

3 1 2
-1 0 4

OBS AGAINST ANY MIXED STRAT,
 \exists BEST COUNTER MIXED-STRAT
 THAT IS PURE

SOLVE FOR R $\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$

$1-\alpha$ $\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$

C'S BEST RESPONSE
 PURE COL1 PAY 2α
 PURE COL2 PAY $\alpha + 3 - 3\alpha$

C CHOOSES $\min\{2\alpha, 3 - 2\alpha\}$ R ^{GUARANTEED} ~~CHOOSES~~ $\leq 2\alpha, \leq 3 - 2\alpha$

FOR 2x2 NON-DOM GAME, SOLVE
 MAXIMIZE Z ST.

$$\left. \begin{aligned} Z &\leq 2\alpha \\ Z &\leq 3 - 2\alpha \\ 0 &\leq \alpha \leq 1 \end{aligned} \right\} \Rightarrow \alpha = \frac{3}{4}$$

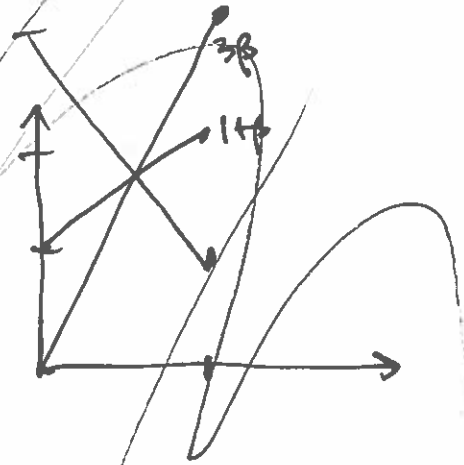
~~$2\alpha = 3 - 2\alpha$
 $4\alpha = 3$
 $\alpha = \frac{3}{4}$~~

PAYOFF $2 \cdot \frac{3}{4} = \frac{3}{2}$

$\frac{1}{4}$ $\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$
 $\frac{3}{4}$

FUN FACT ANY $2 \times n \geq 2$ GAME REDUCES TO 2x2 GAME
 BY DOMINATION ~~?~~ $\begin{matrix} 1 \\ - \\ 1 \end{matrix}$

$\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$
 $\begin{matrix} 1 & 2 & 4 \\ 4 & 3 & 2 \end{matrix} \Rightarrow \begin{matrix} 4\beta & 0 & 1 & 3 \\ \beta & 3 & 2 & 1 \end{matrix}$
 $\begin{matrix} 1-3\alpha & 3-\alpha & 2+2\alpha \\ 3\beta & 1+\beta & 3-2\alpha \end{matrix}$



$$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \begin{bmatrix} -2 & 3 & 1 \\ 0 & -3 & 4 \\ 1 & 2 & -1 \end{bmatrix}$$

MAX z ^{EXP. PAYOFF}

S.T. $z \leq -2x_1 + x_3$

$z \leq 3x_1 - 3x_2 + 2x_3$

$z \leq x_1 + 4x_2 - x_3$

FIND
R-MINIMAX
STRAT

SOLVE 2P MATRIX GAME
USING LINEAR PROGRAM

$x_1 + x_2 + x_3 = 1$

$0 \leq x_1, x_2, x_3 \leq 1$

→ SAGEMATH:

$x_1 \quad 0.045 \quad = \quad 5/110 = \frac{2}{44}$

$x_2 \quad 0.2954 \quad = \quad 13/44$

$x_3 \quad 0.6590 \quad = \quad 29/44$

$z = \frac{25}{44}$

CHECK YOUR ANSWER: ① ~~$\frac{2}{44}$~~ ~~$\frac{13}{44}$~~ ~~$\frac{29}{44}$~~

2) FIND C-MINIMAX STRAT
(EXERCISE)

$-2\left(\frac{2}{44}\right) + \frac{29}{44} = \frac{25}{44}$

$3\left(\frac{2}{44}\right) + -3\left(\frac{13}{44}\right) + 2\left(\frac{29}{44}\right) = \frac{25}{44}$

$\frac{2}{44} + 4\left(\frac{13}{44}\right) - \frac{29}{44} = \frac{25}{44}$

CONCLUSION: VON NEUMANN EQUIL. .. FOR $\begin{bmatrix} -2 & 3 & 1 \\ 0 & -3 & 4 \\ 1 & 2 & -1 \end{bmatrix}$?

• R-STRAT $\left(\frac{2}{44}, \frac{13}{44}, \frac{29}{44}\right)$

• C-STRAT $\left(\frac{15}{44}, \frac{13}{44}, \frac{16}{44}\right)$

• R-EXP-PAYOFF $\frac{25}{44}$



x_3