2018 Coast Combinatorics Conference (28th)

24 – 25 March 2018

University of Victoria
Victoria, BC, Canada
DTB A102

Program and Abstracts

28th Coast Combinatorics Conference

SATURDAY, March 24, 2018 – DTB A102

10:00 AM **Leslie Hogben**, Iowa State University and American Institute of Mathematics
Throttling for Cops and Robbers and Zero Forcing

10:30 AM **Sean McGuinness**, Thompson Rivers University
The toric ideal of a frame matroid

11:00 AM **Skye Dore-Hall**, Kwantlen Polytechnic University
Gracefulness of mK_4-snakes for $m = 3, 4, 5$

LUNCH BREAK

Choices are available on or near campus

1:30 PM **John Gimbel**, University of Alaska
Some Defective Ramsey Numbers for Certain Classes of Graphs

2:00 PM **Hal Sudborough**, University of Texas at Dallas
New results on permutation arrays

2:30 PM **Andrei Bulatov**, Simon Fraser University
The complexity of the finite constraint satisfaction problem

3:30 PM **Boting Yang**, University of Regina
The One-Cop-Moves Game

4:00 PM **Farzaneh Piri**, University of Semnan
Italian dominating set of trees

4:30 PM **Bojan Mohar**, Simon Fraser University
The Genus of a Random Bipartite Graph

Group dinner: Romeos Downtown
1703 Blanshard Street
7:30 PM

26th Coast Combinatorics Conference

SUNDAY, March 25, 2018 – DTB A102

10:00 AM **Kieka Mynhardt**, University of Victoria
On k-Total Dominating Graphs

10:30 AM **Ryan Hayward**, University of Alberta
The puzzlist and the poet: the story of the birth of Hex

11:00 AM **Patrick Fowler**, University of Sheffield
Simple models of molecular conduction - a new use for old characteristic polynomials

11:30 AM **Richard Nowakowski**, Dalhousie University
Diffusion, or Alms for the Poorer

Abstracts of the 2018 Coast Combinatorics Conference

- **Andrei Bulatov**, Simon Fraser University
  The complexity of the finite constraint satisfaction problem

  In a constraint satisfaction problem (CSP, for short) the goal is to find an assignment to a set of variables subject to constraints imposed on specified collections of variables. This framework allows one to express a wide variety of computational problems, from colorings to linear equations, and beyond. The general CSP is NP-complete, so certain restricted versions of the problem are often considered. The most natural way of restricting the CSP is by allowing only constraints of certain type. One of the major open questions about the CSP has been so-called Feder-Vardi Dichotomy conjecture suggesting that every such restricted CSP is either solvable in polynomial time or is NP-complete. We settle this problem and outline a solution algorithm that applies in the polynomial time solvable cases.

- **Skye Dore-Hall**, Kwantlen Polytechnic University
  Gracefulness of mK_4-snakes for $m = 3, 4, 5$

  Whether mK_4-snakes, graphs in which $m \geq 2$ copies of K_4 are connected in a path, can be gracefully labeled has been an open problem for at least 18 years. With the $2K_4$-snake already proven to not be graceful, we consider the $m = 3, 4$, and 5 cases in this talk. In particular, we prove that a graceful labeling for the $3K_4$-snake does not exist, and present graceful labelings for the $4K_4$-snake and $5K_4$-snake that were discovered by taking advantage of graceful labelings of French m-windmills.

- **Patrick Fowler**, University of Sheffield
  Simple models of molecular conduction - a new use for old characteristic polynomials

  Chemists have often used simplified models based on graph theory to predict or rationalise molecular properties. In these models, the characteristic polynomial of the molecular graph is a key object. This talk will show how such models are being adapted to deal with conduction of current through a molecular electronic device, leading to simple predictions of the transmission function for each device, selection rules for the placement of leads, and new definitions of general classes of molecular conductor.

The talk includes joint work with Barry Pickup, Irene Sciriha (Malta), Martha Borg, Victoria Seville, Tsanka Todorova, Tomo Pisanski (Ljubljana)) and Wendy Myrvold (Victoria).

