Clobber: Does $G = H$?

Recall Def'n: $G = H$ iff

\[ \forall X, G + X, H + X \text{ same O.C.} \]

Useful Thm: $G = H$ iff $G + -H = \emptyset$ is a P-PSN

$G + -H \text{ tree?}$

$H = *$

So $-H = *$

So $G + -H = G - * = G *$

\[ G * \]

\[ * \]

\[ G \]

\[ *++ \]

\[ = \emptyset \]

\[ G \]

\[ \bar{\text{Notice:}} \]

One player (R) has option to $\emptyset$,

So $G * \not\text{ not a P-PSN}$

So $G + -H \neq \emptyset$

So $G \neq H$ 😊
**SOME NAMES**

1

-1

* -1

↓

**TREE**

1

**NOT'N**

\{1\}

\{0, 3\}

\{0\}

\{0, 0\}

\{0, x\}

\{x, 0\}

1+ MEANS 1 + *

1+ TREE

* 1 1

\[ G = 1 \times = \{1, x | 1\} \]

**NOTICE** LEFT WOULD ALWAYS PREFER OPTION 1 INSTEAD OF " " 1

SO WE SUSPECT \[ G = \{1 | 1\} \]

EXERCISE: PROVE \[ G = H \]

i.e. PROVE \[ G + -H = 0 \]
**EXERCISE**

Find two unequal games in the same outcome class.

**Notice:** Any two R-PSNS are both $=0$ so EQUAL.

- What about L-PSNS?
- Are these equal?

$G \rightarrow H$

$H = G$ iff $H + -G = \emptyset$

\[ -G = -1 \]

**Notice:**
R has 0 option, so $H + -G$ not a P-PSN
So $H + -G \neq \emptyset$
so $H \neq G$
RECALL "= 0" MEANS "is a P-PSN."

EXTEND THIS TO \( <, >, \parallel \) INCOMPARABLE OR FUZZY.

"< 0" IS A R-PSN

"> 0" IS A L-PSN

"\parallel 0" IS A N-PSN

Similarly

\[ \begin{align*}
G < H & \iff G + H < 0 \\
G > H & \iff G > H > 0 \\
G \parallel H & \iff G \parallel H \parallel 0
\end{align*} \]

\(<, >, \parallel = \text{ALL TRANSITIVE} \)

E.g. \( A < B \) AND \( B < C \)

THEN \( A < C \)

E.g. \( A \leq B \) AND \( B \leq C \)

THEN \( A \leq C \)

\( \parallel \) NOT NEC. TRANSITIVE.

EXERCISE FIND \( A, B, C \) S.T. \( A \parallel B, B \parallel C \) BUT \( A \parallel C \).

EXERCISE SHOW \( \parallel \) COMMUTATIVE.
# Pile Nim Game

<table>
<thead>
<tr>
<th># Stones</th>
<th>Tree</th>
<th>Game Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>* 0</td>
<td>X 2</td>
</tr>
<tr>
<td>3</td>
<td>0 * 2 2 * 1 0</td>
<td>X 3</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>X^n</td>
</tr>
</tbody>
</table>

**WARNING!**

* $\frac{n}{2}$ is above number $n$.
* $\frac{n}{2}$ is game $\frac{n}{2} + \ast$

E.G.

* $1 = \ast$

$1\ast = 1 + \ast$

* $2 = \{ \varnothing, \ast \mid \varnothing, \ast \}$

$2\ast = 2 + \ast$
DOES $ \times 2 = \times \ ?$

$\times 2 = \times \ IFF \ \times 2 - \times = 0$
$IFF \ \times 2 + \times = 0$

SINCE $\times = \times$

$\times 2 + \times$ TREE

$0 + \times$ $\times + \times$ $\times 2$

$\Delta$

L HAS 0 OPTION, SO $\times 2 + \times \neq 0$

$\therefore \times 2 \neq \times$

😊
**VALUE**

Dom. $H$ \[ \xrightarrow{\text{GAME 1}} \]

Dom $G$

L would never really prefer this option.

**This game $H$ is L-posn.**

Does it have some "value"?

**Notice** \[ H + H - G = \emptyset \]

So \[ \text{val}(H) + \text{val}(H) - 1 = \emptyset \]

So ... \[ \text{val}(H) = \frac{1}{2} \]?

This would make sense.

In fact, we call \[ \frac{1}{2} \]!!

**Exercise:**
Prove $H + H - G$ is P-posn.