stepless Planner*Theorem Appendix B.4*

(Stepless Soundness Theorem) All plan costs produced by the stepless planner are lower bounds on the cost of the true optimal plan.

Proof

Case 1. There are sufficient occurrences of fluents and actions to construct the optimal plan: In this case, these occurrences constitute a solution so the minimal solution to this instance will have a cost which is no greater.

Otherwise pick an arbitrary sequentialization of the optimal plan. Now we have two cases:

Case 2. The first missing occurrence in the plan is an action occurrence. In this case, consider the plan cut where all actions up to this point occur (since we have enough occurrences) and the state at this cut form the subgoals. By Lemma Appendix B.3 again, we can put the remainder of the plan into the suffix layer. The missing action occurrence will be a starting action and that action will also be saturated so the saturation rules are satisfied.

Case 3. The first missing occurrence in the plan is a fluent occurrence. In that case, use the plan cut up to (but not including) the action occurrence which *adds* this fluent occurrence (putting the remainder of the plan including the adding action into the suffix layer by Lemma Appendix B.3). This action will be a starting action so the added fluent will be a saturated starting fluent which also satisfies the saturation requirement. \square

The ‘make progress’ rule below refers to the stepless ‘make-progress’ rule.

Lemma Appendix B.4

Given a collection of action occurrences in a plan P , they may be ordered such that for each consecutive pair there is an (s,t) -cut which puts the first one on the s -side and the other one on the t -side.

Proof

Pick a serialization of P . Order the actions according to the sub-order in that serialization. Place the cuts anywhere between them. \square

Lemma Appendix B.5

For any action A there exists a (finite) count k such that the stepless planner will not add more than k occurrences of A .

Proof

In order to add another occurrence of A , it must be the case that A is saturated in some plan that makes progress. This means that k occurrences of A are used in the plan. Order the occurrences of A as $(A_0, A_1, A_2, \dots, A_k)$ such that there exists a cut between each consecutive pair (by Lemma Appendix B.4; also because of our symmetry-breaking rule this can be the natural ordering by action index). It follows by the ‘make-progress’ rule that the state at each of the cuts must be distinct from the state at any other cut. Thus, k cannot exceed the number of possible states (which is finite). \square

Corollary Appendix B.5

For any fluent F there exists a finite count k such that the stepless planner will not add more than k occurrences of F .

Proof

When F is saturated, each occurrence must be caused by some action occurrence. Since Lemma Appendix B.5 bounds the number of action occurrences which a progress-making plan can have, it follows that the number of fluent occurrences is also bounded. \square

Theorem Appendix B.6

(Stepless Completeness Theorem) The stepless planner will eventually find the solution if it exists or produce an UNSAT instance if it doesn’t.

Proof

In the proof of Theorem Appendix B.4 we show that if a plan exists, then there will always be a solution to any stepless instance constructed from that problem (either the plan itself if there are enough occurrences, or a partial plan with a suffix layer since some fluent or action does not have enough occurrences and can therefore be saturated). Lemma Appendix B.5 and Corollary Appendix B.5 together ensure that the process of alternately solving instances and then including any saturated fluents or actions will eventually halt (since there are at most a finite number of actions and fluents that can be included before finding a progress-making plan which saturates something becomes impossible). \square

Appendix C Delete-Free Planning

Recall that delete-free planning can be modeled as a graph problem: Given a directed bipartite graph $G = (X, Y, E)$ with weights on X and a goal set $Y_F \subseteq Y$, find a minimum *acyclic* subgraph $G^* = (X^*, Y^*, E^*)$ such that

1. $Y_F \subseteq Y^*$
2. If $x \in X^*$ and $(y, x) \in E$, then $y \in Y^*$ and $(y, x) \in E^*$
3. For all $y \in Y^*$, E^* contains at least one edge (x, y) (and $x \in X^*$).

