1. (i) Give a short proof that each white cell on the 3x3 board below left is a losing opening move for 3x3 Hex. Hint: use a mustplay argument. (ii) On the empty 3x3 board, assume that Black plays first in the centre. Prove that Black then has a winning virtual connection. (iii) On the empty 3x3 board, assume that Black plays first in one of the obtuse corners. From this position with White to move, give a complete winning Black strategy, and prove that it wins. (In your proof, use the word ”bridge” to mean a virtual connection that uses exactly 2 empty cells.)

2. For 4x4 Hex, give a winning 1st-player strategy with first move the centermost cell on the main diagonal (obtuse corner to obtuse corner). Hint: use virtual connections.

3. Repeat the previous question for first move an obtuse corner. Hint: to begin, show that on Black’s second move, Black has a mustplay region with only 5 cells.

4. (i) For the position above right, explain why the cell set \{b3, c3\} is Black-captured. (ii) Combine the capture-strategy from (i) with a mirror strategy, and give a short proof that on 3x3 Hex, the obtuse corner is a winning first move.

5. (i) Two different pairings give a 432-connection: one is below left, give another. (ii) Below middle, explain why \{a4, b4\} is black-captured. After black-coloring \{a4, b4\}, explain why \{c4, b4\} is black-captured. (iii) Below right, explain why the pairing is a winning black virtual connection. Give a simpler winning pairing.

6. (i) Let \(X\) be a Hex position with a black stone at a cell \(c\). Assume White wins \(X\) when playing first. Let \(X^-\) be the position obtained by removing the black stone at \(c\). Prove that White wins \(X^-\) when playing first. (ii) Recall: a P-position is one where Left (Black) wins when Right (White) starts and Right wins when Left starts. Prove or disprove: there is some Hex P-position.

7. For these positions (i) find any dead cells (ii) find any captured cells (iii) use mustplay analysis to find a winning Black move. (iv) repeat (iii) for White.