Claim: $1 + 1 = 2$

Proof:

1. \( \emptyset \)

2. \{1\}

By definition of \( \emptyset \), \( 1 + 1 = 2 \)

Show \( G \) is \( \text{P-PSN} \) if and only if \( 1 + 1 + -2 = \emptyset \).

Only L-opt'N

\( \emptyset + 1 + -2 \)

Only R-opt'N

\( 1 - 2 \)

Only R-opt'N

\( 1 - 1 = \emptyset \)

Only L-opt'N

\( 1 + 1 - 1 = 1 \)

\( 1 + 1 - 2 \) is \( \text{R-PSN} \) so \( 1 + 1 - 2 = \emptyset \)

So \( 1 + 1 = 2 \).
CLAIM $\frac{1}{2} + \frac{1}{2} = 1$

PROOF WANT TO SHOW $G = \frac{1}{2} + \frac{1}{2} - 1$ is a P-PSN

ONLY 1 OPP'N

$\frac{1}{2} - 1$

$1 - 1 = 0$

$\frac{1}{2} + 1 - 1 = 0 \frac{1}{2}$

$\frac{1}{2} + \frac{1}{2} - 1 = 0 \frac{1}{2}$

$\frac{1}{2} + \frac{1}{2} = 1$

$\{0, 1\}$

$\{0, 1\}$

$\frac{1}{2} + \frac{1}{2} - 1$ is a P-PSN

SO

$\frac{1}{2} + \frac{1}{2} = 1$

😊
CLAIM  \( \forall n \geq 1 \quad \frac{1}{2^{n+1}} + \frac{1}{2^{n+1}} = \frac{1}{2^n} \)

DEF'N  \[ \frac{1}{2^t} = \{ 0 \mid \frac{1}{2^t} \} \]  \[ -\frac{1}{2^t} = \{ \frac{1}{2^t} \mid 0 \} \]

PF  W.T.S.  \[ G = \frac{1}{2^{n+1}} + \frac{1}{2^{n+1}} - \frac{1}{2^n} \]  IS A P-PSN.

BY IND'N ASSUMPTION:  \[ \frac{1}{2^n} + \frac{1}{2^n} = \frac{1}{2^{n-1}} \]  so  \[ \frac{1}{2^n} \rightarrow \frac{1}{2^{n-1}} = -\frac{1}{2^n} \]

so  \[ \frac{1}{2^n} + \frac{1}{2^{n+1}} - \frac{1}{2^{n-1}} = \frac{1}{2^{n+1}} + \left( \frac{1}{2^n} \cdot \frac{1}{2^{n-1}} \right) = \frac{1}{2^{n+1}} - \frac{1}{2^n} \]

BY IND'N

PROVE  \( \frac{1}{2^n} \rightarrow \frac{1}{2^{n+1}} \)  \[ \Rightarrow \]  \[ \frac{1}{2^{n+1}} - \frac{1}{2^n} < 0 \]

AFTER WE DO THIS,  OUR PROOF IS DONE.
\[ \frac{M}{2^t} = \frac{2^k \cdot m}{2^{t+k}} \]

**Corollary**

For any numbers \( x = \frac{a}{2^t} \) and \( y = \frac{b}{2^{t+k}} \),

\[ x + y = \frac{2^k \cdot a + b}{2^{t+k}} \]
**EXERCISE**

**HEX AS A COMB'L GAME**

**DEFN:**
- Every posn where a player has already won has value 0

**E.C. 2x2 HEX**

```
  O
  O
  O
  O
  O
```

O = LEFT already won

So this psn is 0.

O = RIGHT already won

So this psn is 0.

What about this?

```
  O
  O
  O
  O
  O
```

O can move to O

```
  O
  O
```

This is \*.

**EXERCISE:** For each 2x2 HEX position on next page, find position's canonical form.

(Use CGSuite to check your answers).
\[
N = \{0, 1, 2, 3\} = \{0, 1, 2, 3\} = \{0, 1, 2, 3\}
\]

\[
M = \{0, 1, 2, 3\} = \{0, 1, 2, 3\}
\]

\[
\mathbf{x} = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3
\]
- FOREIGN, SELF-NEG.
- IMPORTE, SELF-NEG?
- Can form of \(1^*, \frac{1}{2}, \uparrow, \downarrow, \ast, \ast^2, \pm 1\)?
- Hasse diagram of these 13 games?
WHAT HAVE YOU BEEN DOING DURING THIS "BREAK"?

ME: • LOOKING FOR A "SIMPLE" WINNING 10X10 HEX STRAT.
• TRYING TO PROVE A HEX CG. CONJECTURE USING REVERSIBLE MOVES.

CONJECTURE: HERE, THE O-MOVE CAN BE REVERSED AS SHOWN

ASSUME

THIS POSN O TO PLAY 0 TO PLAY
0 WINS WWWW

THEN (?)

THIS POSN O WINS
CONJECTURE

PF

CONSIDER OWN STRAT S FOR PEN P.

MODIFY TO OBT. OW-S S' / P'.

WHAT CAN GO WRONG?

S ASKS 0 TO MAKE MOVE ... - OCC. BY 0? ONLY CASE

IN THIS TREE ...

- CAN ASSUME 1ST MOVE IS TO v3 (STW 0 CAPTURES ALL)

- SEQUENCE IN REGION R = \{b2-c2, a4\}?

W SEQ b2-c2 ... FOLLOW S-STRAT b2 d3

(Will never require us to play c1, prefer c2!)