Recall its connection to delete-free planning: X is the set of actions, and Y is the set of fluents, the (x, y) edges are add-effects and the (y, x) edges are preconditions. Y_F is the goal set and the initial set has been removed (together with all corresponding preconditions) from the graph. Rule 1 means the goal fluents must be true. Rule 2 means an action implies its preconditions. Rule 3 means every fluent must have a causing action. The graph must be acyclic to ensure the actions can occur in some order. The entire problem of a delete-free planning problem can be encoded in a single “one-shot” ASP program. This is possible because there is no incentive to ever take an action or cause a fluent more than once. As soon as any fluent is true, it is permanently true.

Let us write an ASP program to solve the problem of delete-free planning. Here we do not worry about makespan; thanks to the NP-ness of delete-free planning, we can solve this problem all in one go. Note that we can trivially add an extra rule to make our plans cost-optimal. (The code below is a complete ASP program: run it on the problem and get an optimal solution.)

```
holds(F) :- init(F).
{happens(A)} :- holds(F) : pre(A,F); action(A).
holds(F) :- add(A,F); happens(A).
:- goal(F); not holds(F).
:~ happens(A); cost(A,C) . [C,A]
```

We have again encountered a five-line program which, magnificently, does everything. It handily encodes the problem of delete-free planning. To be supported, an action’s preconditions must hold independently of that action itself and a fluent’s causing action must not require that fluent.

However, we have lost something by encoding planning “from the ground up”. Earlier, we mentioned how the state-space for solving a planning problem was reduced when we started from the goal, and built support up backwards. That is, an action should only happen if something needs it. Let us fix that.

If we build up the plan backwards, we must be careful to ensure that the actions can happen in some order. As such, we need to explicitly include atoms whose only purpose is to ensure supportedness.

```
% Delete-free planning
holds(F) :- goal(F).
{happens(A) : add(A,F)} >= 1 :- holds(F), not init(F).
holds(F) :- pre(A,F); happens(A).

supportFluent(F) :- init(F); holds(F).
supportAct(A) :- supportFluent(F) : pre(A,F), holds(F); happens(A).
supportFluent(F) :- supportAct(A); happens(A); add(A,F); holds(F).
```

```
:- holds(F); not supportFluent(F).
:~ happens(A); cost(A,C). [C,A]
```

Now the first three rules encompass neededness. We add actions and fluents in working backwards from the goal until we encounter the initial fluents. Meanwhile the second three rules indicate whether an action or fluent is supported. Together, with the restriction that all the fluents must be supported, these guarantee a correct plan. Essentially, for an action or fluent to occur, it now must have support both from the bottom and from the top.

