1. Circle all of the combinatorial game conditions that apply to Hex.

(a) usual terminating condition (b) alternate turn (c) deterministic (d) 2-player (e) finite.

In Go, a player’s net score is their stones-plus-territory minus the opponent’s stones-plus-territory. (i) Black has 25 stones plus points territory, White has 24 stones plus points territory, so Black’s net score is . (ii) From the diagram, assume White makes one move and then both players pass. A best move is since it increases her net score by . (iii) From the diagram, assume Black makes an unlimited number of moves and White passes after each. Then the maximum score (stones plus territory) she can reach is . Explain briefly.

2. In Go, a player’s net score is their stones-plus-territory minus the opponent’s stones-plus-territory. (i) Black has 25 stones plus points territory, White has 24 stones plus points territory, so Black’s net score is . (ii) From the diagram, assume White makes one move and then both players pass. A best move is since it increases her net score by . (iii) From the diagram, assume Black makes an unlimited number of moves and White passes after each. Then the maximum score (stones plus territory) she can reach is . Explain briefly.

3. Find a winning 1st-player move for nim(45,35,21,16) or explain why there are none. Show your work.

1 0 1 1 0 1 45  Remove ______ stones from the pile with ______ stones.
1 0 0 0 1 1 35
1 0 1 0 1 21
1 0 0 0 0 16

4. Complete the following proof.

Claim. For all positive integers $a$, nim($a$, $a$) is a 1st-player loss.

Proof. Let Left be the first player and Right the second. It suffices for Right to always mirror Left’s previous move on the other pile. If Left’s first move removes $b$ stones, this leaves nim($a - b$, $a$), so Right removes ______ stones from the other pile, leaving nim( ______ ). If $b = a$ then we are done because

If $b < a$ then we are done because