1. (i) Give a short proof that each white cell on the 3x3 board below left is a losing opening move for 3x3 Hex. Hint: use a mustplay argument. (ii) On the empty 3x3 board, assume that Black plays first in the centre. Prove that Black then has a winning virtual connection. (iii) On the empty 3x3 board, assume that Black plays first in one of the obtuse corners. From this position with White to move, give a complete winning Black strategy, and prove that it wins. (In your proof, use the word ”bridge” to mean a virtual connection that uses exactly 2 empty cells.)

2. For 4x4 Hex, give a winning 1st-player strategy with first move the centermost cell on the main diagonal (obtuse corner to obtuse corner). Hint: use virtual connections.

3. Repeat the previous question for first move an obtuse corner. Hint: to begin, show that on Black’s second move, Black has a mustplay region with only 5 cells.

4. (i) For the position above right, explain why the cell set \{b3, c3\} is Black-captured. (ii) Combine the capture-strategy from (i) with a mirror strategy, and give a short proof that on 3x3 Hex, the obtuse corner is a winning first move.

5. (i) Two different pairings give a 432-connection: one is below left, give another. (ii) Below middle, explain why \{a4, b4\} is black-captured. After black-coloring \{a4, b4\}, explain why \{c4, b4\} is black-captured. (iii) Below right, explain why the pairing is a winning black virtual connection. Give a simpler winning pairing.

6. (i) Let \(X\) be a Hex position with a black stone at a cell \(c\). Assume White wins \(X\) when playing first. Let \(X^-\) be the position obtained by removing the black stone at \(c\). Prove that White wins \(X^-\) when playing first. (ii) Recall: a P-position is one where Left (Black) wins when Right (White) starts and Right wins when Left starts. Prove or disprove: there is some Hex P-position.

7. For these Jens Lindhard puzzles (i) find any dead cells (ii) find any captured cells (iii) use mustplay analysis to find a winning Black move. (iv) repeat (iii) for White. One answer on next page.
(i) A4 is dead, all other empty cells are live.

(ii) \{ A1, B1 \} is black-captured. This is the only captured set.

(iii) After filling black-captured cells A1, B1, obvious that B5 is winning move for black (after that move, Black has virtual connection using cells \{ B2, C2, C1, D1 \}). There are many other winning Black moves.

(iv) This is harder. First, simplify the position: fill in dead and captured cells (below left). Next, Black has a winning semi-connection by playing at B5, so White’s mustplay is \{ B2, B5, C1, C2, D1 \} (below right).

Let’s examine these moves. 1.W[B2] is killed by 2.B[C2], as shown at left, so if White can win, it has some other winning move. (As shown at right, 1.W[B2] loses, as move 2 forces move 3, and move 4 forces move 5.) In similar fashion, 1.W[C1] and 1.W[D1] are each also killed by 2.B[C2].

What about 1.W[B5]? By mustplay analysis, Black must reply at one of the five dots (below left). Four of these moves lose for Black (exercise: confirm this), but 2.B[B4] wins: the principal variation starts below right.

Finally, what about 1.W[C2]? After this move, White has a winning semi-connection shown below, so if 1.W[C2] does not win, Black has a winning reply at one of these 7 cells. But each of these moves loses for Black. (Exercise: show this.) So 1.W[C2] wins, and is the only winning move.

Two exercises. (left) Black to play and win. (right) White to play and win.