Appendix D Stepless Planner with Suffix Layer

Requires an external program to detect which fluents and actions are saturated each time the suffix layer is used and feed in more occurrences.

```
is(fluentOcc(F,1)) :- fluent(F).
is(actOcc(A,1)) :- action(A).
is(fluentOcc(F,0)) :- init(F).

% ===== Problem Description =====

% Helper function to recognize subsequent occurrences of fluent/action.
nextOcc(fluentOcc(F,0),fluentOcc(F,1)) :- fluent(F).
nextOcc(fluentOcc(F,M),fluentOcc(F,M+1)) :- is(fluentOcc(F,M)).
nextOcc(actOcc(A,N),actOcc(A,N+1)) :- is(actOcc(A,N)).

% Fluent occurrence which is not initial (M > 0) must have exactly one
% causing action
{causes(actOcc(A,N),fluentOcc(F,M)) : add(A,F), is(actOcc(A,N))}=1 :-
  holds(fluentOcc(F,M)); M > 0.
% If an action causes a fluent, it happens.
happens(AO) :- causes(AO,_).
% An action cannot cause more than one occurrence of the same fluent.
:- {causes(AO,fluentOcc(F,M))} > 1; is(AO); fluent(F).

% For each precondition an action occurrence has, some occurrence of
% that fluent must permit it.
{permits(fluentOcc(F,M),actOcc(A,N)) : is(fluentOcc(F,M))}=1 :-
  happens(actOcc(A,N)); pre(A,F).
% A fluent occurrence which permits an action must hold.
holds(FO) :- permits(FO,_).
% A fluent which is used to satisfy a subgoal condition "permits" it.
% For each subgoal condition, exactly one occurrence of that fluent
% permits it.
{permits(fluentOcc(F,M),subgoal(F)) : is(fluentOcc(F,M))}=1 :-
  subgoal(F).
% A fluent which permits a subgoal condition cannot be deleted.
```

```

:- deleted(F0); permits(F0,subgoal(_)).

% An occurrence of an action deletes an occurrence of a fluent if
% it permits it and that action has the fluent as a delete effect.
deletes(actOcc(A,N),fluentOcc(F,M)) :-
    permits(fluentOcc(F,M),actOcc(A,N)); del(A,F).
% No fluent may be deleted by more than one action.
:- {deletes(_, F0)} > 1; is(F0).

% An action which deletes a fluent, but doesn't have it as a precondition
% follows some occurrence of that fluent. Can possibly follow occurrence
% index 0 even if the fluent is not an initial fluent (indicating this
% action occurs before any occurrence of that fluent).
{follows(actOcc(A,N),fluentOcc(F,M)) : holds(fluentOcc(F,M));
 follows(actOcc(A,N),fluentOcc(F,0))}=1 :-
    del(A,F); not pre(A,F); happens(actOcc(A,N)).

% Fluent occurrences 0 which aren't initial fluents count as "deleted".
deleted(fluentOcc(F,0)) :- fluent(F); not init(F).
% A fluent is deleted if something deletes it.
deleted(F0) :- deletes(_, F0).
% A fluent is deleted if something follows it.
deleted(F0) :- follows(_, F0).

% Weak constraint charging the cost of an action occurrence.
:~ happens(actOcc(A,N)); cost(A,V).[V,A,N]

% An occurrence of a fluent doesn't hold if its previous occurrence
% doesn't hold.
:- holds(fluentOcc(F,M+1)); not holds(fluentOcc(F,M));
    is(fluentOcc(F,M)); M > 0.
% An occurrence of an action doesn't happen if its previous occurrence
% didn't happen.
:- happens(B0); not happens(A0); nextOcc(A0,B0).

% ===== Plan Event Graph =====

% Events in the graph; these will be grouped into vertices
event(start(F0)) :- holds(F0).
event(end(F0)) :- holds(F0).
event(end(fluentOcc(F,0))) :- fluent(F).
event(A0) :- happens(A0).
% subgoals are events
event(subgoal(F)) :- subgoal(F).

```

```

% Triggering actions
% The start of a fluent by its causing action.
actionTriggers(AO,start(FO)) :- causes(AO,FO).
% The end of a fluent by its deleting action.
actionTriggers(AO,end(FO)) :- deletes(AO,FO).

% Vertices
% If no action triggers an event, then it gets a vertex by itself.
vertex(V) :- event(V); not actionTriggers(A,V) : is(A).
% Otherwise it belongs to the vertex for its trigger action.
inVertex(E,V) :- actionTriggers(V,E).
% Every event which is the name of a vertex belongs to that vertex.
inVertex(V,V) :- vertex(V).

% Graph edges
% A fluent ends after it starts
edge(start(FO),end(FO)) :- holds(FO).
% If a fluent permits an action, then the action happens after
% the start of the fluent
edge(start(FO),AO) :- permits(FO,AO).
% If a fluent permits an action but the action doesn't delete the
% fluent, then the action happens before the end of the fluent.
edge(AO,end(FO)) :- permits(FO,AO); not deletes(AO,FO).

% An action happens after the fluent it follows
edge(end(FO),AO) :- follows(AO,FO).
% but before the next occurrence
edge(AO,start(GO)) :- follows(AO,FO); nextOcc(FO,GO); holds(GO).
% The start of the next occurrence of a fluent happens after the
% end of the previous occurrence
edge(end(FO),start(GO)) :- holds(GO); nextOcc(FO,GO).
% The next occurrence of an action happens after the previous
% occurrence
edge(AO,BO) :- happens(AO); happens(BO); nextOcc(AO,BO).

% And now we use stable models to assert that the graph is acyclic; sup(X)
% indicates that X has acyclic support going back to the root of the graph.

% The input for a given event has support if all events joined
% by any incoming edge have support.
sup(in(E)) :- sup(D) : edge(D,E); event(E).
% A vertex has support if all of its events' inputs have support.
sup(V) :- sup(in(E)) : inVertex(E,V); vertex(V).
% An event has support if its vertex has support.
sup(E) :- sup(V); inVertex(E,V).
% Every vertex must have support.