- **John Gimbel**, University of Alaska
  Some Defective Ramsey Numbers for Certain Classes of Graphs

  We will say a set of vertices is k-sparse if it induces a graph with maximum degree at most k. Likewise, it is k-dense if the vertices form a k-sparse set in the complement. Such notions generalize independent sets and cliques. Given i, j and k let $R_k(i, j)$ be the minimum N where every graph on at least N vertices contains a k-sparse set of order i or a k-dense set of order j. These numbers are studied in several places. We consider such Ramsey numbers when restricted to certain families of graphs such as perfect graphs and cacti.

  Joint work with Tinaz Ekim and Oylum Seker (Bosphorus University).

- **Ryan Hayward**, University of Alberta
  The puzzlist and the poet: the story of the birth of Hex

  From December 1942 through August 1943, a series of columns in the Danish newspaper Politiken — many with challenging unique-solution puzzles — introduced a new board game by Piet Hein. But Hein was not a strong player, so who created the puzzles?

- **Leslie Hogben**, Iowa State University and American Institute of Mathematics
  Throttling for Cops and Robbers and Zero Forcing

  Cops and Robbers and zero forcing (and their variants) are games played on graphs. In (standard) Cops and Robbers the cops and one robber alternate turns moving along the edges of the graph. A round is one turn for the cops and one turn for the robbers. When the cops are placed on a multiset S of vertices of the graph G, and the robber plays to avoid capture as long as possible, the minimum number of rounds needed for the cops to catch the robber is the capture time of S, $\text{capt}(G; S)$; if the robber is not caught the capture time is infinite.

  Thottling involves minimizing the sum of the number of resources used to accomplish a task (e.g., cops) and the time needed to accomplish the task (e.g., capture time). Specifically, the throttling number of

G for Cops and Robbers is $\text{th}_c(G) = \min_{S \subseteq V(G)}(|S| + \text{capt}(G;S))$. Throttling was first studied by Butler and Young for (standard) zero forcing.

Zero forcing is a coloring game in which each vertex is initially blue or white and the goal is to color all vertices blue. The standard color change rule for zero forcing on a graph G is that a blue vertex v can change the color of a white vertex w to blue if w is the only white neighbor of v in G. If $B^{(i)}$ is the set of blue vertices in G at time step i, $B^{(i+1)}$ is the set of blue vertices in G after the color change rule is applied to every vertex in $B^{(i)}$ independently. The propagation time for a set $B^{(0)}$ of vertices, $\text{pt}(G; B^{(0)})$, is the smallest t such that $B^{(t)} = V(G)$ (and is infinity of this never happens). The throttling number of G for (standard) zero forcing is the $\text{th}(G) = \min_{B \subseteq V(G)}(|B| + \text{pt}(G;B))$.

There are many variants of zero forcing, each of which uses a different color change rule, including positive semidefinite (PSD) zero forcing. PSD zero forcing uses the PSD color change rule: Delete the currently blue vertices from the graph G and determine the components of the resulting graph. Apply the standard color change rule to the graph induced by the vertices in one white component together with the blue vertices (apply to each component). PSD propagation time and throttling are defined analogously to standard, using PSD zero forcing.

This talk will present results on the related problems of throttling for PSD zero forcing and Cops and Robbers.

Based on joint work with J. Breen, B. Brimkov, J. Carlson, J. Kritschgau, K. Lorenzen, K. Perry, C. Reinhart. M.S. Ross, V. Valle Martinez.

- **Sean McGuinness**, Thompson Rivers University
  The toric ideal of a frame matroid

  A conjecture of White dating from 1980 can be stated as follows: if $(A_1, \ldots, A_n)$ and $(B_1, \ldots, B_n)$ are n-tuples of bases which partition a matroid M, then $(B_1, \ldots, B_n)$ can be obtained from $(A_1 \ldots, A_n)$ by a sequence of symmetric exchanges of elements between pairs of bases. Algebraically, the conjecture amounts to showing that the toric ideal associated with a matroid is generated by quadric binomials. Our main result is that Whites conjecture holds for frame matroids. This improves on a result of Blasiak who verified the conjecture for graphic matroids.

