1. **Claim.** For any game $G$, $G + 0 = G$.  

   Complete the following proof by induction. $\phi$ is the empty set.

   For any games $G = \{G^L \mid G^R\}$ and $H = \{H^L \mid H^R\}$ with $G^L$ the set of left options of $G$, and $G^R, H^L, H^R$ defined similarly, the sum $G + H$ is defined as $\{G^L + H, G + H^L \mid G^R + H, G + H^R\}$, where $G^L + H$ is the set, for all $X$ in $G^L$, of all games ________________ . $G + H^L, G^R + H, G + H^R$ are defined similarly. ‘,’ represents set union. 0 is the game $\{|\}$, i.e. $\{0^L \mid 0^R\}$ where $0^L = 0^R = (\text{circle one}) \phi \quad \{0\} \quad \{1\} \quad \{|\}$.

   By definition, $G + 0 = \{G^L + 0, G + 0^L \mid G^R + 0, G + 0^R\}$.  

   (1) But $G^L + 0 = G^L$ because 

   ____________________________________________________________________________ .

   Similarly, $G^R + 0 = G^R$. Also, $0^L$ is empty because ____________________________________________________________________________, so $G + 0^L$ is empty because $G + 0^L$ is defined as ____________________________________________________________________________

   and ____________________________________________________________________________. 

   Similarly, the set $G + 0^R$ is empty. So the set $G^L + 0, G + 0^L$ equals the set $G^L, \phi$ which equals the set $G^L$.

   Similarly, $G^R + 0, G + 0^R$ equals $G^R$. So, from (1) we have $G + 0 = \{G^L \mid G^R\} = G$.

2. Let $A, B, C, D$ be the domineering games below. In each case, give the outcome class. Justify briefly.

   $A$ _____      $B$ _____       $C$ _____   $D$ _____

   $A + B$ _____       $A + C$ _____       $C - B$ _____       $C + D$ _____

   ![Images of games A, B, C, D]