```

```

:- vertex(V); not sup(V).

% ===== Strong Minimality =====

% A counterexample to strong minimality consists of two cuts, cut1 and cut2.
cut(cut1; cut2).

% For each vertex V and each cut C, V is on either the s-side or
% the t-side of V. Note this rule is disjunctive.
onSideOf(V,s,C) | onSideOf(V,t,C) :- vertex(V); cut(C).
% An event belongs to the cut side of its vertex.
onSideOf(E,X,C) :- inVertex(E,V); onSideOf(V,X,C).
% Any subgoal is always on the t-side of cut2.
onSideOf(subgoal(F),t,cut2) :- subgoal(F).
% If there's a directed edge from D to E, but D is on the t-side
% and E is on the s-side, this is not a cut (invalidating this
% counterexample to strong minimality).
not_counterexample :- edge(D,E); onSideOf(D,t,C); onSideOf(E,s,C).
% If a fluent starts on the s-side of cut2 and ends on the t-side,
% then it "holds over" cut2.
holdsOver(F0,cut2) :-
  onSideOf(start(F0),s,cut2); onSideOf(end(F0),t,cut2).
% Similarly if it starts and ends on the same side of cut1, then it
% doesn't hold over cut1.
not_holdsOver(F0,cut1) :-
  onSideOf(start(F0),X,cut1); onSideOf(end(F0),X,cut1).
% Action occurrence AO is not between cut1 and cut2 if it's on the
% s-side of cut1 or the t-side of cut2.
not_betweenCuts(AO) :- onSideOf(AO,s,cut1).
not_betweenCuts(AO) :- onSideOf(AO,t,cut2).
% If no action occurs between the two cuts, then this is not a counterexample.
not_counterexample :- not_betweenCuts(AO) : happens(AO).
% If there exists a fluent for which some occurrence holds over cut2,
% but no occurrence holds over cut1, then this is not a counterexample.
not_counterexample :-
  holdsOver(fluentOcc(F,_),cut2);
  not_holdsOver(fluentOcc(F,M),cut1) : holds(fluentOcc(F,M)).

% There should be no counterexample (sorry for the triple negative).
:- not not_counterexample.
% If this is not a counterexample, all atoms must hold.
onSideOf(V,s,C) :- vertex(V); cut(C); not_counterexample.
onSideOf(V,t,C) :- vertex(V); cut(C); not_counterexample.

```

To see why this works, imagine that we find a plan which satisfies these rules. Consider the candidate model which includes the atom *not_counterexample*. Because all the rules here are strictly positive, the last two rules force all the others to hold. Any other solution is a strict subset.