- **Bojan Mohar**, Simon Fraser University
  The Genus of a Random Bipartite Graph

  Archdeacon and Grable (1995) proved that the genus of the random graph $G \in \mathcal{G}_{n,p}$ is almost surely close to $pn^2/12$ if $p = p(n) \geq 3(\ln n)^2 n^{-1/2}$. In this paper we prove an analogous result for random bipartite graphs in $\mathcal{G}_{n_1,n_2,p}$. If $n_1 \geq n_2 \gg 1$, phase transitions occur for every positive integer i when $p = \Theta((n_1 n_2)^{-\frac{i}{2i+1}})$. A different behaviour is exhibited when one of the bipartite parts has constant size, $n_1 \gg 1$ and n_2 is a constant. In that case, phase transitions occur when $p = \Theta(n_1^{-1/2})$ and when $p = \Theta(n_1^{-1/3})$. The last phase transition leads to and especially interesting problem about genus embeddings of complete 3-uniform hypergraphs.

  This is joint work with Yifan Jing.

- **Kieka Mynhardt**, University of Victoria
  On k-Total Dominating Graphs

  For a graph G, the k-total dominating graph $D_k^t(G)$ is the graph whose vertices correspond to the total dominating sets of G that have cardinality at most k; two vertices of $D_k^t(G)$ are adjacent if and only if the corresponding total dominating sets of G differ by either adding or deleting a single vertex. The graph $D_k^t(G)$ is used to study the reconfiguration problem for total dominating sets: a total dominating set can be reconfigured to another by a sequence of single vertex additions and deletions, such that the intermediate sets of vertices at each step are total dominating sets, if and only if they are in the same component of $D_k^t(G)$.

  There are fundamental differences between k-total dominating graphs and (ordinary) k-dominating graphs $D_k(G)$ as studied by Haas and Seyffarth [The k-dominating graph, *Graphs Combin.* **30** (3) (2014), 609–617] and [Reconfiguring dominating sets in some well-covered and other classes of graphs, *Discrete Math.* **340** (2017), 1802–1817]. For example, for k the upper total domination number $\Gamma_t(G)$, any graph without isolated vertices is an induced subgraph of a graph G such that $D_k^t(G)$ is connected, whereas, for k the upper (ordinary) domination number $\Gamma(G)$, $D_k(G)$ is disconnected whenever G has at least one edge. The talk will consist of a brief discussion of this and related results.

- **Richard Nowakowski**, Dalhousie University
  Diffusion, or Alms for the Poorer

  Diffusion is a variant of the Chip-Firing game, first defined in 2016. In this game, each vertex has some number of chips—think dollars. At each tick of the clock, each vertex sends one dollar down each edge that ends in a vertex with fewer dollars. Vertices are allowed to go into debt. Does the distribution of dollars stabilize in some sense? If you don't know the answer, and even if you do, then I encourage you to try several examples instead of waiting for this talk. I'll discuss the stabilized situations and raise some questions about two standard starting configurations—Millpond and Quantum fluctuation.

  Farzaneh Piri, University of Semnan
  Italian dominating set of trees

- **Hal Sudborough**, University of Texas at Dallas
  New results on permutation arrays

  The Hamming distance between permutations p and q, denoted by $hd(p,q)$, is the number of positions x such that $p(x)$ is not equal to $q(x)$. A permutation array (PA) S has Hamming distance d, denoted by $hd(S) \geq d$, if for every distinct pair $p, q \in S, hd(p,q) \geq d$. $M(n,d)$ denotes the maximum size of any PA S such that $hd(S) \geq d$.

  In the following, let p be prime and $k > 0$ be an integer. We show:
  Theorem 1. For $n = p^{2k},\ M(n+1,n) \geq p^{3k} + p^{2k}$.
  Theorem 2. For $n = 2^{2k+1},\ M(n+1,n) \geq n(2^k + 2^{k-2} + 1)$.
  Theorem 3. For $n = p^{2k+1},\ M(n+1,n) \geq \lfloor\sqrt{p}\rfloor\, p^{3k+1}$.

- **Boting Yang**, University of Regina
  The One-Cop-Moves Game

  We study a variant of Cops and Robbers, known alternately as the one-active-cop game, Lazy Cops and Robbers game or the one-cop-moves game. The corresponding cop number of a graph G in this game variant is called the one-cop-moves cop number of G, denoted by $c_1(G)$, which is the minimum number of cops required to capture the robber in G. We construct a connected planar graph whose structure is specifically designed for a robber to evade 3 cops indefnitely. We give a characterization of graphs with $c_1(G) \leq k$. We investigate the cop number of several classes of special graphs, including graphs with treewidth at most 2, Halin graphs, nested-wheel graphs, and some Cartesian product graphs.