Therefore if some *other* solution exists which does not include the *not_counterexample* atom, then a model including it would be rejected for not being minimal. It follows that the *only* models which include *not_counterexample* (and satisfy the triple-negative rule) are those for which no counterexample exists.

```
% ===== Suffix Layer =====

% All goal fluents hold in the suffix layer.
suffix(holds(F)) :- goal(F).
% If a fluent holds in the suffix layer, either some action causes it
% or it is a subgoal.
{subgoal(F); suffix(causes(A,F)) : add(A,F)} = 1 :- suffix(holds(F)).
% If an action causes a fluent in the suffix, it happens.
suffix(happens(A)) :- suffix(causes(A,_)).
% If an action occurs in the suffix layer, then all of its
% preconditions hold in ths suffix layer
suffix(holds(F)) :- suffix(happens(A)); pre(A,F).

% If any action happens in the suffix layer, then we are using it.
useSuffix :- suffix(happens(_)).

% A fluent is supported in the suffix if it's a subgoal
suffix(sup(holds(F))) :- subgoal(F).
% An action is supported in the suffix if all of its preconditions are
suffix(sup(happens(A))) :-
    suffix(sup(holds(F))) : pre(A,F); suffix(happens(A)).
% A fluent is supported in the suffix if its causing action is
suffix(sup(holds(F))) :- suffix(sup(happens(A))); suffix(causes(A,F)).

% No action happens in the suffix without support
:- suffix(happens(A)); not suffix(sup(happens(A))).
% No fluent holds in the suffix without support
:- suffix(holds(F)); not suffix(sup(holds(F))).

% Actions that happen in the suffix layer impose their cost.
:~ suffix(happens(A)); cost(A,V).[V,A,suffix]
% Very weak preference to avoid using the suffix layer.
:~ useSuffix.[1@-1]

% ===== Saturated =====

% A fluent is saturated if all occurrences of it hold (besides the 0th).
saturated(fluent(F)) :-
    holds(fluentOcc(F,M)) : is(fluentOcc(F,M)),M>0; fluent(F).
% An action is saturated if all occurrences of it happen.
saturated(action(A)) :-
```

```

happens(actOcc(A,N)) : is(actOcc(A,N)); action(A).

% If an action happens in the suffix layer and all of its preconditions
% are subgoals, we designate it a "starting" action.
suffix(start(action(A))) :- subgoal(F) : pre(A,F); suffix(happens(A)).
% Any fluent caused by a starting action is designated a "starting" fluent.
suffix(start(fluent(F))) :- suffix(start(action(A))); suffix(causes(A,F)).

% Guarantees that some starting action or fluent will be saturated.
:- useSuffix; not saturated(X) : suffix(start(X)).

% =====
#show causes/2. #show deletes/2. #show happens/1.
#show holds/1. #show permits/2. #show follows/2.
#show suffix(happens(A)) : suffix(happens(A)).

```

Appendix E An Example of Stepless Planning: Bridge Crossing

We will use a modified version of the bridge-crossing problem from (Eiter et al. 2003).

In the original problem, we have four people, Joe, Jack, William, and Averell, needing to cross a bridge in the middle of the night. The bridge is unstable, so at most two people can cross at a time. The four only have a single lantern between them and since there are planks missing it is unsafe to cross unless someone in your party is carrying the lantern. In the original problem, it takes Joe 1 minute to run across, Jack 2 minutes, William 5 minutes and Averell 10. When two people cross together they must go at the slower speed of the two. What's the fastest all four can get across considering that after each crossing somebody needs to cross back carrying the lantern?

In our version we'll add two more people Jill and Candice for a total of six people. Jill takes 3 minutes to cross and Candice takes 20 (the original problem doesn't make for a very interesting example of stepless planning).

We can now phrase the problem as follows:

```

person(joe;jack;jill;william;averell;candice)
side(side_a;side_b)
crossing_time(joe,1).
crossing_time(jack,2).
crossing_time(jill,3)
crossing_time(william,5).
crossing_time(averell,10).
crossing_time(candice,20).

fluent(lantern_at(S)) :- side(S).
fluent(at(P,S)) :- person(P); side(S).

init(at(P,side_a)) :- person(P).
init(lantern_at(side_a)).

```

```

goal(at(P,side_b)) :- person(P).

action(cross_alone(P,FROM,TO)) :-
  person(P); side(FROM); side(TO); FROM != TO.
pre(cross_alone(P,FROM,TO),at(P,FROM)) :-
  action(cross_alone(P,FROM,TO)).
pre(cross_alone(P,FROM,TO),lantern_at(FROM)) :-
  action(cross_alone(P,FROM,TO)).
add(cross_alone(P,FROM,TO),at(P,TO)) :-
  action(cross_alone(P,FROM,TO)).
add(cross_alone(P,FROM,TO),lantern_at(TO)) :-
  action(cross_alone(P,FROM,TO)).
del(cross_alone(P,FROM,TO),at(P,FROM)) :-
  action(cross_alone(P,FROM,TO)).
del(cross_alone(P,FROM,TO),lantern_at(FROM)) :-
  action(cross_alone(P,FROM,TO)).
cost(cross_alone(P,FROM,TO),C) :-
  action(cross_alone(P,FROM,TO)); crossing_time(P,C).

action(cross_together(P_SLOW,P_FAST,FROM,TO)) :-
  side(FROM); side(TO); FROM != TO;
  crossing_time(P_SLOW,T1); crossing_time(P_FAST,T2); T2 < T1.
pre(cross_together(P_SLOW,P_FAST,FROM,TO),at(P_SLOW,FROM)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
pre(cross_together(P_SLOW,P_FAST,FROM,TO),at(P_FAST,FROM)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
pre(cross_together(P_SLOW,P_FAST,FROM,TO),lantern_at(FROM)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
add(cross_together(P_SLOW,P_FAST,FROM,TO),at(P_SLOW,TO)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
add(cross_together(P_SLOW,P_FAST,FROM,TO),at(P_FAST,TO)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
add(cross_together(P_SLOW,P_FAST,FROM,TO),lantern_at(TO)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
del(cross_together(P_SLOW,P_FAST,FROM,TO),at(P_SLOW,FROM)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
del(cross_together(P_SLOW,P_FAST,FROM,TO),at(P_FAST,FROM)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
del(cross_together(P_SLOW,P_FAST,FROM,TO),lantern_at(FROM)):-
  action(cross_together(P_SLOW,P_FAST,FROM,TO)).
cost(cross_alone(P_SLOW,P_FAST,FROM,TO),C) :-
  action(cross_alone(P_SLOW,P_FAST,FROM,TO)); crossing_time(P_SLOW,C).

```

Let's run the stepless solver on this. On the first iteration we input one occurrence of every fluent and every action as well as a bonus zero'th occurrence of each initial fluent.

```
is(fluentOcc(F,1)) :- fluent(F).
```

```
is(actOcc(A,1)) :- action(A).
is(fluentOcc(F,0)) :- init(F).
```

It gives back a directed graph of action and fluent dependencies. After topologically sorting the graph and throwing out everything that isn't an action we have the plan:

```
cross_together(jack,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
suffix cross_together(candice,averell,side_a,side_b)
suffix cross_alone(joe,side_a,side_b)
suffix cross_together(william,jill,side_a,side_b)
cost: 29
```

In the suffix layer when Candice and Averell cross from *side_a* to *side_b*, the fluent *lantern_at(side_a)* is not deleted (because the suffix layer encodes the delete-free relaxation of the problem), so this is still considered to be achieved when Joe, and then William and Jill cross. Nobody needs to bring the lantern back for them. The use of the suffix layer is allowed because there isn't a second occurrence of the fluent *at(joe(side_b))*, but this is a starting fluent (all 3 suffix actions are starting actions since they do not depend on each other). Since the suffix layer was used, we add a second occurrence of each of the fluents and actions which were saturated by this plan:

Adding:

```
is(fluentOcc(at(joe,side_a),2)).
is(fluentOcc(lantern_at(side_a),2)).
is(fluentOcc(lantern_at(side_b),2)).
is(fluentOcc(at(joe,side_b),2)).
is(fluentOcc(at(jack,side_b),2)).
is(actOcc(cross_together(jack,joe,side_a,side_b),2)).
is(actOcc(cross_alone(joe,side_b,side_a),2)).
```

and run it again:

```
cross_together(william,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
cross_together(jill,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
suffix cross_together(candice,averell,side_a,side_b)
suffix cross_together(jack,joe,side_a,side_b)
cost: 32
```

This time we start by having William and Joe cross together and then Joe carries the lantern back, crosses with Jill and carries it back again. In the suffix layer, Candice and Averell cross together while Jack and Joe cross together (each pair making use of the same undeleted lantern). Again the suffix layer occurs because we don't have enough occurrences of *at(joe(side_b))*.

Interestingly, a cheaper solution seems to have been skipped. Namely the plan which is identical to the cost-29 plan, but with Joe running across and running back first for a total cost of 31.

This is because such a plan fails to make progress. We can produce two cuts, namely the one at the start of the plan and the one after Joe crosses back the first time and see that no new fluents hold between the two cuts. The rules enforcing strong minimality will reject this plan.

Add another occurrence of each saturated item

Adding:

```
is(fluent0cc(at(joe,side_a),3)).
is(fluent0cc(lantern_at(side_a),3)).
is(fluent0cc(lantern_at(side_b),3)).
is(fluent0cc(at(joe,side_b),3)).
is(fluent0cc(at(jill,side_b),2)).
is(fluent0cc(at(william,side_b),2)).
is(act0cc(cross_together(jill,joe,side_a,side_b),2)).
is(act0cc(cross_together(william,joe,side_a,side_b),2)).
is(act0cc(cross_alone(joe,side_b,side_a),3)).
```

and again:

```
cross_together(william,jack,side_a,side_b)
cross_alone(jack,side_b,side_a)
cross_together(jill,jack,side_a,side_b)
suffix cross_alone(jack,side_b,side_a)
suffix cross_alone(joe,side_a,side_b)
suffix cross_together(candice,averell,side_a,side_b)
cost: 33
```

Here we have William and Jack crossing together. Then Jack crosses back alone. Jill and Jack cross together, and now Jack *would* cross back alone again taking the lantern, but there are only two occurrences of the action *cross_alone(jack,side_b,side_a)* in our bag so instead we move into the suffix layer. In the suffix layer he carries the lantern back, but because of the delete relaxation, we don't lose the fluent *at(jack,side_b)* so he doesn't need to cross back again. Candice and Averell use the lantern to cross as does Joe by himself.

The rest of the output from the stepless solver follows:

Adding:

```
is(fluent0cc(at(jack,side_a),2)).
is(fluent0cc(at(jack,side_b),3)).
is(act0cc(cross_together(jill,jack,side_a,side_b),2)).
is(act0cc(cross_together(william,jack,side_a,side_b),2)).
is(act0cc(cross_alone(jack,side_b,side_a),2)).
```

```
cross_together(jill,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
cross_together(william,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
cross_together(jack,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
suffix cross_alone(joe,side_a,side_b)
suffix cross_together(candice,averell,side_a,side_b)
cost: 34
```

Adding:

```
is(fluent0cc(at(joe,side_a),4)).
is(fluent0cc(lantern_at(side_a),4)).
is(fluent0cc(lantern_at(side_b),4)).
is(fluent0cc(at(joe,side_b),4)).
is(act0cc(cross_alone(joe,side_b,side_a),4)).
```

```
cross_together(jill,jack,side_a,side_b)
cross_alone(jill,side_b,side_a)
cross_together(william,jill,side_a,side_b)
suffix cross_alone(jill,side_b,side_a)
suffix cross_alone(joe,side_a,side_b)
suffix cross_together(candice,averell,side_a,side_b)
cost: 35
```

Adding:

```
is(fluent0cc(at(jill,side_a),2)).
is(fluent0cc(at(jill,side_b),3)).
is(act0cc(cross_together(william,jill,side_a,side_b),2)).
is(act0cc(cross_alone(jill,side_b,side_a),2)).
```

```
cross_together(jack,joe,side_a,side_b)
cross_alone(jack,side_b,side_a)
cross_together(jill,jack,side_a,side_b)
cross_alone(jack,side_b,side_a)
cross_together(william,jack,side_a,side_b)
suffix cross_alone(jack,side_b,side_a)
suffix cross_together(candice,averell,side_a,side_b)
cost: 36
```

Adding:

```
is(fluent0cc(at(jack,side_a),3)).
is(fluent0cc(at(jack,side_b),4)).
is(act0cc(cross_alone(jack,side_b,side_a),3)).
```

```
cross_together(jack,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
cross_together(jill,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
cross_together(candice,averell,side_a,side_b)
cross_alone(jack,side_b,side_a)
cross_together(william,joe,side_a,side_b)
cross_alone(joe,side_b,side_a)
cross_together(jack,joe,side_a,side_b)
cost: 37
```

In the last one, the suffix layer is not used so we're done. No other plans need be searched.